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Computationally efficient nonlinear Chebyshev models using common-pole parallel filters with the application to loudspeaker modeling

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ABSTRACT

Many audio systems show some form of nonlinear behavior that has to be taken into account in modeling. For this, often a black-box model is identified, coming from the generality and simplicity of the approach. One such model is the polynomial Hammerstein model, which uses parallel branches that have a polynomial-type nonlinearity and a linear filter in series. For example, Chebyshev models use Chebyshev polynomials as nonlinear functions, making model identification a very straightforward procedure by logarithmic swept-sine measurements. This paper proposes a highly efficient implementation of Chebyshev models by using fixed-pole parallel filters for the linear filtering part. The efficiency comes both from using common-pole modeling and from applying a warped filter design that takes into account the frequency resolution of hearing. Due to its efficiency, the proposed model is particularly well suited for the real-time digital simulation of weakly nonlinear devices, such as amplifiers, nonlinear effects, or tube guitar amplifiers.

1. INTRODUCTION

Many audio systems (loudspeakers, tube amplifiers, audio effects) exhibit some form of nonlinear behavior, which has to be taken into account in modeling. One standard approach is to use the a priori knowledge available about the physical system, and construct a model whose building blocks correspond to the various parts of the system. This usually results in precise nonlinear models, like the loudspeaker models of [1, 2] or models of tube amplifiers [3]. The drawback of the physics-based approach is the loss of generality. The other approach, taken also in this paper, is black-box modeling, where no a priori knowledge is needed. The advantage of this methodology is its generality and simpler parameter estimation.

The classical approach to nonlinear black-box modeling is the use of Volterra series. However, the estimation and implementation of the multidimensional kernels for higher-order models is usually too complicated for practical implementations. Therefore, often some simplifications are made. For example, the generalized polynomial Hammerstein model (sometimes also termed MISO model) uses only the diagonal elements of the Volterra model, resulting in a simpler model structure with one-dimensional convolutions. The model parameters are conveniently estimated using a specially synchronized logarithmic sweep signal [4, 5, 6]. (A review of related literature in nonlinear modeling is also given in [6]). Accordingly, these models simulate the harmonic distortion behavior of the system perfectly at the input level where the measurement was made, while other types of distortions (e.g., intermodulation) are also generated, but not directly under control. So far, the method has been applied to modeling loudspeakers [4, 5] and audio effects, such as a limiter [6] or an overdrive pedal [7]. A variant of the technique has been presented in [7], where the power-law nonlinearities are interchanged with Chebyshev polynomials. As a result, the parameter estimation is simplified, because now the impulse responses of the linear filters are obtained directly from the nonlinear impulse responses measured by the synchronized swept-sine technique. This paper proposes an efficient implementation by using common-pole parallel filters for the linear filtering part.

2. CHEBYSHEV MODELS

The Chebyshev model [7] consists of a set of parallel branches, where each branch has a Chebyshev polynomial $T_r(x)$ and a linear filter $H_r(\vartheta)$ in series, as displayed in Fig. 1. A low-pass filter is also added to the input to eliminate aliasing (not displayed in Fig. 1).

The Chebyshev polynomials are defined as

$$T_r(\cos(\theta)) = \cos(n\theta), \quad (1)$$

meaning that if a sinusoidal function with unitary amplitude and angular frequency ϑ is lead to the r-th order Chebyshev polynomial, its output will be a sinusoidal function with the frequency $r\vartheta$. The Chebyshev polynomials can be computed by the recursive formulas

$$T_r(x) = 2xT_{r-1}(x) - T_{r-2}(x) \quad (2)$$

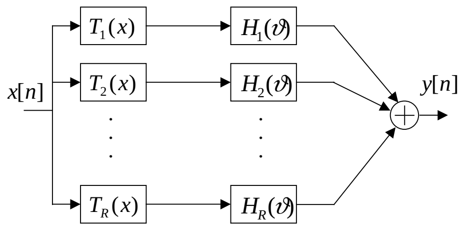

Fig. 1: The Chebyshev nonlinear model.

with $T_1(x) = 1$ and $T_2(x) = x$.

As for parameter estimation, the system is excited by a synchronized logarithmic sweep signal, and the "nonlinear impulse responses" are computed by deconvolving the system output with the synchronized sweep [6]. In the deconvolved signal the linear impulse response $h_1(t)$ and impulse responses corresponding to the harmonic distortion products $h_2(t) \dots h_R(t)$ appear at distinct time instants, thus, they can be easily separated. In Chebyshev models the filters $H_1(\vartheta) \dots H_R(\vartheta)$ directly implement these measured impulse responses $h_1(t) \dots h_R(t)$. The filters $H_r(\vartheta)$ are most straightforwardly implemented as FIR filters. However, in Sec. 4 we will see that a much more efficient implementation is also possible.

3. FIXED-POLE PARALLEL FILTERS

For audio, specialized filter design methods have been developed that take into account the frequency resolution of hearing, including warped [8], Kautz- [9] and parallel filters [10]. In parallel filters, the poles of the second-order sections are fixed prior to filter design, and only the numerators are free parameters for optimization. As a result, the frequency resolution is controlled by the pole frequencies. For example, logarithmic frequency resolution can be achieved by placing the poles on a logarithmic frequency scale [11]. Thus, parallel filters are particularly well suited for audio applications because the allocation of frequency resolution can fit the resolution of human hearing. Comparison to IIR, warped FIR, warped IIR designs have shown that the parallel results in better accuracy at the same filter order [10, 11, 12]. Compared to Kautz filters, the same transfer function is obtained for the same filter order, but the parallel filter requires 33% fewer arithmetic operations [11].

The parallel filter consists in a parallel set of second-order sections and an optional FIR filter path:

$$H(z^{-1}) = \sum_{k=1}^{K} \frac{b_{k,0} + b_{k,1} z^{-1}}{1 + a_{k,1} z^{-1} + a_{k,2} z^{-2}} + \sum_{m=0}^{M} c_m z^{-m} \quad (3)$$

where K is the number of second-order sections and M is the order of the FIR filter. Note that in this study the FIR part is not utilized.

The first step of parallel filter design is the choice of pole frequencies. The simplest option is to set them according to a logarithmic frequency scale, resulting in a logarithmic frequency resolution [11]. Another options include pole positioning based on warped IIR filters [13], the recently developed smoothed multi-band pole positioning method [14] and pole positioning based on custom warping [15].

Once the denominator parameters $a_{k,1}$, $a_{k,2}$ are determined by the poles, the transfer function $H(z^{-1})$ becomes linear in its free parameters $b_{k,0}$, $b_{k,1}$, and c_m. These parameters $b_{k,0}$, $b_{k,1}$, and c_m can be estimated both in the time-domain [10] or in the frequency-domain [12] by the least-squares (LS) normal equations in a closed form.

4. EFFICIENT CHEBYSHEV MODELS USING COMMON-POLE PARALLEL FILTERS

4.1. Model structure

Already using separate parallel filters instead of traditional FIR or IIR filters results in a lower computational complexity for the same sound quality, because parallel filters can take into account the frequency resolution of hearing. The performance benefit is further increased by the proposed common-pole model structure. It can be seen in Fig. 1 that the linear filtering part is basically a multi-input single-output (MISO) system. By choosing the same frequency resolution for the different branches, the pole positions will be the same for all the parallel filters. As a result, the branches can share the same denominators, resulting in a structure where the outputs of the Chebyshev functions are filtered by the first-order FIR filters (numerators) $B_{k,r}(\vartheta)$, summed, and led to the common second-order allpole filters (the common denominators) $A_k(\vartheta)$. Finally, the contributions of the various second-order sections are summed. This is displayed in Fig. 2. In a straightforward (not common-pole) implementation, for an Rth order model with Nth order parallel filters, the total required filter order is NR. In the proposed common-pole implementation the computational complexity is decreased to $N(R/2+0.5)$, which is around the half of the complexity of the straightforward approach.

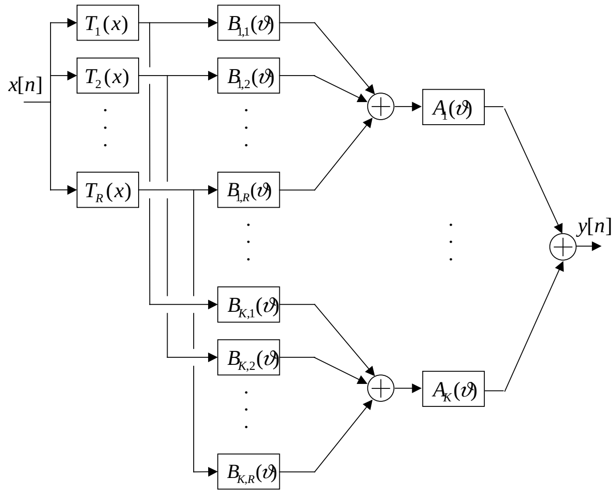

Fig. 2: Structure of the Chebyshev model with common-pole parallel second-order filters.

4.2. Parameter estimation

Once the $h_r(t)$ signals (or their sampled versions $h_r[n]$) have been obtained by the synchronized swept sine technique, the task is to estimate the parameters of $B_{k,r}(\vartheta)$ and $A_k(\vartheta)$ so that the filter impulse responses best match the measured responses. The first step of filter design is the determination of the poles, which should be the same for all the filters $H_r(\vartheta)$. The simplest option is to set the poles to a logarithmic frequency scale, resulting in a strictly logarithmic frequency resolution. However, when modeling relatively low order systems (like the loudspeaker example in Sec. 5), better results are achieved if the pole frequencies are computed based on the measured responses. Since we should still take into account the resolution of the human auditory system, this is done in the warped domain.

The first step of the procedure is warping the measured impulse responses $h_r[n]$ by an allpass chain [8], giving the warped signals $\tilde{h}_r[n]$. Then, similarly to MIMO admittance modeling [16], a common-pole autoregressive filter is identified to these responses. The regression error for the rth warped impulse response is given as

$$E_r = \sum_{n=L}^{N} \left(\tilde{h}_r[n] + \sum_{l=1}^{L} a_m \tilde{h}_r[n-l] \right)^2, \qquad (4)$$

where L is the order of the denominator, and N is the length of the warped impulse response $\tilde{h}_r[n]$. Note that the denominator coefficients a_m are the same for all r in Eq. (4) and the task is to find this common set of a_m coefficients such that the total error

$$e = \sum_{r=1}^{R} W_r E_r \qquad (5)$$

is minimal, where W_r is the weight given to the separate impulse responses. This is a linear least-squares problem that is solved by the normal equations in a closed form. Then, the roots $\tilde{p}_k$ of the denominator are found and "dewarped" by the expression

$$p_k = \frac{\tilde{p}_k + \lambda}{1 + \lambda \tilde{p}_k}, \qquad (6)$$

giving the common set of poles. (Note that instead of "traditional" warping, the custom warping method of [15] – published elsewhere in these proceedings – could also be used. Here traditional warping and time-domain design was used for the sake of simplicity.)

Once the common poles are obtained, the weights of the parallel filter (i.e., the numerators) are obtained by the LS equations for each filter $H_r(\vartheta)$ exactly in the same way as for normal (not common-pole) parallel filters [10].

5. LOUDSPEAKER MODELING EXAMPLE

The method is demonstrated by a loudspeaker modeling example. A three-inch loudspeaker (Hivi B3N) enclosed in a a box with 0.7 liter net volume was measured by the synchronized swept-sine technique with an input level of 10 V_{pp} and a sampling rate of 48 kHz . Its linear impulse response $h_1[n]$ and the nonlinear impulse responses $h_2[n] \ldots h_5[n]$ were computed by deconvolution. The impulse responses were windowed to 1000 tap by a half hanning window. The Fourier transforms $H_{\mathrm{t},r}(\vartheta)$ of the measured impulse responses $h_r[n]$ are displayed in Fig. 3 thin line for the linear part (a) and for the nonlinear responses of order 2 to 5 in (b)–(e).

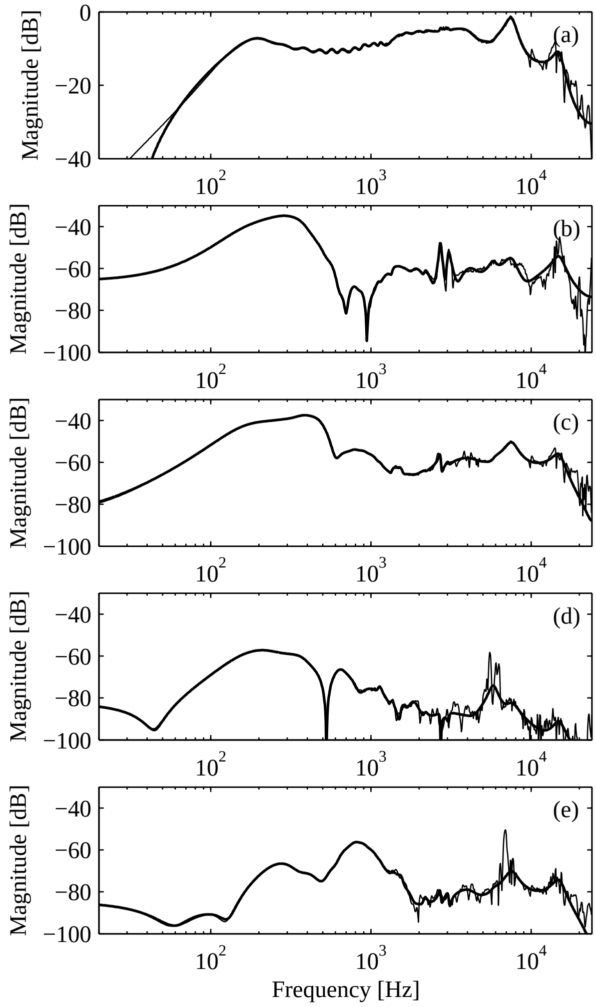

Fig. 3: The measured transfer functions (thin line) and modeled responses using common-pole 50th-order parallel filters (thick line) for the linear part (a) and for the nonlinear transfer functions of order 2 to 5 (b)–(e).

These $H_{\mathrm{t},r}(\vartheta)$ form the target specification for parallel filter design.

Next, the common pole-set is obtained by the warped common-pole autoregressive method outlined in Sec. 4.2 with $\lambda = 0.85$ and filter order 50, and the common poles are used as the denominators of the parallel filters. The numerator coefficients (weights) are obtained by minimizing the mean-squared error between the impulse responses of the parallel filters and the measured responses

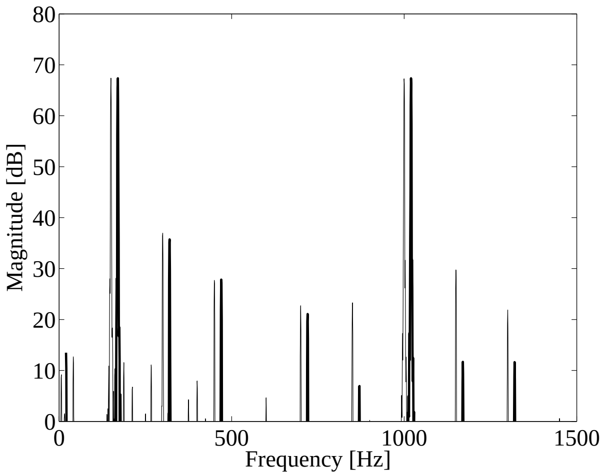

Fig. 4: Measured (thin line) and modeled (thick line) output spectrum of a small loudspeaker to a two-tone input with f_1 =150 Hz and f_2 =1 kHz. The amplitudes of the sinusoids are 5 $\mathrm{V_{pp}}$. The modeled output spectrum is shifted by 20 Hz for clarity.

$h_r[n]$. The frequency responses of the parallel filters $H_r(\vartheta)$ are displayed by thick lines in Fig. 3 (a)–(e). It can be seen that the 50th-order common-pole filters model the measured responses quite precisely. Note that the computational complexity of the filtering is equivalent to an IIR filter of order 150 due to the benefits of common-pole implementation (a straightforward implementation would result in an order of 250).

The model is evaluated by a two-tone input signal with f_1 =150 Hz and f_2 =1 kHz and sinusoidal amplitudes of 5 $\mathrm{V_{pp}}$, displayed in Fig. 4. From theory we expect that the harmonic distortion of the loudspeaker is precisely modeled by the Chebyshev model at the input amplitude with which the sweep measurement was made. Here the input amplitude is half compared to the measurement, but the harmonic distortion products (peaks at 300, 450 and 750 Hz) are still reasonably well represented. On the other hand, the intermodulation products are not correctly modeled. This is inevitable for all the models that use a swept-sine for parameter estimation, since the swept-sine measurement conveys information only about the linear transfer function and harmonic distortion. In other words, this is the price to pay for the simplicity and generality of the model. Whether modeling the linear part and the harmonic distortion products is sufficient for a perceptually plausible model of a nonlinear audio device, can only be determined by listening tests, which is out of the scope of this paper. The message of the paper is that if such a model is sufficient, then it can be very efficiently implemented by common-pole parallel filters. Nevertheless, a listening test in [4] shows that a simplified Volterra model (which is equivalent to the Chebyshev model used here) can simulate the nonlinear behavior of small loudspeakers reasonably well.

6. CONCLUSION AND FUTURE RESEARCH

Polynomial Hammerstein models are simplified Volterra models using only the diagonal part of the Volterra kernels. They consist of several parallel branches where each branch has a static nonlinearity implemented by a polynomial and a linear filtering part in series. The favorable property of these models is that their parameters can be conveniently estimated by logarithmic sweep measurements. In addition to the linear transfer function, these models simulate the harmonic distortion behavior of nonlinear systems, thus, they can be used for modeling weakly nonlinear audio devices, such as loudspeakers, tube amplifiers or guitar effects. A special variant of polynomial Hammerstein models is the Chebyshev model, where the nonlinearities are implemented by Chebyshev polynomials. This paper has proposed an efficient implementation for the Chebyshev nonlinear model by using a common-pole parallel filter for the linear part. The efficiency comes both from the fact of common-pole modeling and by using a frequency-warped filter design that takes into account the frequency resolution of hearing. The same approach can also be used for other nonlinear models with similar model structure, such as those using power-law functions [4, 5, 6]. Due to its high computational efficiency, the model is well suited for the real-time digital simulation of nonlinear audio devices.

Future research may include the efficient implementation of polynomial Wiener models where the signal flow of Fig. 2 is reversed. An efficient polynomial Wiener-Hammerstein model could be obtained by nesting nonlinear functions (e.g., Chebyshev polynomials) between a single-input multiple-output and a multiple-input single output parallel filter. While parameter estimation would be much more complicated, such a model would go one step beyond the “harmonic distortion modeling” paradigm of polynomial Hammerstein models, thus, would allow the more realistic black-box modeling of audio devices.

7. ACKNOWLEDGEMENT

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