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Expressive physical modeling of keyboard instruments: from theory to implementation

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ABSTRACT

Physics-based algorithms for sound synthesis have been extensively studied in the past decades. Nevertheless, their use in commercial synthesizers is still limited due to the difficulty in achieving realistic and easily controllable sounds with current technology. In this Engineering Brief we present an overview of the models used in Physis Piano, a digital piano recently introduced in the market with dedicated physics-based algorithms for acoustic piano, electric pianos (e.g. Rhodes, Wurlitzer and Clavinet) and chromatic percussions (e.g. Vibraphone, Marimba, Xylophone). The synthesis algorithms, which are based on standard techniques such as Modal Synthesis and Digital Waveguides, have been highly customized in order to faithfully reproduce the sound features of the original instruments and are easily controllable by a set of meaningful, user-friendly parameters.

1. INTRODUCTION

Physics-based synthesis algorithms try to imitate the sound of acoustic instruments at the *source* level instead that at the *signal* level. The mechanical components of the original instrument are modeled and efficient numerical schemes are derived in order to synthesize the sounds in real-time.

Compared to traditional techniques, such as sampling or wavetable synthesis, physical models are usually more expressive in terms of dynamic playing. In the case of keyboard instruments, this means better response to key velocity, restrikes and interaction phenomena such as sympathetic string resonance. Moreover, they are very flexible in terms of sound editing, since they are parameterized by a meaningful set of parameters with physical meaning (e.g. mass, hardness) that can be changed by the user to obtain a particular sound.

In this Engineering Brief we review the technology behind a recently introduced digital piano that employs physics-based algorithms for most of its sound generation [4]. For each class of instruments

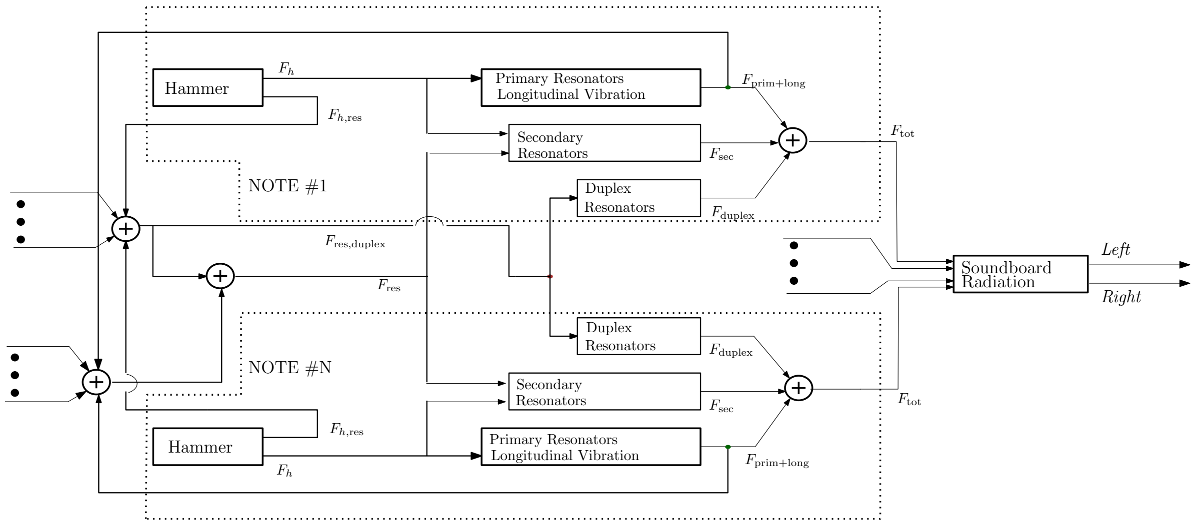

Fig. 1: Synthesis architecture of the acoustic piano model, showing the interactions between the notes and the soundboard model. Adapted from [7].

(acoustic pianos, electric pianos, chromatic percussions) the theory behind the mathematical models is shortly reviewed and then the necessary simplifications of the engineering process are discussed.

2. ACOUSTIC PIANO

In order to derive a complete model for the synthesis of a complex instrument such as the piano, it is typical to follow a *block-based* approach, where various signal-processing blocks are defined starting from the mechanical components of the acoustic instrument. One of the most often used decomposition is made up by the hammer (*exciter*), the string (*resonator*) and the instrument body (*radiator*) blocks [3].

A critical choice in defining a model resides in the discretization technique used for the numerical solution of the partial differential equation which describes the motion of the string. Most of the proposed algorithms in the last years are based on finite differences or digital waveguides [5]. The academic research behind the model presented in this Engineering Brief led instead to the definition of a complete piano model based on *Modal Synthesis* [1]. While more computationally expensive compared to digital waveguides, this technique was preferred because it is highly flexible for the calibration and permits a more accurate modeling of the nonlinear longitudinal string vibrations which characterize the timbre in the low register. In addition, modal synthesis is particularly well suited to code parallelization.

In brief, the algorithm is based on the decomposition of the string displacement $y(x,t)$ into its orthogonal normal modes [3]

$$y(x,t) = \sum_{n=1}^{N} y_n(t) \sin\left(\frac{n\pi x}{L}\right), \qquad (1)$$

where $y_n(t)$ are the instantaneous amplitudes of the modes, or *partials*. Substituting Eq. (1) into the partial differential equation of the string motion results in an ordinary second-order differential equation for each partial, and thus the impulse response of the string becomes a sum of exponentially decaying sinusoidal functions.

After discretization, the input-output relation of the string block is realized as a parallel connection of N second-order all-pole resonators:

$$F_{\mathrm{string}}(z) \quad = \quad H_{\mathrm{string}}(z)\, F_{\mathrm{h}}(z)$$

$$H_{\mathrm{string}}(z) = \sum_{k=1}^{N} W_{\mathrm{in},k}\, H_{\mathrm{mode},k}(z)\, W_{\mathrm{out},k} \quad (2)$$
$$H_{\mathrm{mode},k} = \frac{b_{1,k} z^{-1}}{1 + a_{1,k} z^{-1} + a_{2,k} z^{-2}},$$

where $F_{\mathrm{string}}(z)$ is the transversal force at the bridge, $F_{\mathrm{h}}(z)$ is the force coming from the hammer and $H_{\mathrm{mode},k}(z)$ are the transfer functions of the normal modes. The conversion between the physical variables (i.e. force, displacement) and the modal variables is regulated by a set of input and output weights $W_{\mathrm{in},k}$, $W_{\mathrm{out},k}$.

The engineering process that led to the actual industrial product had to cope with several issues, such as limited computing power, stability under all conditions and ease of calibration. Most of the details are covered in the related international patent [7], while here we just summarize the overall synthesis architecture, depicted in Fig. 1.

The main difference between the model described in [1] and the final product resides in the hammer-string interaction. In the former prototype [1], the hammer was modeled as a pointless mass connected in series with a nonlinear spring, bidirectionally coupled with the string model. While relatively simple and quite accurate, this kind of model has several drawbacks when it comes to the ease of calibration. As it can be seen in Fig. 1, the actual hammer model has a *feedforward* connection to the string model, in a way similar to previous works in literature [6]. We designed a parametric synthesis algorithm which is able to qualitatively reproduce the signals of the former feedback version, but whose parameters are decoupled from the string parameters.

A similar transformation was applied for the synthesis of the spectral components coming from the *longitudinal vibration*, which in [1] were derived from a nonlinear intermodulation of the transversal partials and now are generated from another set of resonators whose parameters are precomputed from the physical laws but are then easily adjustable if needed. Among the other blocks, the *secondary resonators* is a slightly detuned set of resonators which has the double function of simulating the beatings of piano partials and the sympathetic resonances among the notes. The synthesis architecture is designed in a way that there is no feedback loop between the notes, as the signal coming from the primary string resonators is fed into the secondary resonators, summed with a filtered version of the hammer force signal used to give a broadband, impulse-like excitation to the string register, which in real pianos is particularly noticeable in the higher notes. Finally, the *duplex resonators* model the so called *duplex scale* [3], a portion of the strings above their speaking length giving a particular brilliance piano sound.

Sound synthesis parameters are organized in a hierarchical way. The user is presented with a short set of *Macro Parameters* (hammer, tuning, string type, resonance, size) which control a larger set of 15 *Micro Parameters* (e.g. hit position, string stiffness). These in turn can modify the internal parameters used by the sound designers, which are around 100 and in some cases can be controlled note-by-note. Finally, the last layer of transformation generates the approximately 300'000 synthesis micro-parameters which are used by the real-time engine.

3. ELECTRIC PIANOS

A different synthesis engine has been developed in the Physis Piano, able to model several families of electro-mechanical pianos. The electric piano models differ from the acoustic piano one in several regards: the actual implementation of the *exciter* and *resonator* blocks as well as the presence of a *pickup* block in stead of the *soundboard* one, which is neglected due to the low sound radiation produced by typical electric piano soundboards. In electric pianos the sound is instead captured by electromagnetic pickups. Indeed, vintage electric pianos are all electromechanical in their design, often accompanied by additional electronics to filter, amplify or enhance the pickup signal. In general their keyboard extension is lower or equal to that of the acoustic piano. Mechanical action designs are rather different:

- in pianos like the Rhodes, forks are struck by plastic hammers covered with a rubber tip. The forks are asymmetrical and, unlike a tuning fork, create several quickly decaying inharmonic modes in addition to the fundamental.
- In Wurlitzer and Pianet instruments, flat reeds are struck by hammers or *plucked* by adhesive rubber tips. The inharmonic modes are often negligible.

- In the Clavinet family, metal strings are struck by a rubber tip hammer (or *tangent*). Differently from metal beams and forks they do not produce inharmonic modes, but they may have slightly inharmonic overtones.

All the electric piano resonators are based on a modal synthesis engine, with the only exception of stringed ones, which are based on Digital Waveguide (DWG) synthesis [5]. The exciter is able to reproduce struck reeds, struck tuning-forks, plucked reeds and tangent-struck strings based on different hammer models derived from a general force-feedback hammer model, not unlike the ones seen in literature for piano hammer-string interaction [1]. The general block scheme for the electric pianos engine is shown in Figure 1, highlighting the feedback formulation of the hammer with the principal resonator and additional modal resonators added in parallel when needed.

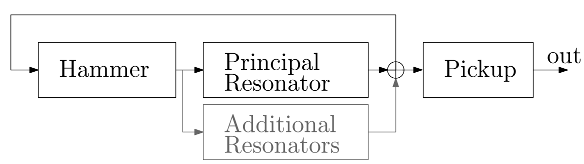

Fig. 1: Electric pianos synthesis engine architecture, showing the feedback hammer model, the resonators and the pickup.

The discretization of the hammer leads to a delay-free loop in the numerical scheme, which is solved using the K-method[2]. Given the relatively lower complexity of the electric pianos with respect to the acoustic piano, a complete decoupling of the parameters is not needed and the feedback hammer model is employed successfully. Normal modes in the electric piano are of lower number compared to the acoustic piano, yielding a lower computational cost.

The string resonator differs from the fork and reed resonators as its implementation is based on a waveguide structure rather than an all-pole filter. Features such as the tuning drop at release time, typical of the Clavinet, are more easily obtained with DWG modeling rather than other techniques. The modeling of the string-tangent interaction also allows to control the attack hardness and the release noise, the latter being particularly prominent in poorly maintained instruments.

All the vintage electric pianos are equipped with pickups. In fork and reed pianos these alter significantly the spectral content by adding overtones. Each has a distinct sound provided by its specific combination of mechanical components (physical components placement, fork or reed design) as well as the electromagnetic components (pickups). The reed and fork pickups are modeled as nonlinear components with memory.

Each instrument has its own set of physical *Macro Parameters* that are exposed to the user, which in turn control a large set of internal synthesis controls in pre-determined ranges specified by the sound designer.

4. CHROMATIC PERCUSSIONS

The mallet percussion instruments included in the synthesizer are vibraphone, marimba, xylophone and glockenspiel [3]. The sound generation mechanism of all of these instruments is based on a vibration of a bar hit by a mallet. Thus, the same basic model structure can cover them all. Modal synthesis is especially well suited for this kind of instruments, since typically only a few tens of partials have to be modeled, but they are inharmonically distributed. (The heavy inharmonicity would make modeling with digital waveguides impractical.)

The simplest of these instruments is the glockenspiel, or orchestra bells, which has rectangular steel bars, and is played with brass or plastic mallets [3]. The model of the bar is based on the closed-form solution of the partial differential equation describing the ideal bar [3]. Once the modal shapes are derived, the ordinary second-order differential equations for the modal amplitudes are obtained similarly to the case of the string, again resulting in an impulse response composed of exponentially decaying sinusoidal functions. Thherefore the vibration of the bar can be modeled in the same way as the string by using Eq. (3). The hammer pulse generated by the mallet strike is modeled by a parametric model similarly to the piano synthesizer.

Other percussion instruments, such as the xylophone, marimba, and vibraphone all comprise a bar with an undercut, as opposed to the rectangular bar of the glockenspiel. The purpose of the undercut is to tune the second and third harmonic of the bar so that it is harmonically related to the fundamental f_0 [3]. The second harmonic is typically tuned to $4f_0$ (sometimes $3f_0$), and the third harmonic is roughly around $10f_0$. While the undercut also changes the modal shapes and thus the harmonic amplitudes, the sonic difference is mainly coming from the change in modal frequencies. Therefore we have decided to apply the modal shapes of the ideal bar also for this case. The modal frequencies are determined according to the analysis of real instrument sounds. A further difference of these more sophisticated mallet percussion instruments compared to the glockenspiel is the presence of tubular resonators which are used to increase the radiation efficiency of the fundamental [3]. This results in higher amplitude and faster decay of the first harmonic compared to the sole bar, which can be easily taken into account by changing the parameters of the first mode, without the need of changing the model structure.

The characteristic timbre of the vibraphone is coming from a valve that can open or close the tubes beneath the bars. This valve is either turned by hand to set the decay time and amplitude of the fundamental to the musicians taste, or it is rotated by an electric motor resulting in a vibrating amplitude of the fundamental [3]. The sound of a real vibraphone with various angles of the valve has been analyzed and a model was fit to amplitude and decay time variations. As a result, the synthesis engine is continuously updating the decay time and amplitude of the first mode based on the valve angle, which is either fixed (set by the user) or changes according to the rotor speed parameter.

When analyzing the spectrum of vibraphone sounds, additional frequency components were observed which are not explained by the linear model of the bar. It has turned out that these components are actually at the sum- and difference-frequencies of normal (linear) harmonics, thus, they are generated by a nonlinear phenomenon, possibly due to the rubber supports of the bar. While the precise explanation is still unknown and left for future research, this does not prohibit us to construct a physically inspired model for this phenomenon. The model simply multiplies pairs of harmonics (e.g., the first and second harmonic) similarly to a ring modulator to produce the required sum and difference components.

5. CONCLUSION

This paper has presented the methodology behind a recently introduced digital piano synthesizer. Most of the physical models of the instrument, including acoustic and electric pianos and mallet percussion instruments, are based on modal synthesis. Modal synthesis simulates the motion of the string or bar as a parallel set of second-order digital filters, allowing great flexibility for parametric control. Besides describing the basic model structure, the paper has outlined the additional blocks needed to implement the special features of the various instruments.

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