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A Mathematical Approach to Derive Optimum Detector Configurations for UWB Radio Applications

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Abstract—A systematic approach for the derivation of UWB detectors cannot be found in the literature. The detectors published up to now have been developed by inspection, using a heuristic approach. However, that solution prevents the optimization of UWB detector performance. A new approach is proposed here for the derivation of optimum waveform detectors that is valid for each type of carriers. To illustrate the application of the new method, the derivation of two detectors are shown.

1. Introduction

Due to their low power spectral density, the Ultra Wide-Band (UWB) carriers allow the reuse of the already occupied frequency bands. The typical bandwidth of a UWB signal is 2 GHz, the carriers may be either fixed waveforms as in UWB impulse radio or continuously varying wideband chaotic signals.

In conventional telecommunications fixed, mostly sinusoidal waveforms are used as carriers [1]. UWB communications is completely different, the UWB regulations give only the rule under which the frequency band may be accessed and say nothing about the carrier and modulation scheme [2]. Even varying waveforms, where the carrier varies continuously, may be used when the same symbol is transmitted repeatedly.

The conventional communication theory does not cover the case of UWB and continuously varying carriers. As a result, the UWB detectors are developed using a heuristic approach, where the detection algorithms cannot be optimized and matched to the channel conditions. This paper proposes a new systematic approach for the derivation of detector configurations. The method is valid for each class of communications, the cases of sinusoidal, UWB and continuously varying chaotic carriers are each covered.

First the mathematical model of waveform communications that has been published as Fourier analyzer concept earlier [3] is surveyed. Then the *a priori* information, exploited by the detector to separate the useful signal from channel disturbances, is quantified. Finally, a new systematic approach is proposed for the derivation of different detector configurations.

2. Model of UWB Waveform Communications

The transmitter encodes the information to be transmitted into analog waveforms of finite duration. The receiver observes the received corrupted waveforms for the observation time period and gives an estimate on the transmitted information. To get a model for UWB waveform communications a mathematical model must be given for both the transmitter and receiver.

2.1. Model of Transmitter: Elements of Signal Set

Each symbol to be transmitted is mapped into a signal vector $\mathbf{s}_m = [s_{mn}]$. The elements of signal set are bandpass waveforms generated as a weighted linear combination of *n basis functions* $g_n^q(t)$

$$s_m(t) = \sum_{n=1}^{N} s_{mn}\, g_n^q(t), \qquad \begin{cases} 0 \le t < T \\ m = 1, 2, \ldots, M \\ n = 1, 2, \ldots, N \le M \end{cases} \tag{1}$$

The basis functions may be either fixed or continuously varying waveforms but they are orthonormal at least in mean and their values differ from zero only over the symbol duration [3]. Consequently, $s_m(t)$ is also zero outside the time interval $0 \le t < T$. The continuously varying property of chaotic and random basis functions is reflected by the upper index q in (1). For fixed waveform communications the upper index q is omitted.

2.2. Model of Receiver: Received Signal Space

Symbol m is transmitted by sending the analog waveform $s_m(t)$ to the receiver via the analog radio channel where it is corrupted by an additive Gaussian white noise $n(t)$ as shown in Fig. 1. The received signal $r_m(t)$ is bandlimited by the bandpass channel filter having an RF bandwidth of $2B$ and the noisy filtered signal $\tilde{s}_m(t) + \tilde{n}(t)$ is observed by the detector over the symbol duration T to generate the *observation variable* $\mathbf{z}_m$, a random quantity. The elements of signal set are *a priori* known, this information is exploited by the detector designer to suppress channel noise, interference, etc.

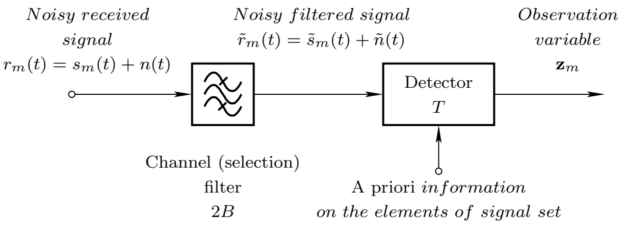

Figure 1: General block diagram of a digital waveform communications receiver.

To get a mathematical model for the detection problem first a *received signal space* has to be constructed in which each signal, either deterministic or random, appearing at the detector input and observed over the symbol duration T can be fully represented. The Fourier analyzer concept [3] derives a *finite dimensional discrete* received signal space in the frequency domain.

Assume that the channel filter passes the transmitted signal without distortion, i.e., $\tilde{s}_m(t) = s_m(t)$. Because the detector observes the received signal only on the time interval $(0, T)$, the input signal can be substituted by a periodic signal

$$s_{T,m}(t) = \begin{cases} s_m(t), & \text{for } 0 \le t < T \\ s_m(t - CT), & \text{otherwise} \end{cases} \tag{2}$$

where C is an arbitrary nonzero integer. Due to the periodicity, the received signal space becomes discrete. The introduction of the periodic signal in (2) does not cause any distortion since the two signals *coincide* each other over the observation time period, i.e., the symbol duration.

In the Fourier analyzer concept [3], the received signal space is a Hilbert space spanned by the harmonically related sinusoidal functions

$$\cos\left(k\frac{2\pi}{T}t\right) \quad \text{and} \quad \sin\left(k\frac{2\pi}{T}t\right) \tag{3}$$

where T denotes the observation time period which is identical to the symbol duration. The detector projects the received waveform $s_m(t)$ over $0 \le t < T$ into this Hilbert space and returns its Fourier coefficients

$$a_{mk} = \frac{2}{T}\int_0^T s_m(t)\cos\left(k\frac{2\pi}{T}t\right)dt$$
$$b_{mk} = \frac{2}{T}\int_0^T s_m(t)\sin\left(k\frac{2\pi}{T}t\right)dt$$

Over the observation time period the received bandpass signal may be reconstructed from its Fourier coefficients

$$s_m(t)\Big|_{0\le t<T} = s_{T,m}(t) = \sum_{k=K_1}^{K_2}\left[a_{mk}\cos\left(k\frac{2\pi}{T}t\right) + b_{mk}\sin\left(k\frac{2\pi}{T}t\right)\right]$$

where K_1 and K_2 are determined by the center frequency and bandwidth of channel filter [3]. The signal dimension is determined by the product of symbol duration T and channel filter bandwidth $2B$

$$S_D = 2(K_2 - K_1 + 1) = 4BT$$

Note, the received signal space is fully specified by the always known receiver parameters, i.e., the observation time period and receiver bandwidth.

2.3. Quantifying *'a priori'* information

After channel filtering, the detector projects the received waveform into the Hilbert space and returns its Fourier coefficients. These Fourier coefficients are compared against the *a priori* information to get the observation variable.

To quantify the *a priori* information, the basis functions $g_n^q(t)$ are also projected into the received signal space

$$\alpha_{nk}^q = \frac{2}{T}\int_0^T g_n^q(t)\cos\left(k\frac{2\pi}{T}t\right)dt$$
$$\beta_{nk}^q = \frac{2}{T}\int_0^T g_n^q(t)\sin\left(k\frac{2\pi}{T}t\right)dt \tag{4}$$

These Fourier coefficients quantify the *a priori* information. In the Fourier analyzer concept the amount of *a priori* information relates to how precisely α_{nk}^q and β_{nk}^q are known at the receiver.

3. Comparison of received signal against the *a priori* known elements of signal set

To get the detection algorithms, the detector compares the Fourier coefficients of basis functions known *a priori* against the Fourier coefficients of received waveform determined in the received signal space, a Hilbert space. In mathematics, correlation gives the measure of similarity of two vectors.

Cross-correlation of two functions is closely related to convolution. This is why the outputs of correlation and matched filter receivers are identical in the decision time instants [1]. This equivalence shows that the derivation technique proposed here provides an optimum solution for the Additive White Gaussian Noise (AWGN) channel.

To illustrate the application of new derivation method proposed here two detector configurations known from the literature are developed: (i) coherent correlation detector also referred to as coherent correlation receiver [1] and (ii) averaged optimum noncoherent FM-DCSK detector [4] where two arbitrary basis functions are applied. To simplify the problem only binary modulation schemes are considered.

3.1. Coherent Detection Algorithm

All the Fourier coefficients α_{nk}^q and β_{nk}^q are exactly known in pure coherent detection. Consider the case where one basis function, $n = 1$, with antipodal modulation scheme is used, then the elements of signal set are obtained from (1)

$$s_m(t) = \pm\sqrt{E_b}\, g_1^q(t)$$

The a priori information, carried by the Fourier coefficient vector of the single basis function $g_1^q(t)$

$$\left(\mathbf{g_1^q}\right)^T = \left(\alpha^q_{1,K_1}\ \beta^q_{1,K_1}\ \cdots\ \alpha^q_{1,K_2}\ \beta^q_{1,K_2}\right)$$

is totally exploited in coherent detection. In the above vector, the Fourier coefficients are given by (4) and the upper index T denotes the transpose.

The detector projects the noisy filtered waveform $\tilde{r}_m(t)$ into the received signal space. Because of the channel noise, only estimates of Fourier coefficients of transmitted waveform $s_m(t)$, denoted by hats, are available

$$(\mathbf{\hat{r}_m})^T = \left(\hat{a}_{m,K_1}\ \hat{b}_{m,K_1}\ \cdots\ \hat{a}_{m,K_2}\ \hat{b}_{m,K_2}\right)$$

where

$$\begin{aligned} \hat{a}_{mk} &= \frac{2}{T}\int_0^T \tilde{r}_m(t)\cos(k\frac{2\pi}{T}t)dt \\ \hat{b}_{mk} &= \frac{2}{T}\int_0^T \tilde{r}_m(t)\sin(k\frac{2\pi}{T}t)dt \end{aligned} \tag{5}$$

Quality of fitting of the two Fourier coefficient vectors is given by their cross-correlation

$$C^{corr}_{\mathbf{\hat{r}_m g_1^q}} = (\mathbf{\hat{r}_m})^T\,\mathbf{g_1^q} = \sum_{k=K_1}^{K_2}\left(\hat{a}_{mk}\,\alpha^q_{1k} + \hat{b}_{mk}\beta^q_{1k}\right) \tag{6}$$

Substituting (5) into (6) and changing the order of sum and integration, the one-dimensional observation variable, indicated in Fig. 1, is obtained as

$$\begin{aligned} z_m &= \frac{T}{2}C^{corr}_{\mathbf{\hat{r}_m g_1^q}} \\ &= \int_0^T \tilde{r}_m(t)\sum_{k=K_1}^{K_2}\left[\alpha_{1k}\cos\left(k\frac{2\pi}{T}t\right)+\beta_{1k}\sin\left(k\frac{2\pi}{T}t\right)\right]dt \end{aligned}$$

Recognizing that the sum on the RHS is the Fourier series representation of the basis function $g_1(t)$ over the observation time period we get the detection algorithm for the coherent receiver as

$$z_m = \int_0^T \tilde{r}_m(t)g_1(t)dt \tag{7}$$

Figure 2 shows the block diagram of coherent detector constructed from (7). The decision circuit is a comparator with zero threshold and it generates the estimate $\hat{b}_m$ of transmitted bit. The coherent detector can be used in both fixed and varying waveform communications, however, the basis function $g_1^q(t)$ must be recovered from the modulated and noisy received signal. Note, the block diagram depicted in Fig. 2 is identical with the coherent correlation receiver well known from the literature [1].

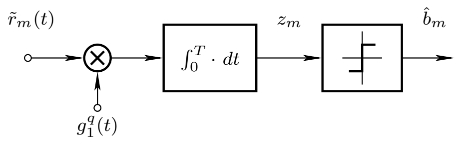

Figure 2: Block diagram of a coherent detector.

3.2. Averaged Optimum Noncoherent Algorithm

To get the optimum noncoherent detector, the harmonic form of Fourier series representation is used

$$s_m(t)\Big|_{0\le t<T} = s_{T,m}(t) = \sum_{k=K_1}^{K_2} C_k\cos\left(k\frac{2\pi}{T}t-\theta_k\right)$$

where each harmonic component is defined by its harmonic amplitude C_k and phase angle θ_k. In the optimum noncoherent approach, the phase information is neglected and only the harmonic amplitudes are used to derive the detection algorithm.

In varying waveform communications the basis functions are continuously varying, consequently, only the *averages* of *harmonic amplitudes* of basis functions are available

$$\overline{C_{nk}} = \mathrm{E}\left[C^q_{nk}\right] = \mathrm{E}\left[\sqrt{(\alpha^q_{nk})^2+(\beta^q_{nk})^2}\right] \tag{8}$$

where E[·] denotes averaging.

Both the neglected phase information and averaging reduce the amount of exploited *a priori* information, consequently, the noise performance of averaged optimum noncoherent detector will be worse than that of the coherent one.

Projecting the waveform $\tilde{r}_m(t)$ into the received signal space, the estimates of harmonic amplitudes of transmitted waveform $s_m(t)$ are obtained as

$$\hat{R}_{mk} = \sqrt{\hat{a}^2_{mk}+\hat{b}^2_{mk}} \tag{9}$$

where $\hat{a}_{mk}$ and $\hat{b}_{mk}$ are given by (5).

From (8) and (9), respectively, the harmonic amplitude vectors of basis functions, $\overline{\mathbf{C_n}}$, and received signal, $\mathbf{\hat{R}_m}$, are constructed. The elements of observation vector are determined from the cross-correlation of these vectors

$$z_{mn} = \frac{T}{2}C^{corr}_{\mathbf{\hat{R}_m}\overline{\mathbf{C_n}}} = \frac{T}{2}\left(\hat{R}_{mK_1}\cdots\hat{R}_{mK_2}\right)\left(\overline{C_{nK_1}}\cdots\overline{C_{nk}}\right)^T \tag{10}$$

Substituting (5) into (9), then substituting (8) and (9) into (10), the observation variable is obtained as

$$\begin{aligned} z_{mn} = \sum_{k=K_1}^{K_2}\overline{C_{nk}}\Bigg(&\left[\int_0^T \tilde{r}_m(t)\cos\left(k\frac{2\pi}{T}t\right)dt\right]^2 \\ &+ \left[\int_0^T \tilde{r}_m(t)\sin\left(k\frac{2\pi}{T}t\right)dt\right]^2\Bigg)^{\frac{1}{2}} \end{aligned} \tag{11}$$

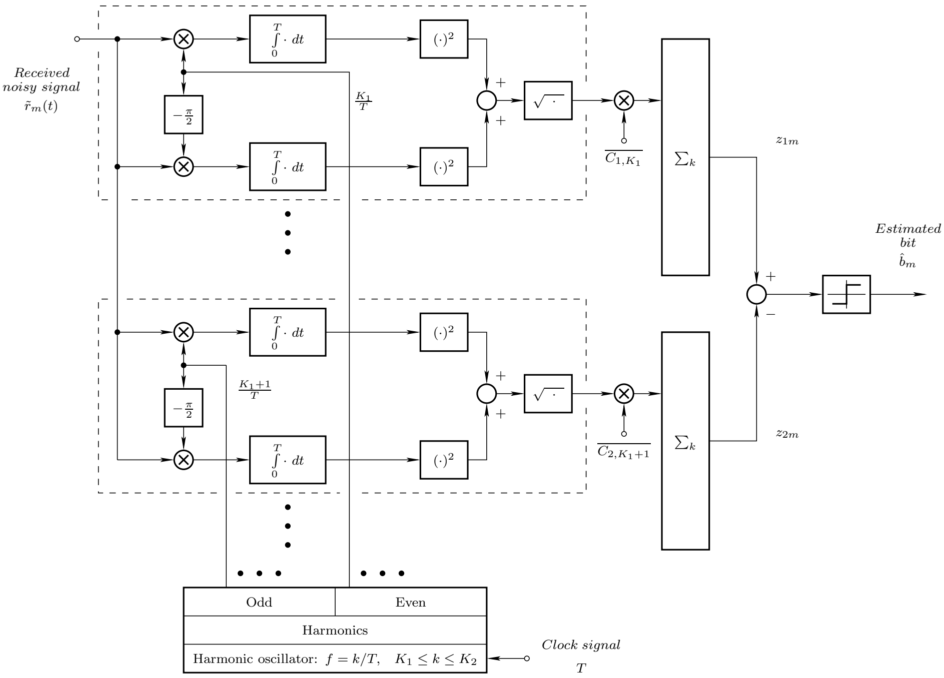

Figure 3: Block diagram of an averaged optimum noncoherent detector.

As an example, starting from (11), let us develop the optimum noncoherent detector configuration for the FM-DCSK modulation scheme. Two basis functions, $g_1(t)$ and $g_2(t)$, are used in FM-DCSK. The elements of signal set are $s_1(t) = \sqrt{E_b} g_1(t)$ and $s_2(t) = \sqrt{E_b} g_2(t)$, respectively, for bits 1 and 0 [4]. The decision is done in favor of bit 1 if

$$z_{m1} > z_{m2} \quad \text{or} \quad z_{m1} - z_{m2} > 0 \tag{12}$$

To get the optimum noncoherent FM-DCSK detector, the weights $\overline{C_{nk}}$ in (11) must be found.

The two FM-DCSK basis functions are constructed from an FM chaotic waveform and the first two Walsh functions [4]. Each basis function is divided into two chips in time, where the first chip serves as a reference. For bits 1 and 0 the second chips are the delayed repeated and delayed inverted, respectively, reference chip.

The averaged harmonic amplitudes of the two FM-DCSK basis functions were derived in [4]. As shown by (3), the fundamental period of the harmonically related sinusoidal base spanning the received signal space is the bit duration T. In the received signal space the basis function $g_1(t)$ has only *even* harmonics, while the other basis function $g_2(t)$ contains only *odd* harmonics. Due to their special structure, $g_1(t)$ and $g_2(t)$ are totally separated in the received signal space.

For the case of an even K_1, Fig. 3 shows the optimum noncoherent FM-DCSK detector configuration constructed from (12) and (11). Note, the complete separation of the two FM-DCSK basis functions in received signal space has been exploited. In varying waveform communications only the averaged values $\overline{C_{nk}}$ of harmonic amplitudes of basis functions are available, in fixed one the weights calculated from the basis functions are real constants.

A *noncoherent matched filter* includes a matched filter followed by an envelope detector. The circuits included in dashed boxes in Fig. 3 are the quadrature receiver equivalents of noncoherent matched filters [1], each matched to one harmonic component of basis functions. These outputs are weighted according to the shape of basis functions and summed to get the observation variable.

4. Conclusions

A new mathematical approach has been proposed for the systematic derivation of optimum waveform detector configurations. The new method is valid for each kind of waveform communications systems, the carrier may be a sinusoidal signal as in conventional communications, a UWB impulse or even a chaotic waveform.

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