Chaotic Communications with Autocorrelation Receiver: Modeling, Theory and Performance Limits

Géza Kolumbán and Tamás Krébesz

Abstract Chaotic signals are ultra-wideband signals that can be generated with simple circuits in any frequency bands at arbitrary power level. The ultra-wideband property of chaotic carriers is beneficial in indoor and mobile applications where multipath propagation limits the attainable bit error rate. Another possible application is the ultra-wideband (UWB) radio, where the spectrum of transmitted signal covers an ultra-wide frequency band (a few GHz) and the power spectral density of transmitted UWB signal is so low that it does not cause any noticeable interference in the already existing conventional telecommunications systems sharing the same RF band. The UWB technology makes the reuse of the already assigned frequency bands possible. This chapter provides a unified framework for modeling, performance evaluation and optimization of UWB radios using either impulses or chaotic waveforms as carrier. The Fourier analyzer concept introduced provides a mathematical framework for studying the UWB detection problem. The autocorrelation receiver, the most frequently used UWB detector, is discussed in detail and an exact closed-form expression is provided for the prediction of its noise performance. Conditions assuring the best bit error rate with chaotic UWB radio are also given.

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Introduction

Since 1990, much research effort has been devoted to the study of communications using chaotic carriers. The earliest works, which were inspired by the synchronization results of Pecora and Carroll [Pecora and Carroll(1990)], exploited synchronization and proposed analog modulation schemes with coherent receivers [Cuomo et al(1993), Cuomo and Oppenheim(1993), Kocarev and Parlitz(1995), Papadopoulos et al(1995)]. Digital modulation using chaotic carriers and a coherent receiver was first introduced in 1992 [Parlitz et al(1992)] and called chaos shift keying (CSK) [Dedieu et al(1993)]. Several other chaotic digital modulation schemes were proposed in the following years, a survey of the state of the art in 1995 can be found in [Hasler(1995)].

The intensive study of synchronization-based coherent communications systems have shown that they are very sensitive to channel noise and distortion. A robust noncoherent technique called differential chaos shift keying (DCSK) [Kolumbán et al(1996)] was introduced in 1996, and later optimized as FM-DCSK [Kolumbán et al(1997b)] where FM means that the power of chaotic basis functions is kept constant by frequency modulation. Since then, the methods of communications theory [Kolumbán et al(1997a), Kolumbán et al(1998), Kolumbán and Kennedy(2000)] and statistical analysis [Abel et al(2000), Sushchik et al(2000)] have been applied to chaotic digital modulation schemes, culminating in the development of chaotic counterparts for conventional modulation schemes [Kolumbán(2000)], and in a theoretical classification and modeling of all chaotic modulation schemes [Kolumbán et al(2005)]. The state of the art has been summarized in three recent publications [Kennedy et al(2000), Hasler et al(2002a), Lau and Tse(2003)].

Chaotic signals are ultra-wideband signals that can be generated with simple circuits in any frequency bands at arbitrary power level. The ultra-wideband property of chaotic carriers is beneficial in indoor and mobile applications where multipath propagation limits the attainable bit error rate. Recovery of chaotic basis functions independently of the modulation and in a noisy channel is difficult to achieve; failure to solve this problem to date has impeded the development of coherent demodulators for chaotic communications. Consequently, the noncoherent modulation schemes are preferred in chaotic communications.

To day everything goes wireless, consequently, there is a huge demand for cheap low-data rate wireless networking devices that can operate for years using the same AAA battery. These requirements can only be satisfied by CMOS technology. Unfortunately, the radio frequency (RF) bands where CMOS can be used is already occupied by conventional narrowband radio communications systems. To overcome this problem frequency re-use must be used.

A recently elaborated tool for frequency reuse is the ultra-wideband (UWB) radio [Siwiak and McKeown(2004)] where the spectrum of transmitted signal covers an ultra-wide frequency band. The power spectral density (psd) of transmitted UWB signal is extremely low and does not cause any noticeable interference in the conventional telecommunications systems sharing the same RF band. The UWB technology has nothing in common with the spread spectrum (SS) systems since in UWB technology

- RF badwidth may be as wide as a few GHz;
- it is not allowed to disrupt the already existing radio channels sharing the same RF band, consequently, contrary to slow frequency hopping the psd of UWB signal must be always kept below the specified limit;
- there is no spreading sequence, instead, the modulation is applied directly to an ultra-wideband carrier.

The chaos-based communications systems are inherently ultra-wideband systems and satisfy the UWB requirements listed above. The noncoherent receiver configurations are preferred in the built UWB systems [Arsalan et al(2006)] since the recovery of UWB carriers with cheap CMOS circuitry is a very difficult task, especially if the ultra-low power consumption is a must. If so then coherent receivers cannot be used and chaos-based communications schemes offer a competitive alternative to UWB radio implementation.

This chapter provides a unified framework for modeling, performance evaluation, optimization and comparison of UWB radios using either impulses or chaotic waveforms as carrier.

Section 1 generalizes the idea of basis functions to varying waveform communications, provides a model for waveform communications and discusses the estimation problem, a special problem arising in chaos-based communications.

Starting from the general receiver model, Sec.2 develops a signal model in order to provide a mathematical tool for the waveform detection problem. By means of the Fourier analyzer concept, a signal space, referred to as received signal space, is defined in which each received signal, either deterministic or random, can be represented. Finally, the hierarchy of waveform communications systems is established.

Section 3 surveys the UWB radio regulations and discusses the UWB modulation schemes.

In transmitted reference (TR) systems the reference signal used by the correlation receiver is not recovered at the receiver by a carrier recovery circuit but it is transmitted via the radio channel. This approach makes the TR systems very robust against the channel distortion. Furthermore, a very simple circuit, the autocorrelation receiver can be used for reception of a TR signal. Section 4 determines the special properties of TR basis functions and discusses the operation principle of TR autocorrelation receiver. An exact closed-form expression is provided for the calculation of noise performance of TR autocorrelation receivers and the condition assuring the best bit error rate is determined.

1 Basis Functions: Model for Waveform Communications

Since only analog waveforms may be transmitted over a radio channel and the data rate R is given by the specification, the modulator of a digital telecommunications system maps the symbols to be transmitted into analog waveforms of finite duration $T = 1/R$. These analog waveforms of duration T constitute the *signal set*. This technique is referred to as *waveform communications* where T denotes the signalling time interval. To get the simplest mathematical model, the elements of signal set are represented by a minimum number of *basis functions* in the basis function approach [Haykin(1994)].

Each type of digital demodulators has more or less *a priori* information on the basis functions. This knowledge is exploited to recover the digital information transmitted and to suppress channel noise and interference. The more the amount of *a priori* knowledge exploited, the better the system performance.

The type of basis functions gives an upper bound on the *a priori* information that may be exploited by the demodulator. Based on the basis functions, three classes of waveform communications are distinguished [Kolumbán et al(2005)], namely, communications with

- Fixed waveforms [Haykin(1994), Proakis(1995), Simon et al(1995)];
- Chaotic waveforms [Hasler et al(2002b)];
- Random waveforms [Basore(1952)].

In *fixed waveform communications*, the basis functions are fixed. Consequently, every time when the same symbol is sent then the same waveform is transmitted. The basis functions and the elements of signal set are exactly known. In the built coherent receivers the fixed basis functions are recovered from the received signal (see the correlator receiver including a carrier recovery circuit) or stored at the receiver (see the matched filter approach) [Proakis(1995)].

Note, the type of generator used to produce the fixed basis functions is irrelevant. Even a windowed part of chaotic or random signal may be used in fixed waveform communications as basis function provided that it is stored at both the transmitter and receiver.

In chaotic communications, each basis function is the actual output of a chaotic signal generator. The chaotic signals are predictable only in short run, because the chaotic systems have an extremely high sensitivity to the initial conditions and the parameters of chaotic attractor [Parker and Chua(1989)]. Since the shape of chaotic basis functions is not fixed, the radiated waveform varies even if the same symbol is transmitted repeatedly. A unique feature of chaotic communications systems is that the transmitted signal is never periodic.

Communications techniques where chaotic and random waveforms are used as carrier are referred to as *varying waveform communications* systems.

1.1 Basis Functions in Fixed Waveform Communications

Consider a fixed waveform communications system using M symbols for the data communications. First the symbol m is mapped into a signal vector $\mathbf{s}_m = [s_{mn}]$. From each signal vector an analog waveform $s_m(t)$ is generated, these waveforms constitutes the signal set. In order to get the simplest mathematical model for the modulator, the elements of signal set are expressed as a linear combination of N basis functions [Haykin(1994)]

$$s_m(t) = \sum_{n=1}^{N} s_{mn} g_n(t), \quad \begin{cases} 0 \le t < T \\ m = 1, 2, \ldots, M \\ n = 1, 2, \ldots, N \end{cases} \tag{1}$$

where $N \le M$. Note, each symbol is characterized by a distinct signal vector $\mathbf{s}_m = [s_{mn}]$, $m = 1, 2, \ldots, M$ and, according to (1), by a distinct waveform. To avoid intersymbol interference (ISI), the real-valued fixed basis functions $g_n(t),\ n = 1, 2, \ldots, N$ must be zero outside the signalling time interval T.

By definition, basis functions are orthonormal

$$\int_0^T g_i(t) g_n(t) dt = \begin{cases} 1, & \text{if } i = n \\ 0, & \text{otherwise} \end{cases}$$

which means that each basis function carries unit energy and each pair of distinct basis functions are orthogonal to each other over the signalling time period $[0, T]$.

1.2 Basis Functions in Varying Waveform Communications

Let the continuously varying property of chaotic basis functions be reflected by an upper index q in (1)

$$g_n^q(t), \quad q = 1, 2, \ldots$$

where q identifies the basis function used to transmit the qth element in a sequence of symbols.

This chapter focuses on chaotic UWB waveform communications. To get compact equations, q will be suppressed in the remaining part of the chapter except when we want to emphasize the time-varying property of basis functions.

The main difference between fixed and chaotic waveform communications is that in the latter the basis functions are orthonormal over the signalling time interval only in the mean

$$E\left[\int_0^T g_i^q(t) g_n^q(t) dt\right] = \begin{cases} 1, & \text{if } i = n \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

where $E[\cdot]$ denotes the expectation operator.

The duration of basis functions is determined by the required data rate $R = 1/T$. Let the chaotic signals with finite duration be referred to as chaotic sample functions. The energy of chaotic sample functions varies from sample function to sample function, and due to their finite duration, two chaotic sample functions are never orthogonal. These properties are reflected by the expectation operator in (2).

As a result, the energy used to transmit a given symbol may vary even if the same symbol is transmitted repeatedly and the actual basis functions are not orthogonal in chaotic communications. The former and the latter are referred to as *auto- and cross-correlation estimation problems*.

The estimation problems must be solved, otherwise, they seriously corrupt the bit error rate. To see the consequence of estimation problem consider chaotic on-off keying (COOK) modulation [Kis et al(1998)] where only one basis function is used, consequently, only the autocorrelation estimation problem appears. Figure 1 shows that the autocorrelation estimation problem manifests itself if $\int_0^T g_1^2(t)dt$ is not constant (see dashed curve) but the problem disappears when $\int_0^T g_1^2(t)dt$ is kept constant (see solid curve). For more details on the estimation problems and solutions to them refer to [Kolumbán et al(2002)].

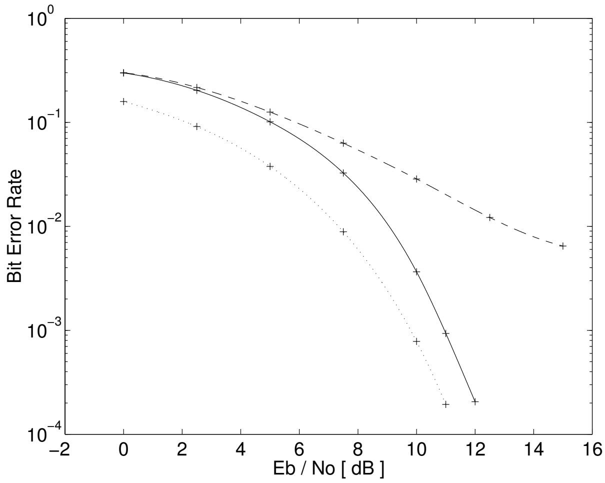

Fig. 1 Noise performance of noncoherent COOK with constant (solid curve) and varying (dashed curve) energy per symbol. BER of coherent COOK is also shown (dotted curve) for comparison.

2 Signal Model for Detection

In digital communications, the elements of signal set carrying the symbols pass through a telecommunications channel. The received signal is corrupted by noise and may suffer from distortion, interference and multipath effect. Observing the corrupted and distorted received analog waveform for the signalling time interval, the detector must decide which message has been most likely transmitted.

According to (1), the elements of signal set are represented by the basis functions that are known, or at least some of their characteristics are known at the receiver. This *a priori* knowledge is exploited to perform the detection and suppress channel noise and interference. The noise and interference suppression capabilities of a waveform communications system depend on the amount of *a priori* information exploited by the demodulator. As a rule of thumb, a detector algorithm exploiting less amount of *a priori* information provides a simpler detection technique.

2.1 General Block Diagram of a Receiver

Figure 2 shows the general block diagram of a waveform communications receiver.

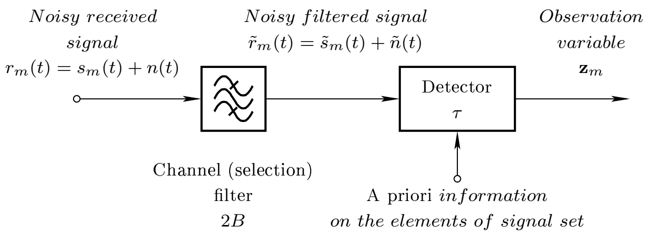

Fig. 2 General block diagram of a digital waveform communications receiver.

The transmitted signal $s_m(t)$, the mth element of signal set, is corrupted by a signal $n(t)$ which represents the sample function of white Gaussian channel noise or interference. The received signal is obtained as

$$r_m(t) = s_m(t) + n(t).$$

To select the signal to be received, $r_m(t)$ is fed into a bandpass channel (selection) filter of RF bandwidth $2B$. The detector observes the filtered received signal $\tilde{r}_m = \tilde{s}_m(t) + \tilde{n}(t)$ over τ and generates the observation variable $\mathbf{z}_m$ which may be either a random scalar number or a random vector.

The decision time instants, the signalling time interval T and the RF bandwidth $2B$ of transmitted signal $s_m(t)$ are always known at the receiver. The receiver parameters are matched to (i.e., identical) these data. Fortunately, these parameters are enough to apply the Fourier analyzer concept that provides a unified theory for the detection problem.

2.2 Fourier Analyzer Concept

To get a mathematical model for the detection problem first a *received signal space* has to be constructed in which each signal, either deterministic or random, appearing at the detector input and observed over the observation time interval τ can be fully represented. Although the Fourier transform is widely used in electrical engineering to represent arbitrary waveforms in the frequency domain, it cannot be used here since (i) it does not provide a discrete space, (ii) its dimension goes to infinity, and (iii) it is not restricted to the observation time interval. The Fourier analyzer concept [Kolumbán et al(2005)] derives a *finite dimensional* and *discrete* received signal space in the frequency domain.

In the Fourier analyzer concept the received signal space is defined as a Hilbert space [Kreyszig(1999)] spanned by the harmonically related $\cos(\cdot)$ and $\sin(\cdot)$ functions. To distinguish the basis of this Hilbert space from the basis functions introduced by (1) let the former be referred to as a Fourier base.

Consider two arbitrary real-valued waveforms denoted by $x_1(t)$ and $x_2(t)$. To get a Hilbert space an inner product

$$\langle x_1(t), x_2(t) \rangle = \int_0^\tau x_1(t) x_2(t) dt$$

and a norm

$$\| x_1(t) \| = \sqrt{\int_0^\tau x_1^2(t) dt}$$

have to be defined.

Now let these ideas be applied to the detection problem depicted in Fig. 2. In a well-designed receiver the channel filter passes the transmitted signal without distortion, i.e., $\tilde{s}_m(t) = s_m(t)$. For the sake of simplicity, consider the noise- and interference-free case where $n(t) = 0$, therefore $\tilde{r}_m(t) = s_m(t)$ and assume that the observation and signalling time intervals are identical, $\tau = T$.

Our goal is to derive a *discrete* received signal space in the frequency domain. Because the detector observes the received signal only on the time interval $[0, \tau)$, the input signal can be substituted by a periodic signal

$$s_{T,m}(t) = \begin{cases} s_m(t), & \text{for } 0 \leq t < \tau \\ s_m(t - C\tau), & \text{otherwise} \end{cases} \tag{3}$$

where C is an arbitrary nonzero integer. Due to the periodicity introduced in (3), the received signal space becomes discrete. The introduction of the periodic signal in (3) does not cause any distortion since the two signals *coincide* each other over the observation time period.

In the Fourier analyzer concept [Kolumbán et al(2005)], the received signal space is a Hilbert space spanned by the harmonically related sinusoidal functions

$$\cos\left(k\frac{2\pi}{\tau}t\right) \quad \text{and} \quad \sin\left(k\frac{2\pi}{\tau}t\right)$$

where τ denotes the observation time interval. Note, the fundamental period of Fourier base is determined by the observation time period and has nothing to do with the center frequency of $s_m(t)$.

During reception, the detector projects the received waveform $s_m(t)$ into this Hilbert space and returns its Fourier coefficients by calculating the inner products

$$\begin{aligned} a_{mk} &= \frac{2}{T}\langle s_{T,m}(t), \cos(k\frac{2\pi}{T}t)\rangle = \frac{2}{T}\int_0^T s_m(t)\cos\left(k\frac{2\pi}{T}t\right)dt \\ b_{mk} &= \frac{2}{T}\langle s_{T,m}(t), \sin(k\frac{2\pi}{T}t)\rangle = \frac{2}{T}\int_0^T s_m(t)\sin\left(k\frac{2\pi}{T}t\right)dt \end{aligned} \tag{4}$$

where we exploited the assumption of $\tau = T$.

Over the observation time period the received bandpass signal may be reconstructed from its Fourier coefficients

$$s_m(t)\Big|_{0\le t<T} = s_{T,m}(t) = \sum_{k=K_1}^{K_2}\left[a_{mk}\cos\left(k\frac{2\pi}{T}t\right) + b_{mk}\sin\left(k\frac{2\pi}{T}t\right)\right] \tag{5}$$

where K_1 and K_2 are determined by the center frequency and bandwidth of channel filter.

Chaotic and random signals may also appear at the detector input. The Fourier series representation introduced in (5) remains valid for these signals, but the Fourier coefficients defined by (4) becomes random variables.

2.3 Dimension of Received Signal Space

Consider an ideal bandpass channel filter with bandwidth $2B$ and center frequency f_0. Assume that the ideal bandpass channel filter is perfectly matched to the transmitted signal and $\tau = T$.

Let $S_m(\omega)$, $m = 1, 2, \ldots, M$, denote the Fourier transform of the elements of signal set. Since $\tilde{s}_m(t) = s_m(t)$, substituting (3) into (4) and applying the definition of Fourier transform we get

$$a_k = \frac{2}{T}\Re\left[S_m(k\frac{2\pi}{T})\right] \qquad \text{and} \qquad b_k = -\frac{2}{T}\Im\left[S_m(k\frac{2\pi}{T})\right].$$

Figure 3 marks the location of Fourier coefficients a_k, b_k of detector input by arrows in the frequency domain. Due to the periodicity introduced in (3), a discrete spectrum is obtained and the distance between two adjacent spectral components is equal to the data rate $R = 1/T$. The bandwidth of detector input is limited by the channel filter, this ideal bandpass filter suppresses each spectral component lying outside the frequency range $(2K_1 - 1)/2T \le f \le (2K_2 + 1)/2T$. Note, since the channel filter limits the bandwidth of received signal, only a finite number of Fourier coefficients differs from zero.

By definition, the signal dimension gives the number of harmonically related $\sin(\cdot)$ and $\cos(\cdot)$ functions along which the receiver collects information on the received signal. In other words, the signal dimension gives the dimension of Hilbert

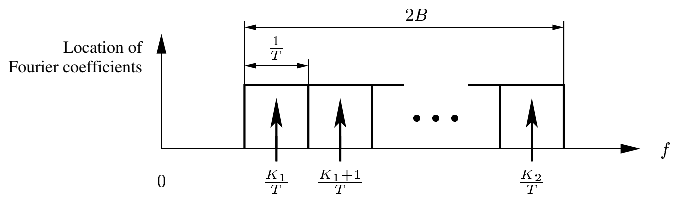

Fig. 3 Determination of the dimension of received signal space: Location of Fourier coefficients of detector input in the frequency domain.

space spanned by the Fourier base which is required to represent *any signal* appearing at the detector input *over the observation time interval in the received signal space*. From Fig. 3 and (5), the signal dimension is obtained as

$$S_D = 2(K_2 - K_1 + 1) = 4BT \tag{6}$$

where

$$K_1 = \frac{2(f_0 - B)T + 1}{2} = \frac{4f_0T - S_D + 2}{4} \tag{7}$$

and

$$K_2 = \frac{2(f_0 + B)T - 1}{2} = \frac{4f_0T + S_D - 2}{4}\,. \tag{8}$$

The signal dimension S_D is independent of the center frequency of telecommunications channel, it is proportional to the product of channel bandwidth $2B$ and observation time interval $\tau = T$. Note, only two receiver parameters, bandwidth of channel filter and observation time interval, are required to construct the received signal space. These parameters are always known.

2.4 Measure of a priori *Information*

As shown in Fig. 2, after channel filtering the detector projects the received waveform into the received signal space and returns either the Fourier coefficients of the noisy received signal (see coherent systems in Sec. 2.4.1) or some other parameter(s) derived from the Fourier coefficients (see noncoherent receiver in Sec. 2.4.4). These Fourier coefficients or derived parameters are compared against the *a priori* information available at the receiver to get the observation variable.

To compare the different modulation schemes and to determine their theoretical performance bounds, an exact measure for the amount of *a priori* information exploited by the demodulator must be found. Let the basis functions $g_n^q(t)$ be projected into the received signal space

$$
\begin{aligned}
\alpha_{nk}^{q} &= \frac{2}{T}\langle g_{T,n}^{q}(t), \cos(k\frac{2\pi}{T}t)\rangle = \frac{2}{T}\int_{0}^{T} g_{n}^{q}(t)\cos\left(k\frac{2\pi}{T}t\right)dt \\
\beta_{nk}^{q} &= \frac{2}{T}\langle g_{T,n}^{q}(t), \sin(k\frac{2\pi}{T}t)\rangle = \frac{2}{T}\int_{0}^{T} g_{n}^{q}(t)\sin\left(k\frac{2\pi}{T}t\right)dt.
\end{aligned} \tag{9}
$$

Recall, in fixed waveform communications the basis functions are fixed and the upper index q has to be dropped.

The Fourier coefficients given by (9) can be used to quantify the amount of *a priori* information. In the Fourier analyzer concept it relates to how precisely the Fourier coefficients α_{nk}^{q} and β_{nk}^{q} are known at the receiver.

Fixed basis functions, mostly sinusoidal signals, are used in conventional digital communications systems. From the detection point of view coherent and noncoherent receivers are distinguished [Haykin(1994)]. In the Fourier analyzer concept, which includes both fixed and varying waveform communications, four cases have to be considered.

2.4.1 Coherent Modulation Technique

If the exact values of Fourier coefficients are available at the receiver then the basis functions may be reconstructed over the observation time period without any error or approximation

$$
g_{n}^{q}(t)\Big|_{0\leq t<T} = g_{T,n}^{q}(t) = \sum_{k=K_1}^{K_2}\left[\alpha_{nk}^{q}\cos(k\frac{2\pi}{T}t) + \beta_{nk}^{q}\sin(k\frac{2\pi}{T}t)\right] \tag{10}
$$

where the Fourier coefficients α_{nk}^{q} and β_{nk}^{q} are obtained from (9) and the constants K_1 and K_2 are given by (7) and (8), respectively.

In fixed waveform communications, the complex Fourier coefficients are constant [consequently, q does not appear in (9) and (10)] and they are recovered by a carrier recovery circuit (see coherent correlation receiver) or stored as the impulse response of a matched filter [Haykin(1994)].

In chaotic waveform communications, the Fourier coefficients vary from symbol to symbol, consequently, the matched filter approach cannot be used. Since a robust solution to the recovery of chaotic basis functions from the received noisy and distorted signal has not yet been published, the coherent modulation technique is not feasible in the built chaotic communications systems.

2.4.2 Optimum Noncoherent Modulation Technique

The optimum noncoherent modulation technique can be used only in fixed waveform communications. Let the basis functions be represented in harmonic form

$$g_n(t)\Big|_{0\le t<T} = g_{T,n}(t) = \sum_{k=K_1}^{K_2} \gamma_{nk}\cos\left(k\frac{2\pi}{T}t - \theta_k\right)$$

where each harmonic component is defined by its harmonic amplitude γ_k and phase angle θ_k. The harmonic amplitudes are derived from the Fourier coefficients (9) as

$$\gamma_{nk} = \sqrt{\alpha_{nk}^2 + \beta_{nk}^2}\,. \tag{11}$$

In the optimum noncoherent modulation, the phase information θ_k is neglected and only the harmonic amplitudes γ_k are used to derive the detection algorithm. A typical example for this approach is the conventional noncoherent frequency-shift keying (FSK) modulation scheme [Haykin(1994)].

The neglected phase information means that the optimum noncoherent receiver exploits less amount of *a priori* information during detection as its coherent counterparts. The loss in exploited *a priori* information results in a worse noise suppression capability, consequently, in a worse bit error rate.

2.4.3 Averaged Optimum Noncoherent Modulation Technique

In the Fourier analyzer concept, basis functions and signals are represented in the received signal space. Since the value of chaotic signals can be predicted only in short run, the random signal model has to be used to both the chaotic and random basis functions [Kis and Kolumbán(1998)]. Recall, random signals have no Fourier transform, they can be characterized only with their psd in the frequency domain which is the Fourier transform of their autocorrelation function [Bendat and Piersol(1966)].

The averaged optimum noncoherent modulation technique is used in chaotic communications. It follows from the definition of psd that the $\left(k\frac{2\pi}{T}\right)$-frequency components of chaotic and random signals has no phase information in the received signal space and the square root of their psd can be *estimated* from (9)

$$\overline{\gamma_{nk}} = \mathrm{E}\left[\gamma_{nk}^q\right] = \mathrm{E}\left[\sqrt{(\alpha_{nk}^q)^2 + (\beta_{nk}^q)^2}\right] \tag{12}$$

where $\mathrm{E}[\cdot]$ denotes averaging.

Both the lost phase information and averaging reduce the amount of exploited *a priori* information, consequently, the noise performance of averaged optimum noncoherent detector is worse than that of the optimum noncoherent one. An example for the chaos-based averaged optimum noncoherent detector is shown in [Kolumbán et al(2004)].

2.4.4 Noncoherent Modulation Technique

The least *a priori* information is exploited in the noncoherent modulation technique. This approach can be used only if the basis functions are *completely separated* in the received signal space. The basis functions of transmitted reference system introduced in Sec. 4.1 satisfy this condition, the operation principle of the TR autocorrelation receiver is discussed in Sec. 4.2.

In the Fourier analyzer concept the detector collects the components of received signal in the received signal space to generate the observation variable. The power of channel noise is uniformly distributed over the Fourier base, while the distribution of transmitted waveform is *a priori* known. During the generation of observation variable the parameters (9), (11) and (12) are used as weights in signal summation in coherent, optimum noncoherent and averaged optimum noncoherent, respectively, modulation techniques. As a result, the signal-to-noise ratio (SNR) measured at each $\left(k\frac{2\pi}{T}\right)$ frequency of Fourier base is accounted; the higher the SNR, the larger the weight.

In noncoherent modulation the basis functions are completely separated and *only the presence* of signal components is checked. Consequently, the weights have only two distinct values, their value is 1 and 0 if signal component is and is not, respectively, transmitted

$$W_{nk} = \begin{cases} 1, & \text{if } E\left[(\alpha_{nk}^{q})^2 + (\beta_{nk}^{q})^2\right] > 0 \\ 0, & \text{if } E\left[(\alpha_{nk}^{q})^2 + (\beta_{nk}^{q})^2\right] = 0\,. \end{cases}$$

Since the same weights are used for each component of observation variable, the noise contribution of a component is not controlled by its SNR. This causes a further loss in *a priori* information and results in the worst noise performance.

Although the noncoherent modulation technique offers the worst noise performance in an AWGN channel, it is very robust against the channel distortion. Examples in fixed waveform communications are the autocorrelation reception of differential phase shift keying (DPSK) signals [Okunev(1997)] and the autocorrelation reception of UWB TR impulses [Hoctor and Tomlinson(2002)].

In chaotic communications two kinds of noncoherent receivers have been developed, the autocorrelation receiver [Kolumbán et al(1996)] and the energy detector [Kolumbán and Kis(2003)].

3 UWB Waveform Communications

3.1 UWB Radio Regulations

The conventional and UWB radio communications systems share the same RF band and operate simultaneously. To avoid the interference caused in the already existing conventional systems, the transmitted spectrum of UWB radio is spread over an *ultra-wide frequency band* as shown in Fig. 4. In UWB radio the psd of equivalent isotropically radiated power (EIRP), measured with a resolution of 1 MHz, must be less than -41.3 dBm [Siwiak and McKeown(2004)] where EIRP is the product of the power supplied to the antenna and the antenna gain in a given direction referred to an isotropic antenna.

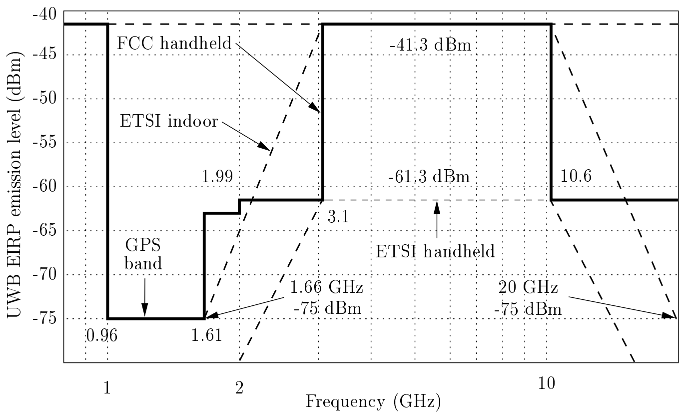

Fig. 4 Emission limits for handheld and indoor UWB radio systems allowed by the Federal Communications Commission (FCC, USA), solid curve, and the European Technical Standards Institute (ETSI), dashed curve [Siwiak and McKeown(2004)].

The frequency band allocated to the UWB devices goes from 3.1 GHz to 10.6 GHz. By definition, the UWB transmitter is an intentional radiator that, at any time instant, has a fractional bandwidth greater than 20% or a UWB bandwidth greater than 500 MHz. The UWB regulations specify only the maximum emission limit and minimum bandwidth and say nothing about the type of carrier and the technique used to generate the modulated UWB waveform. The UWB regulations give only the rule under which the assigned frequency band may be accessed, the UWB carrier may be either (i) a fixed waveform, typically an impulse, or (ii) a chaotic waveform.

3.2 Structure of Modulated UWB Waveforms

The digital information to be transmitted is mapped to wideband wavelets of very short duration in UWB radio. The wavelets have a fixed waveform in UWB impulse radio and they are chaotic signals in chaotic UWB radio. In the latter, the shape of transmitted wavelets is continuously varying even if the same information bit is transmitted repeatedly.

For the sake of simplicity, only binary systems are considered in the remaining part of this chapter. Two classes of modulations exist in UWB radio, namely, one information bit may be mapped to (i) one or (ii) two wavelets [Kolumbán and Krébesz(2006)].

3.2.1 UWB Modulation Schemes Using One Wavelet

The structure of UWB modulations using one wavelet is shown in Fig. 5, where $c(t)$ denotes the wavelet having an arbitrary waveform, T_{ch} is the wavelet duration, t_{pos} denotes the pulse positioning and T_f is the frame repetition time from which the data rate is obtained as $R = 1/T_f$. As shown in Fig. 5, a guard time, $T_f > t_{pos} + T_{ch}$, may be inserted after wavelet $c(t)$ in UWB radio to prevent the intersymbol interference in multipath channels.

Note, the signalling time interval is called frame repetition time in UWB radio, that is, $T_f = T$. In conventional communications guard time is not used. To emphasize the special structure of UWB signal that allows the insertion of a guard time we will refer the signalling time interval to as frame repetition time T_f in the remaining part of this chapter.

To carry the digital information pulse positioning, amplitude and polarity of a wavelet may be varied in accordance with the modulation.

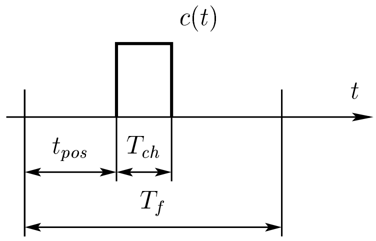

Fig. 5 Structure of UWB modulation using one wavelet where $c(t)$ denotes the carrier wavelet.

3.2.2 UWB Modulation Scheme Using Two Wavelets

The recovery of basis function(s) with CMOS circuitry featuring extremely low power consumption is a hard task to solve because of the extremely short wavelet duration in UWB impulse and the continuously varying carrier in chaotic UWB radios. However, each correlator-based receiver requires a reference signal to perform demodulation. If the reference signal cannot be recovered from the received noisy signal, than it must be transmitted. In transmitted reference UWB systems the same radio channel is used to transmit both the reference and information bearing wavelets.

To provide the reference waveform required by the correlation receiver, each bit to be transmitted is mapped into two wavelets, called chips, in the TR modulator. The first chip serves as a reference, while the second one carries the information.

The structure of a modulated TR signal is shown in Fig. 6, where $c(t)$ denotes an arbitrary wavelet, T_{ch} is the chip duration and $\Delta T \geq T_{ch}$ gives the delay between the reference and the information bearing chips. Note, a guard time may be inserted after both the reference and information bearing chips. The best noise performance is achieved by antipodal modulation scheme, where the information bearing wavelet is equal to the delayed reference one for bit "1," and to the inverted and delayed reference wavelet for bit "0." As wavelet $c(t)$, frequency-shifted bell-shaped Gaussian pulse, monocycle and doublet pulse are used in UWB impulse radio [Siwiak and McKeown(2004), Ghavani et al(2006)] while chaotic waveforms are applied in chaotic UWB radio.

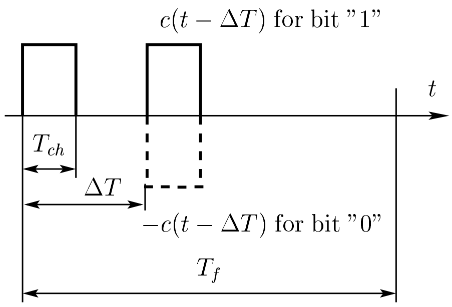

Fig. 6 Structure of modulation using two wavelets. This modulation scheme is also referred to as transmitted reference (TR) system.

The unique feature of a TR system, namely that both the reference and information bearing chips are transmitted via the same telecommunications channel, makes the TR radio system very robust against the channel distortions.

In case of channel distortion, the modulated signal has to be correlated with a reference signal distorted in the same manner as the modulated one to get the best system performance. A correlation with the original distortion-free reference results in a performance degradation. The reference chip transmitted in TR systems serves as a test signal used to measure the actual channel characteristics. Consequently, the TR modulation scheme may be used even in a time-varying channel.

TR system suffers from two drawbacks:

- since two chips are required to transmit one bit information, half of the energy per bit E_b is "lost" in that sense that the reference chip is not used directly to transmit information, it is used "only" to measure the actual channel characteristics, and serves as a reference for the correlation receiver
- both the reference and information bearing chips are corrupted by channel noise. As shown in Sec. 4.3, the noisy reference chip results in a noise performance degradation.

To select the best solution to a given data communications problem the actual channel conditions have to be evaluated. In case of AWGN channel and a very simple sinusoidal carrier which is easy to recover by a phase-locked loop, the correlation receiver with carrier recovery circuit offers the best system performance. When the channel suffers from distortion, the TR system offers a better system performance [Kolumbán and Krébesz(2006)] provided that the loss caused by the noisy reference chip is less than the gain arising due to the perfect correlation between the reference and information bearing chips. In chaos-based UWB systems, or if the duration of UWB impulse is extremely short then the carrier recovery is not feasible and the TR approach is the only solution.

4 UWB TR Modulation with Autocorrelation Receiver

The TR modulation scheme offers a very robust solution to the UWB radio communications and does not require a carrier recovery circuit for demodulation. A further advantage follows from the special structure of modulated TR waveform, as shown in Fig. 6 the binary information transmitted may be recovered from the sign of correlation measured between the reference and information-bearing chips. Autocorrelation receiver exploits this property of TR modulation to generate observation signal.

4.1 TR Basis Functions

According to Fig. 6, the signal set of binary TR modulation includes two waveforms

$$s_m(t) = s_{m1} g_1(t) + s_{m2} g_2(t), \quad m = 1, 2 \tag{13}$$

where the two basis functions are defined as

$$\begin{aligned} g_1(t) &= \begin{cases} +\frac{1}{\sqrt{E_b}} c(t), & 0 \le t < T_{ch} \\ +\frac{1}{\sqrt{E_b}} c(t - \Delta T), & \Delta T \le t < \Delta T + T_{ch}, \end{cases} \\ g_2(t) &= \begin{cases} +\frac{1}{\sqrt{E_b}} c(t), & 0 \le t < T_{ch} \\ -\frac{1}{\sqrt{E_b}} c(t - \Delta T), & \Delta T \le t < \Delta T + T_{ch} \end{cases} \end{aligned} \tag{14}$$

and the two signal vectors are

$$\mathbf{s}_1 = [s_{1n}] = \begin{pmatrix} s_{11} \\ s_{12} \end{pmatrix} = \begin{pmatrix} \sqrt{E_b} \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{s}_2 = [s_{2n}] = \begin{pmatrix} s_{21} \\ s_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ \sqrt{E_b} \end{pmatrix} \tag{15}$$

for bit "1" and "0," respectively. In (15), E_b denotes the energy per bit that is used to transmit one bit information. Wavelets $c(t)$ in (14) may be either a fixed waveform as in UWB impulse radio or may denote sample functions of a chaotic signal as in chaotic UWB radio.

Recall, by definition basis functions are orthonormal over the bit duration at least in mean. To avoid estimation problem that corrupts the noise performance seriously, the othonormality must be assured.

Observe the special structure of basis functions given by (14) orthogonality of basis functions is assured by the first two Walsh functions [Tzafestas(1985)]

$$W_1 = [1 \;\; 1] \quad \text{and} \quad W_2 = [1 \;\; -1].$$

independently of the actual shape of $c(t)$.

The energy of fixed basis functions is constant. However, in chaotic communications some extra signal processing is necessary to fix the varying energy of chaotic basis functions. For example, since the power of FM signals is constant, in FM-DCSK a chaotic signal is applied to the baseband input of an FM modulator and the modulator output is used as $c(t)$. This approach is referred to as FM-DCSK modulation [Kolumbán et al(1997b)].

In FM-DCSK

$$\frac{T}{2} = \Delta T = T_{ch}$$

while in chaotic TR UWB systems

$$\Delta T > T_{ch} \quad \text{and} \quad \Delta T + T_{ch} < T_f.$$

Note, the maximum data rate is achieved by FM-DCSK, while chaotic TR UWB modulation provides guard time for multipath.

By the definition introduced in Sec. 2.4.4, the noncoherent modulation technique can be used only if the spectra of basis functions are completely separated in the received signal space. To illustrate this property of TR modulation defined by (13) and (14), the spectra of TR basis functions with the parameters $T_{ch} = 1\ \mu$s, $\Delta T = 1\ \mu$s and $T_f = 2\ \mu$s were determined by computer simulation. The center frequency of the modulated TR signal was 2.4 GHz.

Figures 7 and 8 show the spectra of basis functions $g_1(t)$ and $g_2(t)$, respectively. Observe, the fundamental harmonic frequency is $1/T_f = 500$ kHz and only the even $k=0,2,\dots$ and odd $k=1,3,\dots$ harmonics appear in the spectra of $g_1(t)$ and $g_2(t)$, respectively. For a detailed analytical proof refer to [Kolumbán(2003)].

Note that the spectra of two basis functions are completely separated, although they overlap each other. The two spectra may be interpreted as the teeth of two combs fitted into each other.

4.2 Autocorrelation Receiver

The special structure of TR basis functions — each consists of a reference chip followed by a non-inverted or inverted copy of itself — can be exploited to perform the

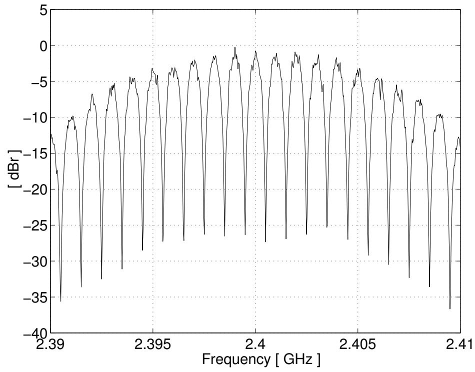

Fig. 7 Spectrum of TR basis function $g_1(t)$. As shown by (15) and (13), except a constant this spectrum is identical with the transmitted signal when a pure bit "1" sequence is transmitted.

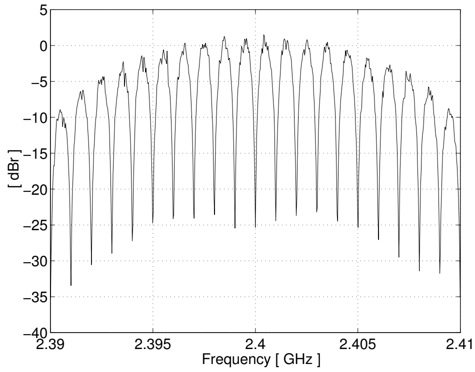

Fig. 8 Spectrum of TR basis function $g_2(t)$. Observe, the fundamental harmonic frequency is $1/T_f = 500$ kHz, but only the odd harmonics are present in the spectrum.

demodulation. The TR autocorrelation receiver makes its decision by evaluating the sign of correlation measured between the reference and information-bearing chips as shown in Fig. 9, where $\tau = T_{ch}$ denotes the observation time period, $\hat{b}_m$ is the estimated (i.e. received) bit and $h(t)$ is the impulse response of bandpass channel filter.

From Fig. 9, the observation variable is obtained as

$$z_m = \int_{\Delta T}^{\Delta T + T_{ch}} [\tilde{s}_m^q(t) + \tilde{n}(t)][\tilde{s}_m^q(t - \Delta T) + \tilde{n}(t - \Delta T)]dt. \qquad (16)$$

Recall, the upper index q reflects the continuously varying property of basis functions in varying waveform communications.

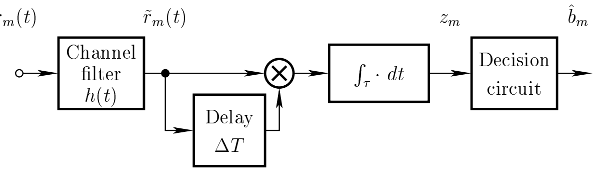

Fig. 9 Block diagram of TR autocorrelation receiver.

Assume that the bandwidth of channel filter is matched to the elements of signal set, that is, $\tilde{c}^q(t) = c^q(t)$. Substituting (15) and (14) into (13), and substituting the result into (16) the observation variable is obtained as a sum of four terms

$$
\begin{aligned}
z_m =&(-1)^{m+1}\int_{\Delta T}^{\Delta T+T_{ch}} \left[c^q(t-\Delta T)\right]^2 dt + \int_{\Delta T}^{\Delta T+T_{ch}} \tilde{n}(t)c^q(t-\Delta T)dt \\
&+(-1)^{m+1}\int_{\Delta T}^{\Delta T+T_{ch}} c^q(t-\Delta T)\tilde{n}(t-\Delta T)dt + \int_{\Delta T}^{\Delta T+T_{ch}} \tilde{n}(t)\tilde{n}(t-\Delta T)dt
\end{aligned}
\tag{17}
$$

where $\tilde{n}(t)$ and $\tilde{n}(t-\Delta T)$ denote the sample functions of filtered noise that corrupt the reference and information-bearing parts of the received signal, respectively.

In noise-free case $\tilde{n}(t) = 0$ and the binary modulation can be recovered by a comparator from the sign of observation signal, see the first term on the RHS of (17). Observe, TR autocorrelation receiver may be used even in a time-varying channel provided that the channel parameters vary slowly compared to the symbol rate.

4.3 Noise Performance of UWB TR Modulation Demodulated by an Autocorrelation Receiver

The observation variable of TR autocorrelation receiver is given by (17). To get the bit error rate (BER), its probability distribution has to be determined.

The observation variable contains four terms, the characteristics of these terms are evaluated independently of one another in [Kolumbán(2000)]. The closed-form expression given for the BER here is valid for both fixed and varying waveform communications. Only the BER expression and the conclusions are given here, for a detailed discussion refer to [Kolumbán(2000)].

The noise performance of a modulation scheme depends on the variance of observation variable z_m; the higher the variance, the worse the BER [Haykin(1994)]. In the general case the source of variance is twofold, the channel noise $n(t)$ and wavelet $c^q(t)$.

The *first term* on the RHS of (17) gives the half of energy per bit. In fixed waveform communications this term is a constant, while in varying waveform communications it may become a random variable. To minimize the contribution of this term to the variance of observation variable, the transmitted energy per bit E_b must be kept constant. However, the application of fixed basis functions is not a necessary requirement, the shape of wavelet $c(t)$ is irrelevant. Any bandpass signal with constant E_b can be used; the shape of basis functions may even vary from bit to bit.

In Section 1.2 the effects of cross- and autocorrelation estimation problems have been discussed. Recall, the source of estimation problems are (i) the non-orthogonal property of chaotic basis functions (cross-correlation estimation) and (ii) the varying energy per bit (autocorrelation estimation). In FM-DCSK [Kolumbán et al(1997b)] the orthogonality of TR basis functions (14) is assured by the Walsh functions, and E_b is kept constant for every transmitted bit by means of FM modulation. Consequently, the estimation problems do not appear in FM-DCSK.

The *second* and *third terms* on the RHS of (17) give the cross-products of filtered channel noise $\tilde{n}(t)$ and wavelet $c(t)$. Let the channel noise be modeled as a stationary zero-mean Gaussian process. Recognize, these terms can be interpreted as linear time-invariant (LTI) integral transformations of filtered channel noise, consequently, the outputs of these integral transformations are stationary random variables with Gaussian distributions.

Closed-form expressions are provided in [Bendat and Piersol(1966)] to determine the first and second moments of an LTI integral transformation. Since $n(t)$ has zero-mean, the output of LTI transformation is also a zero-mean process.

[Kolumbán(2000)] has shown that the variance of qth integral transformation is obtained as

$$\sigma_q^2 = \frac{N_0}{2} \int_{\Delta T}^{\Delta T + T_{ch}} [c^q(t - \Delta T)]^2 \, dt. \tag{18}$$

Note, except a constant, (18) is identical with the *first term* of (17). During the investigation of the *first term* we concluded that this term must be kept constant. If so then the variance of *second* and *third terms* becomes independent of the type of basis functions

$$\sigma_q^2 = \frac{N_0}{2} \frac{E_b}{2}.$$

A unique feature of TR modulation scheme implemented with an autocorrelation receiver is that it can operate with either fixed or varying basis functions. In AWGN channel the noise performance of a varying waveform communications system may reach, at the best, that of its fixed waveform counterpart, provided that the basis functions are othonormal [Kolumbán(2000)].

When the effect of basis functions is studied then the *fourth term* is irrelevant since it is independent of the basis functions. Results of detailed analysis have shown that the *fourth term* has the most significant contribution to the variance of observation variable and it is responsible for the relatively poor noise performance of autocorrelation receiver in AWGN channel compared to that of a coherent correlation receiver.

Starting from the results of [Gut(1972, in Russian)], the bit error rate of a TR autocorrelation receiver was derived in [Kolumbán(2000)]

$$\mathrm{BER} = \frac{1}{2^{2B\tau}} \exp\left(-\frac{E_b}{2N_0}\right) \sum_{i=0}^{2B\tau-1} \frac{\left(\frac{E_b}{2N_0}\right)^i}{i!} \sum_{j=i}^{2B\tau-1} \frac{1}{2^j} \binom{j+2B\tau-1}{j-i} \tag{19}$$

where τ denotes the observation time period of autocorrelation receiver. Recall, in a well designed receiver $\tau = T_{ch}$. This expression is valid in an AWGN channel for both the fixed and varying waveform communications systems, provided that the energy per bit E_b is kept constant when varying basis functions are used.

Equation (19) shows that the noise performance of TR systems depends on the product of $2B\tau$, that is, on the dimension of received signal space. Figure 9 shows that the TR autocorrelation receiver observes the received signal only over $\tau = T_{ch}$, consequently, the guard time has no influence on the noise performance. The signal dimension is obtained from (6) by substituting $T = 2\tau$.

Figure 10 shows the noise performance of TR autocorrelation receiver, where (from left to right) the signal dimensions $S_D = 4BT = 8B\tau$ are 8.5, 17, 34 and 68. The solid curves show the analytical predictions from (19), while the results of simulations are denoted by '+' marks.

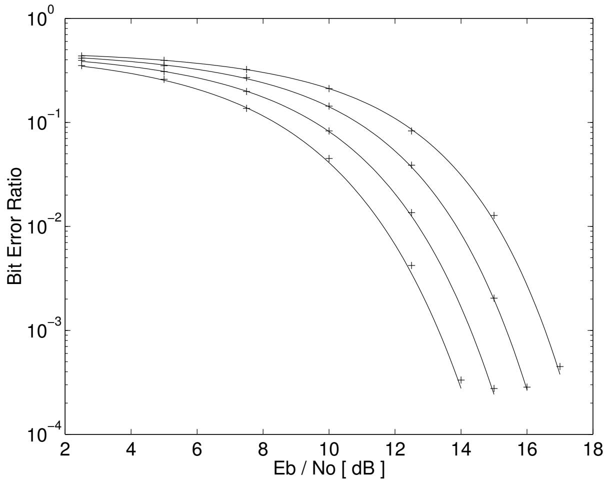

Fig. 10 Effect of signal dimension on the noise performance of TR autocorrelation receiver. From left to right the signal dimensions are 8.5, 17, 34 and 68.

Conclusions

Chaotic signals are ultra-wideband signals that can be generated with simple circuits in any frequency bands at arbitrary power level. The ultra-wideband property of chaotic carriers is beneficial in indoor and mobile applications where multipath propagation limits the attainable BER. Another possible application is the UWB radio, where the spectrum of transmitted signal covers an ultra-wide frequency band and the psd of transmitted UWB signal is so low that it does not cause any noticeable interference in the already existing conventional telecommunications systems sharing the same RF band. The UWB technology makes the frequency reuse possible.

The correlation receivers offer the best noise performance in AWGN channel. However, in case of channel distortion, the modulated signal has to be correlated with a reference signal distorted in the same manner as the modulated one to get the best BER. The TR modulation scheme solves this problem by transmitting the reference and information bearing wavelets via the same radio channel. The autocorrelation receiver determines the correlation of the two wavelets and uses the sign of correlation for making the decision.

This chapter provided a unified framework for modeling, performance evaluation, optimization and comparison of UWB radios using either impulses or chaotic waveforms as carrier. The Fourier analyzer concept introduced provides a mathematical framework for studying the UWB detection problem. The autocorrelation receiver was discussed in detail and an exact closed-form expression was provided for the prediction of its noise performance. Condition assuring the best bit error rate with chaotic UWB radio was also given.

Acknowledgments

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