Analytical Calculation of the Impedance of Lossy Power/Ground Planes

Bertalan Eged and László Balogh

***Abstract*—Power and ground planes are required to have low impedance over a wide range of frequencies. Parallel ground and power planes in multilayer printed-circuit boards exhibit multiple resonances, which increase the impedance. Dissipative loading of the radial transmission line structure of the planes reduces the resonance peaks. The dissipative loads can be realized by resistors distributed on the surface or the edges of plain pairs and lossy dielectric material can be applied for distributed loading, and the characterization of the impedance by simulation is very important during the design phase. Measurement results are compared with analytical solutions and simulation results. Furthermore, in this paper, a new algorithm is given for the more efficient calculation of the impedance of power distribution structures.**

***Index Terms*—Impedance calculation, lossy power/ground planes, resonance effect.**

I. Introduction

The faster the bus signaling, the faster are slopes and the transients in a power-distribution network. Therefore, a proportionally wider bandwidth is required. A high-end system today with single-ended signaling may have 10-A total transient current in the signal-return path of a bus, and may require 50-mV maximum ripple on the power-distribution network. This converts into 5 mΩ of required power-distribution impedance. With a $0.3 \ldots 0.6$ ns signal transition time, the necessary bandwidth for the power-distribution impedance amounts to 0.5–1 GHz. To avoid noise, the power-distribution network (i.e., the power-distribution planes) must exhibit low enough impedance over the full bandwidth of signals. Therefore, during the design phase a fast and accurate characterization of impedance should be done.

This paper reviews the known methods, and proposes a new and more effective one. Finally, simulation and measurement results are shown for validation.

II. Characterization of Power-Ground Distribution System

For digital electronics below the megahertz clock-frequency range, individual traces or metal bus bars were sufficient to distribute ground and power.

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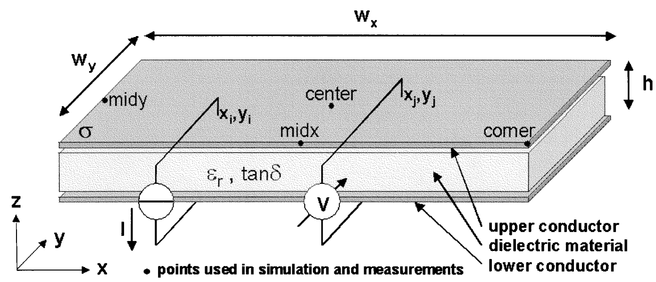

Fig. 1. Pair of parallel planes in the power distribution system (PDS).

Power and ground planes in a multilayer printed circuit board (PCB) of the high-speed application may be considered as two-dimensional transmission lines, where both the x and y dimensions are longer than one tenth of the shortest wavelength of interest. Throughout this paper, we assume that the separation h of the planes along the z axis is still negligible compared to the shortest wavelength.

In this structure, the different impedances are defined in the following way (see Fig. 1):

- the *impedance* of the power distribution system is defined by $Z_{i,j} = (V_{x_j,y_j})/(I_{x_i,y_i}),$;
- this is called *self impedance* if $i = j$, and *transfer impedance* otherwise.

A. Analytical Expression of Lossless Power-Ground Plane Impedances

In contrast to signal traces where the signal travels along the axis of the signal conductor, the wave generated by an injected signal between the planes launches a radially expanding wave. Two-dimensional (2-D) transmission lines are therefore also referred to as radial transmission lines. The self and transfer impedances of radial transmission lines with rectangular or circular shapes can be analytically calculated. Impedances of square-shaped parallel planes are widely analyzed in the literature for planar microwave circuits and printed antennas. Analytical formulation is given, e.g., in [1], [2]. Assuming infinitesimally small port sizes, open boundaries at the edges, and a pair of parallel, rectangular planes with side dimensions w_x and w_y along the x and y axes, with plane separation (dielectric height) of h along the z axis, the generalized transfer impedance between ports i and j (at coordinates x_i, y_i, and x_j, y_j respectively) can be written as [3]

$$Z_{i,j} = \frac{\mu h}{w_x w_y} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\chi_{mn}^2 jw}{k_m^2 + k_n^2 - k^2} C, \tag{1}$$

$$C = \cos k_m x_i \cos k_n y_i \cos k_m x_j \cos k_n y_j, \tag{2}$$

where m represents the mth mode associated with the x-dimensions, n represents the nth mode associated with the y-dimensions, k represents the real wavenumber for lossless case, $k = \omega\sqrt{\mu\epsilon}, k_m = m\pi w_x, k_n = n\pi w_y$. The constant $\chi_{mn} = 1$ for $m = 0$ and $n = 0, \sqrt{2}$ for $m = 0$ or $n = 0, 2$ for $m \neq 0$ and $n \neq 0$. In a low-loss case, k is complex: $k = k_r - jk_i$, where $k_r = k$ above and $k_i = (kr/2)(\tan(\delta) + (r/h))$, where $\tan(\delta)$ is the loss tangent of the dielectric, and r is the skin depth in the metal plane.

The analytical expression is not limited by finite spatial granularity like the transmission line grid model, but it has a double infinite series, which for practical calculations must be truncated, leading to an error in calculation [4]. The above $Z(jw)$ expression is well for numerical calculation, but is not adequate for circuit simulation where the planes are simulated with the whole part of electronic devices. For circuit simulations, either a macro model can be generated [5], or an electrical equivalent circuit of the pair of planes can be formed.

B. Transmission Line Grid Equivalent Circuit Model of the Ideal Power-Ground Plane

A pair of parallel planes can be simulated by an equivalent circuit of a grid of transmission lines, as described in e.g., [6], [7]. The low-frequency equivalent components of the planes can be derived from a quasistatic model. We assume a pair of rectangular planes with dimensions of w_x and w_y. First, we define the u size of the square unit grid cell, which should be equal to or less than 10% of the shortest wavelength of interest. The u cell size is selected such that we have an integer N_x and N_y number of cells along the x and y axes, respectively. For every unit square of the planes with side dimensions of u, plane separation (dielectric height) of h, the propagation delay $t_{\rm pd}$ along the edge of the unit cell and the static plane capacitance C can be calculated as

$$t_{\rm pd} = u\frac{1}{c}\sqrt{\epsilon_r} = u\sqrt{\epsilon_r}\sqrt{\epsilon_0\mu_0} \tag{3}$$

$$C = \frac{u^2}{h}\epsilon_r\epsilon_0. \tag{4}$$

From the capacitance and delay, an equivalent L inductance and Z_0 characteristic impedance of the unit cell can be calculated

$$L = \frac{t_{\rm pd}}{C} = h\mu_0 \tag{5}$$

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{h}{u}\frac{1}{\sqrt{\epsilon_r}}\sqrt{\frac{\mu_0}{\epsilon_0}}. \tag{6}$$

In the above expressions, all input and output parameters are in SI units, $\epsilon_0 = 8.86 \cdot 10^{-12}$ [(As/Vm)] is the dielectric constant in vacuum, $\mu_0 = 4 \cdot 10^{-7}$ [(H/m)] is the permeability of vacuum.

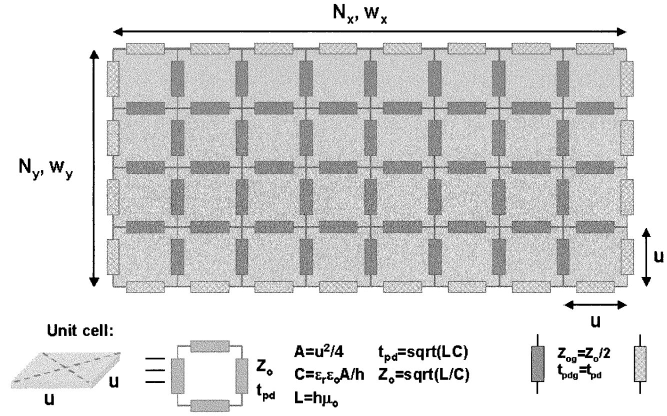

Fig. 2. Equivalent circuit representation of parallel conductive planes with a rectangular grid of transmission lines.

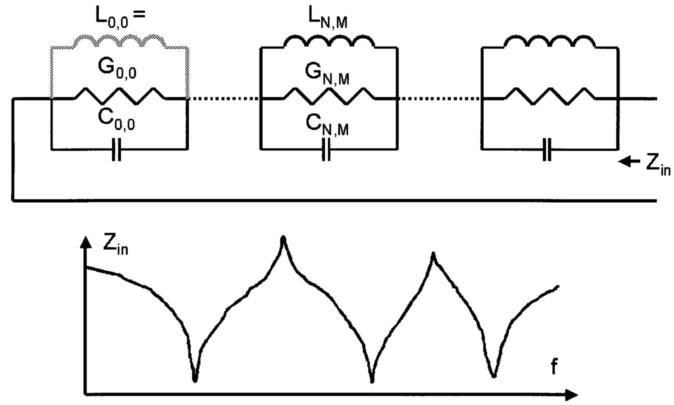

Fig. 3. Equivalent circuit describing the impedance of a given point of the plain.

The unit cells are replaced by four transmission lines along the edges of the unit cells, (Fig. 2), each transmission line representing the same delay and only one quarter of the area, thus having an impedance of $2Z_0$. Inside the equivalent grid, where the sides of unit cells meet, the capacitances of transmission lines are doubled, reducing the characteristic impedance to $(2Z_0/\sqrt{2})$. Whereas along the outer edges, the unit-cell transmission lines are standing alone. The parameters for the edge $(Z_{0e}, t_{\rm pde})$ and grid $(Z_{0g}, t_{\rm pdg})$ transmission lines are [8]

$$Z_{0g} = \sqrt{2}Z_0 \quad t_{\rm pdg} = \frac{t_{\rm pd}}{\sqrt{2}} \tag{7}$$

$$Z_{0g} = 2Z_0 \quad t_{\rm pde} = \frac{t_{\rm pd}}{\sqrt{2}}. \tag{8}$$

The correction factors $\sqrt{2}$ in the delays are used to match the equivalent circuit's delay along the x and y axes [9]. Alternative equivalent circuits may use lossless LC ladder [10] or lossy transmission lines [9], [11] to represent of each transmission-line segment. For all simulations presented in this paper, lossless transmission line grids were used.

Note that the grid takes account of some extent, the effect of the edge discontinuity by using twice the characteristic impedances of transmission lines along the edges. The transmission line model can be used easily for simulating the power planes and the other components of the circuit including the

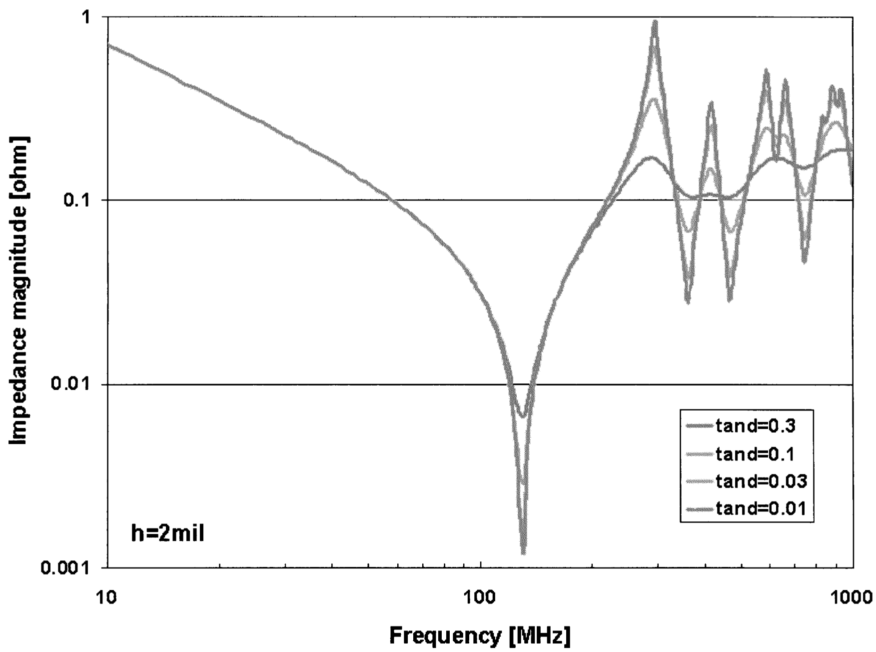

Fig. 4. Calculated impedance profile of a $10'' \times 10''$ plan pairs with different dielectric loss.

dissipative edge termination [11], which is used for reducing the effect of the resonance behavior. The price to be paid for this feature of the model is the spatial granularity of the transmission line grid.

III. Analytical Calculations of Lossy Plain Pairs

The calculation of the impedance profile in the case of lossy dielectric material can be based on the equivalent circuit of the impedance (Fig. 3) at a given point [1]. The equivalent circuit consists of an infinite number of serially connected resonators. The loss of the dielectric material is represented by the $G_{N,M}$ admittance in the circuit. The impedance elements of the equivalent circuit (rectangular plane with dimensions w_x, w_y and point of measurement x_i, y_i) are the following:

$$Z_{\text{in}}(f) = \sum_{n=0}^{N}\sum_{m=0}^{M} \frac{1}{j2\pi f C_{m,n} - j\frac{1}{2\pi f L_{m,n}} + G_{m,n}},$$

$$C_{m,n} = \frac{\epsilon_r \epsilon w_x w_y}{2h}\frac{1}{F_{m,n}},$$

$$L_{m,n} = \frac{2\mu h F_{m,n}}{w_x w_y \left(k_m^2 + k_n^2\right)},$$

$$G_{m,n} = \frac{2\pi f_{0m,n} C_{m,n}}{Q_0(f)},$$

$$F_{m,n} = \cos(k_m x_i)\cos(k_n y_i),$$

$$f_{0m,n} = \frac{\sqrt{\left(\frac{m}{w_x}\right)^2 + \left(\frac{n}{w_y}\right)^2}}{2\sqrt{\epsilon\mu}\sqrt{\epsilon_r}},$$

$$Q_0(f) = \frac{1}{Q_d(f)^{-1} + Q_c(f)^{-1}},$$

$$Q_d(f) = \frac{1}{\tan\delta} \quad Q_c(f) = \frac{h}{r(f)},$$

$$r(f) = \sqrt{\frac{2}{2\pi f 10^6 \mu\sigma}}.$$

The known impedance profile and its different characteristics have also been introduced, and [4] describes this type of equivalent circuit in detail. Calculation with the above described method was done for a $10''$ by $10''$ power/ground plane pair. The impedance profile at the corner of the plane was calculated with different dielectric losses. The results can be seen in Fig. 4. As it was pointed out in [12], the thickness of the dielectric material is important parameter to introduce a wide-band low impedance power distribution system. The impedance was calculated for different dielectric thickness, too. The results are in Fig. 5. and they are very similar to the simulated ones in [12].

IV. Effect of the Mode Number Truncation

In the practice for speeding up the calculations, the mode numbers are truncated. The effect of this truncation is reported and evaluated in [4]. The plane-impedance expression contains a double series of second-order terms. These terms accurately describe the poles (peaks) in the impedance profile, and the frequencies of the peaks do not change as we add or remove terms. The minimum of the impedance profile, however, does change as more terms are added to the series. More importantly, beyond the frequency of the last pole, the truncated series yields an impedance of capacitive downslope, as opposed to the inductive upslope of the plane impedance at high frequencies.

The truncation of the two dimensional modal space can be done in different ways. Some practical ways are illustrated in Fig. 6.

The published calculation methods use the rectangular truncation, but in [13] the elliptical truncation is used and the convergence of this method is reported. The main advantage of

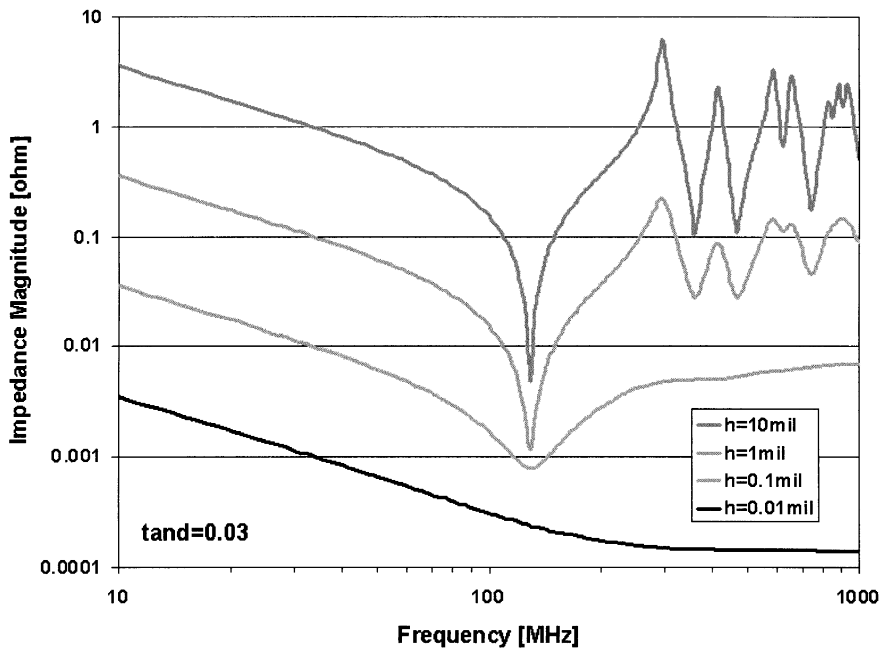

Fig. 5. Calculated impedance profile of a $10'' \times 10''$ plan pairs with different dielectric thickness.

the calculation compared to transmission line grid simulation is that the truncation of the double infinite series can be used. With a different truncation method, the speed of calculation can be increased proportionately to the area of the truncated modal space. However, using truncation, a trade-off has to be made between calculation time and accuracy.

The calculation program is based on the formulas of concentrated elements, lossy equivalent circuit. Rectangular, elliptic, linear, and inverse elliptic mode truncations were implemented, and the calculations of first resonant frequencies were evaluated regarding the speed and accuracy.

As it can be seen in Figs. 7 and 8, the differences of the frequencies of first resonants comparing to the rectangular mode truncation are less than 0.75%, 2.5%, 5.9% for elliptic, linear, and inverse elliptic truncations, respectively.

V. Measurement Results

To correlate calculated and measured impedances, a 10×10-in square pair of planes was selected with 31 mil FR4 dielectric material.

Self impedances were measured with HP8752A vector-network analyzer in the 1 MHz–1 GHz range at the center, corner and edge of the test board (Fig. 9). The probe connections, calibrations, and conversions from S parameters to impedances were according to [14].

To improve the measurement accuracy at low impedance readings, two-port S21-based self-impedance measurement was used. The S21 parameter readings were converted to self and transfer impedance values by the $Z = 25S_{21}$ approximative formula [14].

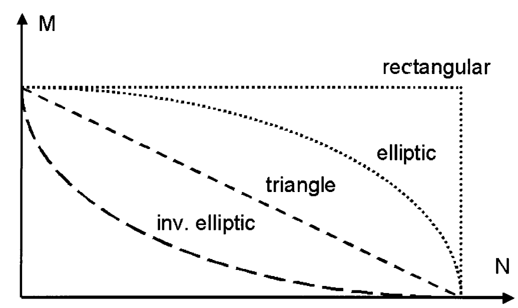

Fig. 6. Possible ways for truncating the mode numbers.

VI. Novelties

This paper introduces a new procedure for calculating the impedance in the case of lossy power/ground planes based on discrete component equivalent circuit. This method can be used for the calculations of distributed loss applied in the dielectric material and metallization, too.

The method can be used for speeding up the calculation by allowing different types of mode truncation. The speed and the error of the calculation of the resonance frequencies were evaluated and presented for rectangular and elliptic truncation of mode numbers. Simulation based on the discrete component equivalent circuit should be used to allow the usage of the mode truncation feature.

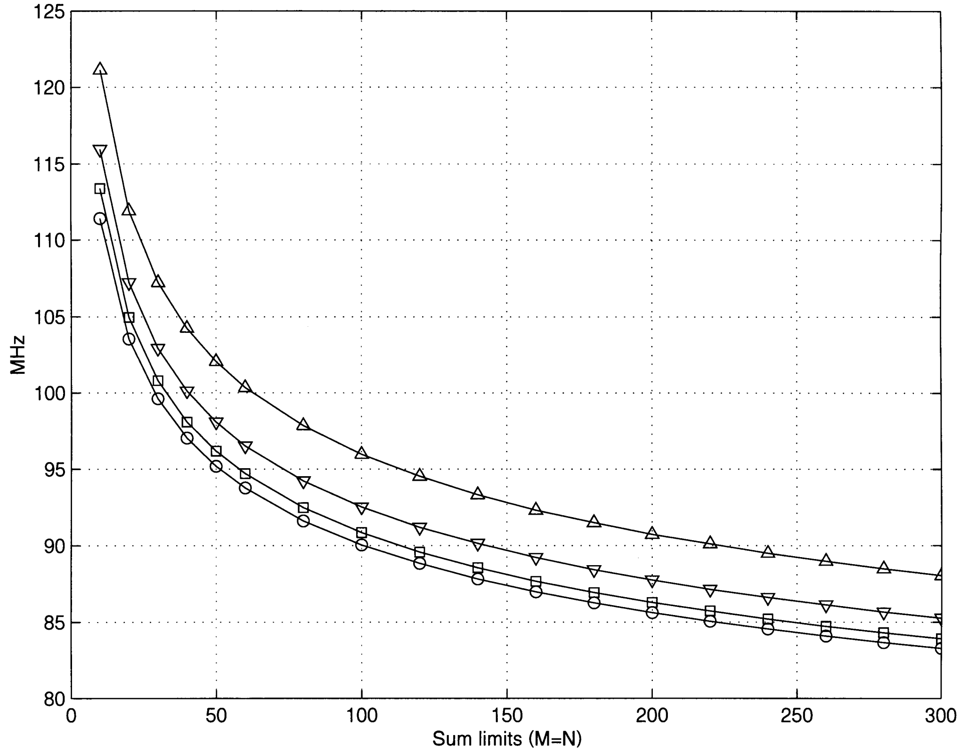

Fig. 7. Calculated first resonant frequency using different truncation modes (circle: rectangular, square: elliptic, triangle down: linear, triangle up: inverse elliptic).

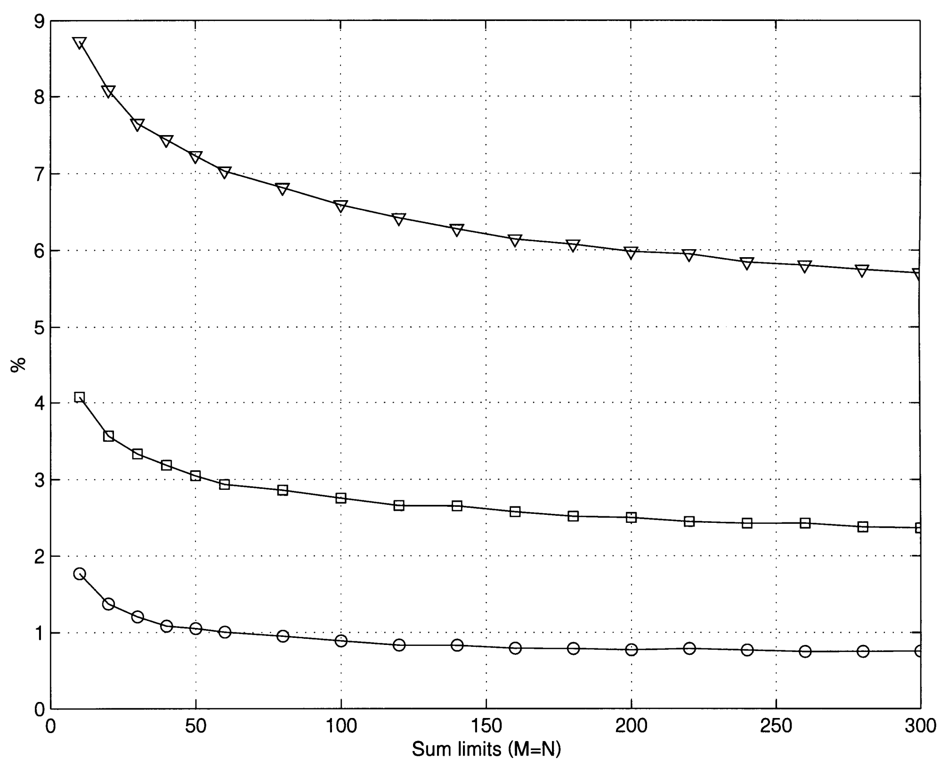

Fig. 8. Relative difference between resonant frequencies calculated with different truncations (circle: elliptic, square: linear, triangle down: inverse elliptic).

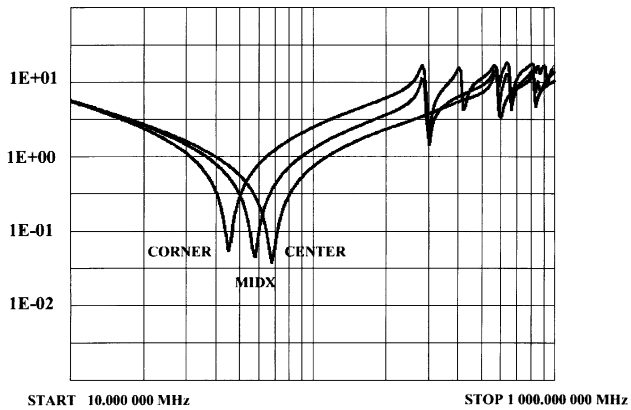

Fig. 9. Measure self impedances of the power plane.

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