IEEE Instrumentation and Measurement
Technology Conference
Budapest, Hungary, May 21–23, 2001

Analytical Calculation Of the Impedance of Lossy Power/Ground Planes

Bertalan Eged
Department of Microwave Telecommunications
Budapest University of Technology and Economics
Goldmann tr 3. 1111 Budapest, Hungary
Phone: + 36 1 463-3614, FAX: +36-1-463-3289, email:beged@mht.bme.hu

László Balogh
Department of Measurement and Information Systems
Budapest University of Technology and Economics
Phone: + 36 1 463-3583, email: lbalogh@mit.bme.hu

Abstract ***– Power and ground planes are required to have low impedance over a wide range of frequencies. Parallel ground and power planes in multilayer printed-circuit boards exhibit multiple resonances, which increase the impedance. Dissipative loading the radial transmission line structure of the planes reducing the peaks of the resonances. The dissipative loads can be realized by resistors distributed on the surface or edges of plain pairs and lossy dielectric material can be applied for distributed loading. Calculations based on the analytical method and measurements results are presented for comparing to the calculated and simulated impedances.***

Keywords ***– lossy power/ground planes, resonance effect, impedance calculation***

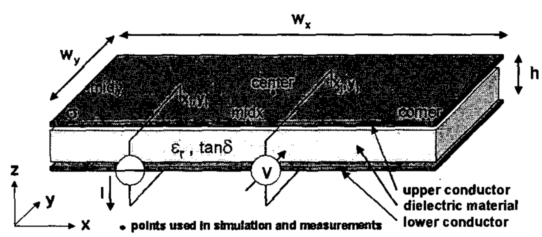

Fig. 1. A pair of parallel planes in the Power Distribution System (PDS)

I. INTRODUCTION

The faster bus signaling of high-speed digital interconnection comes with faster edges and transients, requiring a proportionally wider bandwidth in the power-distribution network. A high-end system today with single-ended signaling may have 10A total transient current in the signal-return path of buses, and may require 50mV maximum ripple on the power-distribution network. This converts into 5 milliohms of required power-distribution impedance. With a 0.3 ... 0.6nsec signal transition time, the necessary bandwidth for the power-distribution impedance is 0.5-1GHz. To avoid excessive simultaneous switching noise, the power-distribution network (the power-distribution planes) must exhibit low enough impedance over the full bandwidth of signals.

II. CHARACTERIZATION OF POWER-GROUND DISTRIBUTION SYSTEM

For digital electronics below the MHz clock-frequency range, individual traces or metal bus bars were sufficient to distribute ground and power.

Power and ground planes in a multilayer PCB of the high-speed application may be considered as two-dimensional transmission lines, where both the x and y dimensions are longer than one tenth of the shortest wavelength of interest. Throughout this paper we assume that the h separation of the planes along the z axis is still negligible compared to the shortest wavelength.

A. *Analytical expression of lossless power-ground plane impedances*

In contrast to signal traces where the signal travels along the axis of signal conductor, the wave generated by an injected signal between the planes launches a radially expanding wave. Two-dimensional transmission lines are therefore also referred to as radial transmission lines. The self and transfer impedances of radial transmission lines with rectangular or circular shapes can be analytically calculated. Impedances of square-shaped parallel planes are widely analyzed in the literature for planar microwave circuits and printed antennas. Analytical formulation is given, e.g. in [1], [2]. Assuming infinitesimally small port sizes, and open boundaries at the edges, the generalized transfer impedance between ports i and j (at coordinates x_i,y_i and x_j, y_j respectively) of a pair of parallel, rectangular planes with side dimensions w_x and w_y along the x and y axes, with plane separation (dielectric height) of h along

0-7803-6646-8/01/$10.00 ©2001 IEEE

the z axis, can be written as.

$$Z_{i,j} = \frac{\mu h \chi_{mn}^2}{w_x w_y} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{jw}{k_m^2 + k_n^2 - k^2} \quad (1)$$

$$C = \cos k_x x_i \cos k_y y_i \cos k_x x_j \cos k_y y_j \quad (2)$$

where m represents the mth mode associated with the x-dimensions,n represents the nth mode associated with y-dimensions, k represents the real wavenumber for lossless case, $k = \omega\sqrt{\mu\epsilon}$, $k_m = m\pi w_x$, $k_n = n\pi w_y$. The constant $\chi_{mn} = 1$ for $m = 0$ and $n = 0$, $\sqrt{2}$ for $m = 0$ or $n = 0$, 2 for $m \neq 0$,$n \neq 0$. When considering a low-loss case k is complex: $k = k_r - jk_i$, where $k_r = k$ above and $k_i = \frac{kr}{2}(\tan(\delta) + \frac{r}{h})$, where $\tan(\delta)$ is the loss tangent of the dielectric, r is the skin depth in the metal plane.

The analytical expression is not limited by finite spatial granularity like transmission line grid model, but is has a double infinite series, which for practical calculations must be truncated which leads to introducing error in calculation [3]. The above $Z_(jw)$ expression is well for numerical calculation, but not well suited for circuit simulation where the planes have to be simulated with the whole part of electronics. For circuit simulations, either a macro model can be generated [4], or an electrical equivalent circuit of the pair of planes is formed.

B. Transmission line grid equivalent circuit model of ideal power-ground plane

A pair of parallel planes can be simulated by an equivalent circuit of a grid of transmission lines, as described in e.g., [5], [6]. The low-frequency equivalent components of the planes can be derived from a quasi-static model. We assume a pair of rectangular planes with dimensions of w_x and w_y. First we define the u size of the square unit grid cell, which should be equal to or less than 10% of the shortest wavelength of interest. The u cell size is selected such that we have an integer N_x and N_y number of cells along the x and y axes, respectively. For every unit square of the planes with side dimensions of u, plane separation (dielectric height) of h, the t_{pd} propagation delay along the edge of the unit cell and the C static plane capacitance can be calculated.

$$t_{pd} = u\frac{1}{c}\sqrt{\epsilon_r} = u\sqrt{\epsilon_r}\sqrt{\epsilon_0\mu_0} \quad (3)$$

$$C = \frac{u^2}{h}\epsilon_r\epsilon_0 \quad (4)$$

From the capacitance and delay, an equivalent L inductance and Z_0 characteristic impedance of the unit cell can be calculated.

$$L = \frac{t_{pd}}{C} = h\mu_0 \quad (5)$$

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{h}{u}\frac{1}{\sqrt{\epsilon_r}}\sqrt{\frac{\mu_0}{\epsilon_0}} \quad (6)$$

In the above expressions, all input and output parameters are in SI units, $\epsilon_0 = 8.86 * 10^{-12}[\frac{\mathrm{As}}{\mathrm{Vm}}]$ the dielectric constant in vacuum, $\mu_0 = 4 * 10^{-7}[\frac{\mathrm{H}}{\mathrm{m}}]$ is the permeability of vacuum.

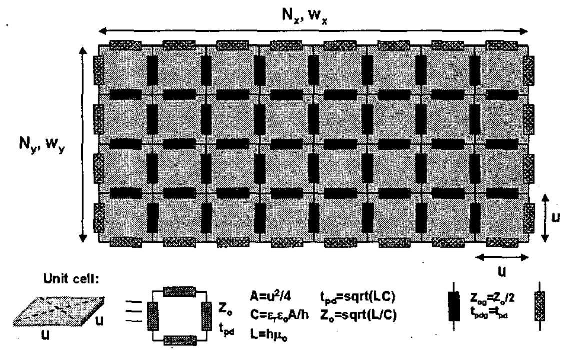

Fig. 2. The equivalent circuit representation of parallel conductive planes with a rectangular grid of transmission lines

The unit cells are replaced by four transmission lines along the edges of unit cells, (Figure 2), each transmission line representing the same delay but only one quarter of the area, thus having an impedance of $2Z_0$. Inside the equivalent grid, where the sides of unit cells touch, the capacitance of transmission lines are doubled, reducing the characteristic impedance to $\frac{2Z_0}{\sqrt{2}}$. Along the outer edges, the unit-cell transmission lines are standing alone. The parameters for the edge (Z_{0e}, t_{pde}) and grid (Z_{0g}, t_{pdg}) transmission lines are. ([7])

$$Z_{0g} = \sqrt{2}Z_0 \quad t_{pdg} = \frac{t_{pd}}{\sqrt{2}} \quad (7)$$

$$Z_{0g} = 2Z_0 \quad t_{pde} = \frac{t_{pd}}{\sqrt{2}} \quad (8)$$

The $\sqrt{2}$ correction factor in delays are used to match the equivalent circuit's delay along the x and y axes [8]. Alternative equivalent circuits may use lossless LC ladder [9] or lossy transmission lines [8], [10] representation of each transmission-line segment. For all simulations presented in this paper, lossless transmission line grids were used.

Note that the grid takes the effect of edge discontinuity into account to some extent by using twice the characteristic impedance of transmission lines along the edges. The transmission line model can be used easily for simulation of power planes and the other components of the circuit including the dissipative edge termination [10] used for reducing the effect of the resonance behavior. The price of this feature of the model is the spatial granularity of the transmission line grid.

III. ANALYTICAL CALCULATION OF LOSSY PLAIN PAIRS

The calculation of the impedance profile in the case of lossy dielectric material can be based on the equivalent circuit of the impedance (Figure x.) at a given point. [1] The equivalent circuit consist of infinite number of serial connected parallel resonator. The loss of the dielectric material is represented by the $G_{N,M}$ admittance in the circuit. The impedance and the

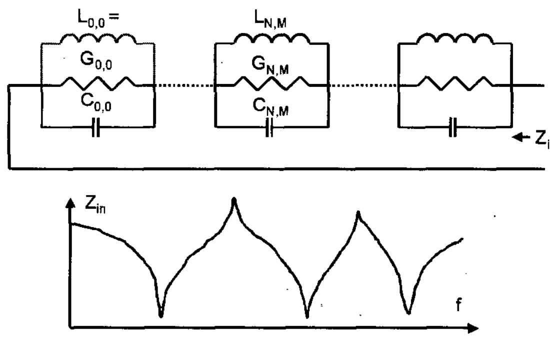

Fig. 3. The equivalent circuit describing the impedance of a given point of the plain

element of the equivalent circuit for rectangular shaped plane with dimensions w_x, w_y and point of measurement x_i, y_i are the following.

$$Z_{\text{in}}(f) = \sum_{n=0}^{N} \sum_{m=0}^{M} \frac{1}{j2\pi f C_{m,n} - j\frac{1}{2\pi f L_{m,n}} + G_{m,n}}$$

$$C_{m,n} = \frac{\epsilon_r \epsilon w_x w_y}{2h} \frac{1}{F_{m,n}}$$

$$L_{m,n} = \frac{2\mu h F_{m,n}}{w_x w_y \left(k_x^2(m) + k_y^2(n)\right)}$$

$$G_{m,n} = \frac{2\pi f_{0\,m,n} C_{m,n}}{Q_0(f)}$$

$$F_{m,n} = \cos\left(k_x(m)x_i\right)\cos\left(k_y(m)y_i\right)$$

$$f_{0\,m,n} = \frac{\sqrt{\left(\frac{m}{w_x}\right)^2 + \left(\frac{n}{w_y}\right)^2}}{2\sqrt{\epsilon\mu}\sqrt{\epsilon_r}}$$

$$Q_0(f) = \frac{1}{Q_d(f)^{-1} + Q_c(f)^{-1}}$$

$$Q_d(f) = \frac{1}{\tan\delta} \quad Q_c(f) = \frac{h}{r(f)}$$

$$r(f) = \sqrt{\frac{2}{2\pi f 10^6 \mu\sigma}}$$

The known impedance profile and its different characteristic introduced in [3] is exactly showed with this type of equivalent circuit. Calculation with the above described method was done for a 10" by 10" power/ground plane pairs. The impedance profile at the corner of the plane was calculated with different dielectric loss. The results can be seen on Fig. 4. As it was

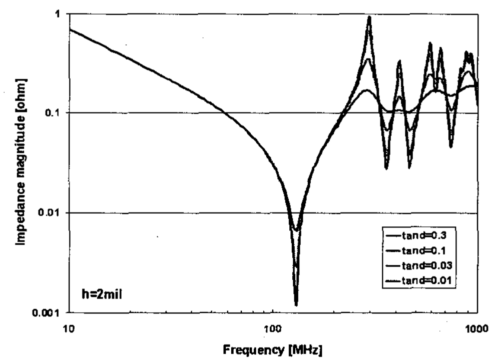

Fig. 4. Calculated impedance profile of a 10" by 10" plan pairs with different dielectric loss

pointed out in [11] the thickness of the dielectric material is important parameter to introduce a wide-band low impedance power distribution system. The impedance was calculated for different dielectric thickness too. The results are on Fig. 5. And they are very similar to the simulated ones in [11].

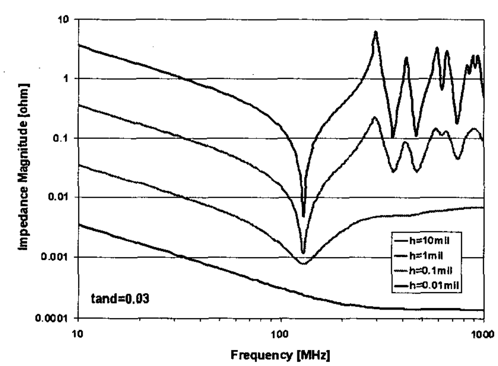

Fig. 5. Calculated impedance profile of a 10" by 10" plan pairs with different dielectric thickness

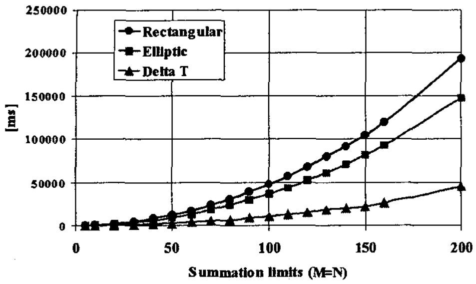

Fig. 6. The calculation time of frequency of the first resonant with rectangular and elliptic mode truncations

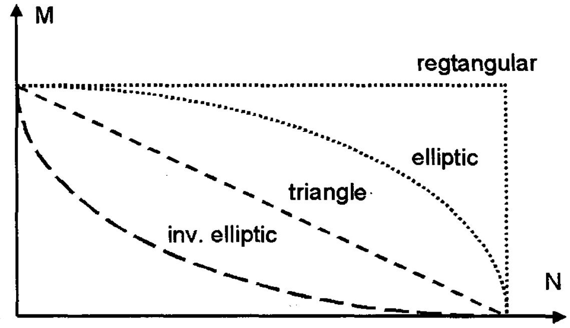

Fig. 8. Possible ways for truncating the mode numbers

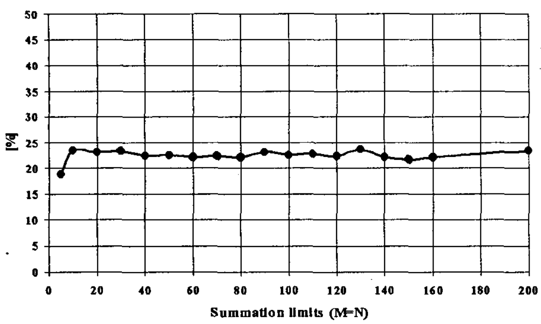

Fig. 7. Relative difference between the calculation time of the resonant using rectangular and elliptic truncation modes

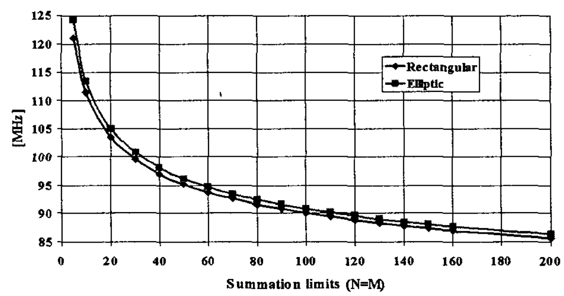

Fig. 9. Calculated first resonant frequency using different truncation modes

IV. EFFECT OF MODE NUMBER TRUNCATION

In the practice for speeding up the calculations the mode numbers are truncated. The effect of this truncation is reported and evaluated in [3]. The plane-impedance expression contains a double series of second-order terms. These terms accurately describe the poles (peaks) in the impedance profile, and the frequencies of the peaks do not change as we add or remove terms. The minima of the impedance profile, however, do change as more terms are added to the series. More importantly, beyond the frequency of the last pole of the truncated series, as opposed to the inductive upslope of the plane impedance at high frequencies, the truncated series yields an impedance of capacitive downslope.

The truncation of the two dimensional modal space can be done in different ways. Some practical ways are illustrated on Fig. 6.

The published calculation methods use the rectangular truncation, but in [12] the elliptical truncation is used and the convergence of this method is reported. The main disadvantage of the simulation of the lossy plains instead of calculation that there is the rectangular truncation of the double infinite series is available. With different truncation method the speed of calculation cab be increased.

The calculation program based on the formulas for lossy structure using rectangular and elliptical mode truncation was implemented and the speed and error in the calculation of frequencies was evaluated.

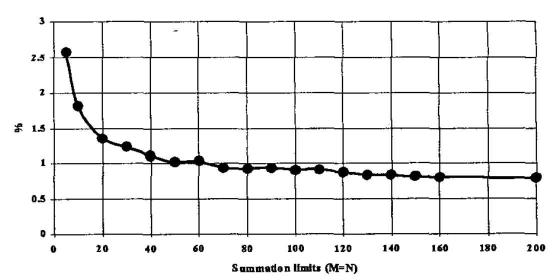

Fig. 10. Relative difference between resonant frequencies calculated with different truncations

As it can be see from the figures (Figure 7.-10.) the elliptical truncation of mode numbers speed up the calculation more than 20%. The difference of the resonant frequencies is less than

1% comparing to the rectangular mode truncation.

V. MEASUREMENT RESULTS

To correlate calculated and measured impedance, a 10 by 10 inch square pair of planes was selected with 31 mil FR4 dielectric material.

Self impedances were measured with HP8752A vector-network analyzer in the 1Mhz-1GHz range at the center, corner and edge of the test board (Figure 11). The probe connections, calibrations, conversions from S parameters to impedances were according to [13].

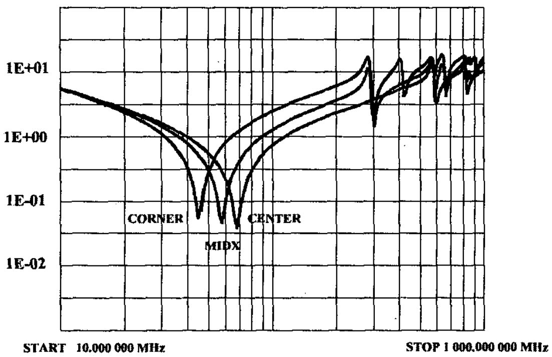

Fig. 11. Measure self impedances og the power plane.

To improve the measurement accuracy at low impedance readings, two-port S21-based self-impedance measurement was used. The S21 parameter readings were converted to self and transfer impedance values by the $Z = 25S_{21}$ approximative formula [13].

VI. NOVELTIES

The paper introduces a new procedure for calculating the impedance in the case of lossy power/ground planes based on discrete component equivalent circuit. This method can be used for calculation in distributed loss applied in the dielectric material and metallization too.

The method can be used for speeding up the calculation with enabling the usage of different type of mode truncation. The calculation speed and error difference in the calculation of resonance frequencies with the rectangular and elliptic truncation of mode numbers is evaluated and presented. Simulation based on the discrete component equivalent circuit should be used to enable the usage of the mode truncation feature.

References

[1] T. Miyoshi T. Okoshi. The planar circuit - an approach to microwave integrated circuit. *IEEE Transaction on Microwave Theory and Techniques*, 20(4):245–252, April 1972.

[2] J. W. Mink K. R. Carver. Microstrip antenna technology. *IEEE Transactions on Antennas and Propagation*, pages 2–24, January 1981.

[3] I. Novák. Accuracy considerations of power-ground plane models. In *Proceedings of the 8th Topical Meeting on Electrical Performance of Electronic Packaging*, pages 153–156, San Diego, CA, October 1999.

[4] Nanju Na. Madhavan swaminathan, "modeling and simulation of planes in electronic packages for ghz systems. In *Proceedings of the International Conference on Computer Design*, pages 149–152, October 1999.

[5] P. Bajor E. Leroux. Modeling of power planes for electrical simulations. In *Proceedings of the 1996 Wroclaw EMC Symposium*, pages 664–668.

[6] A. Barber K. Lee. Modeling and analysis of multichip module power supply planes. *IEEE Transactions on Components, Packaging, and Manufacturing Technology - Part B*, 18(4):628–639, November 1995.

[7] Larry D. Smith. Packaging and power distribution design considerations for a sun microsystems desktop workstation. In *Proceedings of the 6th Topical Meeting on Electrical Performance of Electronic Packaging*, pages 19–22, October 1997.

[8] K. Lee A. Barber H. H. Wu et al., J. W. Meyer. Accurate power supply and ground plane pair models. In *Proceedings of the 7 th Topical Meeting on Electrical Performance of Electronic Packaging*, pages 163–166, October 1998.

[9] D. W. Forehand C. B. O'Sullivan, L. D. Smith. Developing a decoupling methodology with spice for multilayer printed circuit boards. In *Proceedings of the 1998 International Symposium on Electromagnetic Compatibility*, pages 562–565, Denver, CO, Aug. 1998.

[10] I. Novák. Reducing simultaneous switching noise and emi on ground/power planes by dissipative edge termination. *IEEE Tr. CPMT*, 22(3):274–283, August 1999.

[11] I. Novák. Design considerations and modeling accuracy of lossy power ditribution networks. *Submitted for IEEE Transaction CPMT*, 23(3), August 2000.

[12] B. K. Gilbert G.-T. Lei, R. W. Techentin. High-frequency characterization of power/ground-plane structures. *IEEE Transaction on Microwave Theory and Techniques*, 47(5):562–569, May 1999.

[13] I. Novák. Probes and setup for measuring power-plane impedances with vector network analyzer. In *Proceedings DesignCon99, High-Performance System Design Conference*, Santa Clara, CA, 1999.