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Application of common-pole parallel filters to nonlinear models based on orthogonal functions

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ABSTRACT

Different nonlinear models are exploited to model real-world devices. Among them, an effective technique is based on the combination of orthogonal nonlinear functions and frequency-domain adaptive filtering algorithm for nonlinear system identification. In this paper, first the independence of the model from the orthogonal basis is demonstrated by complementing the previously obtained results. Then, a highly efficient model implementation is presented by taking advantage of fixed pole parallel filters for the linear filtering part. The efficiency comes both from using common-pole modeling and from applying a warped filter design that takes into account the frequency resolution of human hearing. Experimental results prove the effectiveness of the proposed approach showing its suitability in real-time digital simulation of nonlinear audio devices.

1. INTRODUCTION

System identification plays an important role in the field of digital audio systems. While linear systems can be fully characterized by their impulse responses (IRs) in the time domain and their transfer functions in the frequency domain, nonlinear models have to be introduced for modeling many real-world devices [1, 2, 3]. Considering this scenario, several efforts have been made in order to emulate the acoustic behaviour of nonlinear electroacoustic devices employing digital signal processing techniques. In the literature, a well-known technique is the dynamic convolution based on the application of signals with different levels to a device under test in order to derive the resulting IRs for each level [4, 5]. A low-cost implementation of dynamic convolution is discussed in [6] exploiting principal component analysis. A different approach is based on the diagonal Volterra kernels and it is typically used for weak nonlinearities with memory: the nonlinear system is described in terms of a simplified form of their Volterra series expansion discarding all multiplications with delayed samples [7]. A novel approach for the estimation of nonlinear systems has been proposed in [1] based on the introduction of suitable orthogonal nonlinear functions and frequency-domain adaptive FIR filtering algorithm. This is a black-box modeling where no a priori knowledge about the physical system is needed.

In this paper, first the approach based on suitable orthogonal nonlinear functions and frequency-domain adaptive FIR filtering algorithm is reviewed and its independence from the orthogonal basis is underlined by complementing the previous results obtained using the Legendre polynomials with novel results obtained using the Chebyshev polynomials. Then, an efficient implementation of the model is discussed based on common-pole parallel filters for the linear filtering part. Various specialized filter design methods have been developed for audio applications fitting the logarithmic frequency resolution of the human auditory system, including warped [8], Kautz [9], and parallel filters [10]. In this paper, parallel filters are utilized, increasing the efficiency significantly compared to FIR filters. Moreover, a further improvement derives from the common-pole model structure as previously discussed in [11]. Indeed, the choice of the same frequency resolution on each branch makes the pole positions the same for all parallel filters.

The paper is organized as follows. The proposed technique is described in Section 2 providing both the nonlinear system identification in Section 2.1 and the efficient model implementation in Section 2.2. Then, Section 3 depicts the tests carried out to illustrate the performance of the approach. Finally, concluding remarks are summarized in Section 4.

2. PROPOSED APPROACH

An efficient implementation of the nonlinear model proposed in [1] is described in this section exploiting common-pole parallel filters for the linear filtering

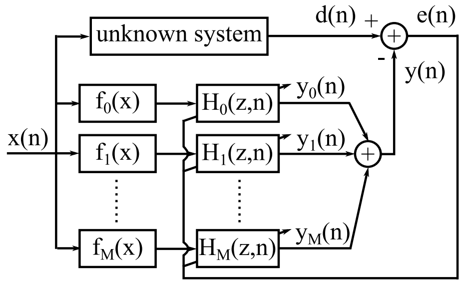

Fig. 1: Proposed system identification.

part. Therefore, the proposed approach can be summarized as follows. First, nonlinear system identification is performed using a model based on a polynomial structure. Then, its efficient implementation is obtained deriving the common-pole parallel filters. In the following, after a review of the nonlinear system identification technique, the model structure and the parameter estimation for the parallel filters are discussed in detail.

2.1. Nonlinear system identification

The nonlinear model previously proposed in [1] is based on a polynomial structure. More specifically, system identification is performed by means of suitable orthogonal nonlinear functions that allow to split the input signal into different branches and to perform a sort of linearization of the problem. In this way, linear system identification can be applied for each known nonlinearity. A schematic diagram of the proposed approach for system identification is reported in Fig. 1: the set of functions $f_0(x), \ldots, f_M(x)$ is a basis of orthogonal nonlinear functions applied to the input signal $x(n)$, where M is the highest order polynomial taken into consideration, while the set of FIR filters $H_i(z, n)$ is used to estimate the response to each known nonlinearity. The approach is independent from the basis of orthogonal nonlinear functions as it will be demonstrated in the following considering both the Legendre polynomials and the Chebyshev polynomials [2]. Regarding the Legendre polynomials, each polynomial is obtained by the following recursive equation:

$$L_{i+1}(x) = \frac{2i+1}{i+1} x L_i(x) - \frac{i}{i+1} L_{i-1}(x), \qquad (1)$$

with $L_0(x) = 1$ and $L_1(x) = x$, whereas for the Chebyshev polynomials, each polynomial is computed as follows:

$$C_{i+1}(x) = 2xC_i(x) - C_{i-1}(x), \quad (2)$$

with $C_0(x) = 1$ and $C_1(x) = x$. Regarding the FIR filters, it is well known that they are characterized by the following transfer function

$$H_i(z,n) = \sum_{m=0}^{N} h_{i_m}(n) z^{-i}, \quad (3)$$

where $h_{i_m}(n)$ for $m = 0, \ldots, N$ are the filter coefficients at time n.

Applying white noise to the unknown nonlinear system and to the set of orthogonal nonlinear functions, the estimated signal $y(n)$ is computed as the sum of the outputs of each branch

$$y(n) = \sum_{i=0}^{M} y_i(n) \quad (4)$$

and the residual error $e(n)$ is obtained from the knowledge of the real output signal $d(n)$, i.e., the signal processed by the nonlinear system to be identified, as follows

$$e(n) = d(n) - y(n). \quad (5)$$

Finally, the adaptation of the filter coefficients is performed using a multichannel frequency-domain adaptive filtering algorithm as described in [12], where the orthogonality among channels is ensured by the introduction of the orthogonal functions.

2.2. Efficient model implementation

Once the set of FIR filters $H_i(z)$ has been obtained through the aforementioned nonlinear system identification technique, an efficient model implementation is obtained using a common-pole parallel filter structure [11] as displayed in Fig. 2.

The parallel filter introduced in this paper is described by the following transfer function:

$$\hat{H}_i(z) = \sum_{k=1}^{K} \frac{B_{k,i}(z)}{A_k(z)} = \sum_{k=1}^{K} \frac{b_{k,i_0} + b_{k,i_1} z^{-1}}{1 + a_{k,1} z^{-1} + a_{k,2} z^{-2}}, \quad (6)$$

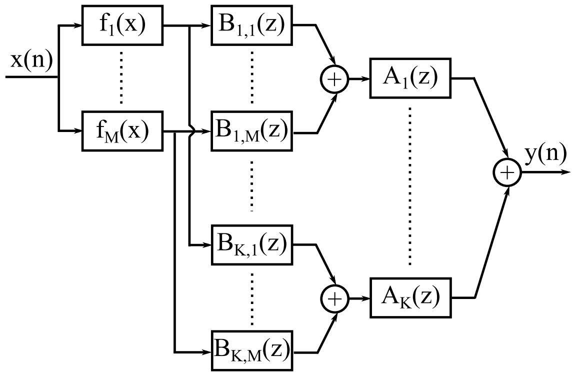

Fig. 2: Proposed efficient model implementation based on common-pole parallel filters.

being K the number of second-order sections. The frequency resolution is controlled by the pole frequencies. Therefore, parallel filters are well suited for audio application since frequency resolution can fit the resolution of human hearing. A further improvement derives from the common-pole model structure as previously discussed in [11]. Indeed, the choice of the same frequency resolution on each branch makes the pole positions the same for all the parallel filters. In this way, different first-order FIR filters $B_{k,i}(z)$ are used for each nonlinearity but common second-order allpole filters $A_k(z)$ are introduced and the contributions of these second-order sections are summed together to provide the signal $y(n)$. This structure is summarized in Fig. 2.

The first step of filter design is the determination of the common poles. Since the goal is that of considering the resolution of the human auditory system, the pole frequencies are computed in the warped domain based on the estimated responses $H_i(z,n)$ with $i = 1, \ldots, M$ using a common-pole autoregressive filter. The regression error for the i-th warped impulse responses $\tilde{h}_i(n)$ is given by the following equation:

$$E_i = \sum_{n=L}^{N} \left(\tilde{h}_i(n) + \sum_{l=1}^{L} a_m \tilde{h}_i(n-l) \right)^2, \quad (7)$$

being L the denominator order and N the warped impulse response length. Then, the common set of denominator coefficients a_m is derived minimizing

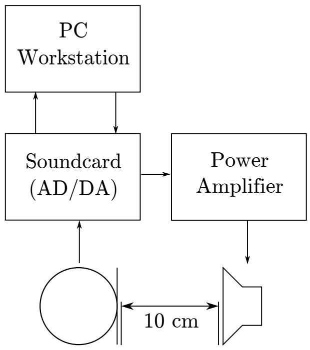

Fig. 3: Measurement setup used for the identification of the loudspeaker model.

the total error

$$e = \sum_{i=1}^{M} W_i E_i, \qquad (8)$$

where W_i is the weight given to the separate impulse responses. This is a linear least-squares (LS) problem solved by the normal equations in a closed form [11]. Finally, the roots $\tilde{p}_k$ of the denominator are found and dewarped using the following equation:

$$p_k = \frac{\tilde{p}_k + \lambda}{1 + \lambda \tilde{p}_k}. \qquad (9)$$

Once the common poles are obtained, the numerator coefficients b_m are derived by solving the normal equations in a closed form [11].

When using normal parallel filters, i.e., not common-pole implementation, and assuming a model with M branches (Fig. 2) and N-th order parallel filters, the total required filter order is MN. The common-pole structure further decreases the computational complexity to $(M/2 + 0.5)\,N$, i.e., the computational complexity is roughly halved with respect to the straightforward implementation.

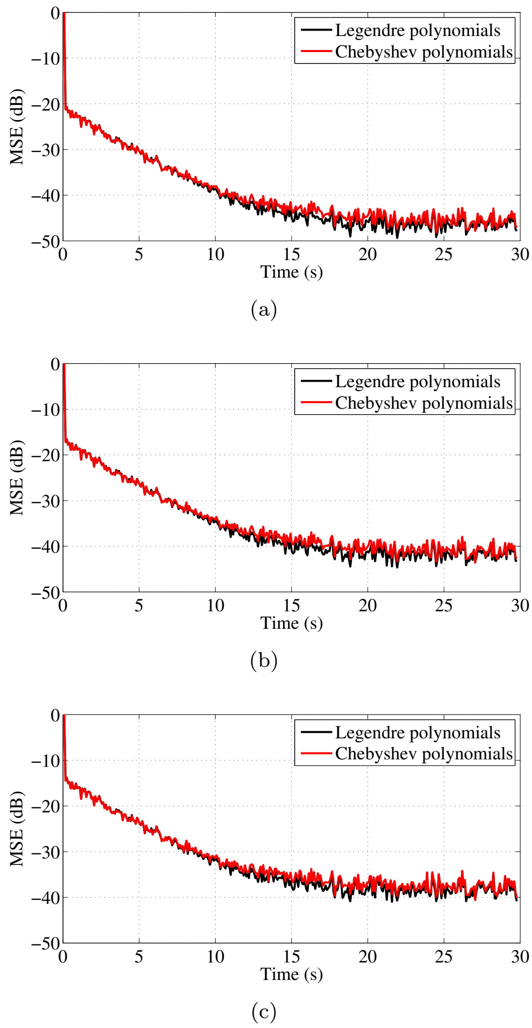

Fig. 4: MSE resulting from the loudspeaker model estimation using both the Legendre and the Chebyshev polynomials as orthogonal functions. (a) RMS input voltage 3 V. (b) RMS input voltage 5 V. (c) RMS input voltage 7 V.

3. EXPERIMENTAL RESULTS

The presented method is suitable for performing real-time digital simulation of nonlinear audio devices due to its high computational efficiency. As an example, its effectiveness in estimating the distortion produced by a real loudspeaker has been evaluated. First, the nonlinear model is obtained using the system identification method discussed in Section 2.1. Then, the common-pole set is obtained by the warped common-pole autoregressive method as previously presented in Section 2.2. The results are shown in terms of mean square error (MSE) and accuracy of the response of the loudspeaker model to a sinusoidal input signal with respect to the recorded output of the loudspeaker. Moreover, for the sake of completeness, the estimated and modeled magnitude frequency responses are reported, considering a 1024 tap FIR and 40th order IIR, to prove the

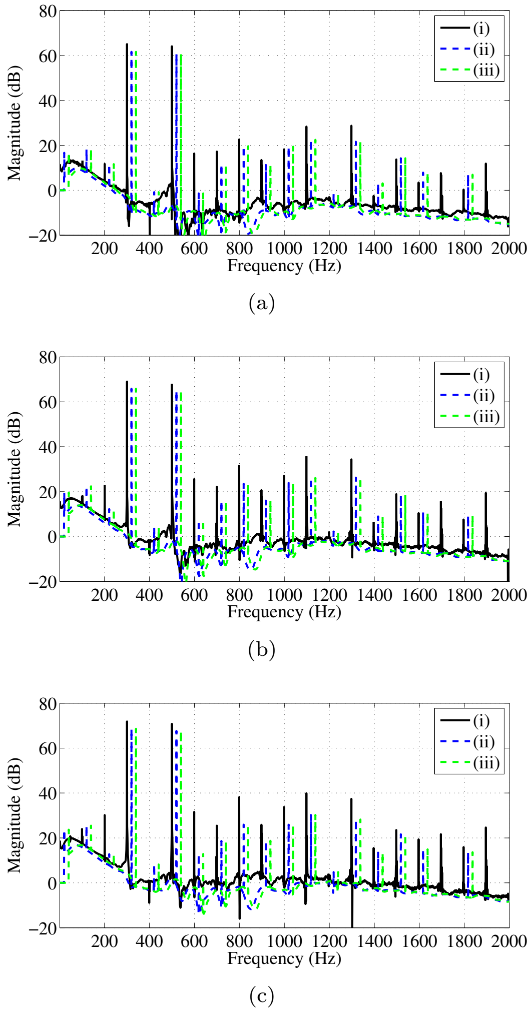

Fig. 5: Spectrum of the measured output signal (i), of the output signal estimated using Legendre polynomials and 1024 tap FIR filters (ii), and of the output signal modeled using Legendre polynomials and 40th order parallel filters (iii) obtained applying to the loudspeaker a two-tone input signal with $f_1 =$ 300 Hz and $f_2 =$ 500 Hz. (a) RMS input voltage 3 V. (b) RMS input voltage 5 V. (c) RMS input voltage 7 V.

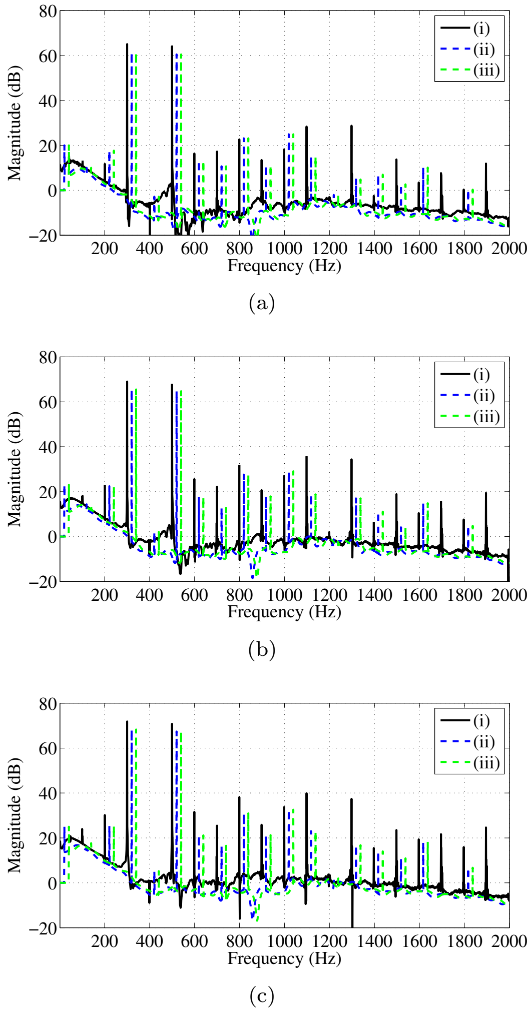

Fig. 6: Spectrum of the measured output signal (i), of the output signal estimated using Chebyshev polynomials and 1024 tap FIR filters (ii), and of the output signal modeled using Chebyshev polynomials and 40th order parallel filters (iii) obtained applying to the loudspeaker a two-tone input signal with $f_1 =$ 300 Hz and $f_2 =$ 500 Hz. (a) RMS input voltage 3 V. (b) RMS input voltage 5 V. (c) RMS input voltage 7 V.

effectiveness of the efficient model implementation.

The identified loudspeaker is a 5″ driver characterized by a nominal impedance of 4 Ω and an AES power handling of 80 W, driven by a professional power amplifier (Alesis RA-100). On-axis responses have been recorded with an AKG condenser microphone (C-480B) at a distance of 10 cm and sampled with a professional MOTU sound card, using the test setup scheme shown in Fig. 3. Three different setups have been evaluated with the speaker driven with RMS input voltage 3, 5, and 7 V to vary the strength of the nonlinearity introduced by the loudspeaker. Considering a nominal impedance of 4 Ω, these voltages correspond to a power of 2.25, 6.25, and 12.25 W.

Regarding nonlinear system identification, white noise sampled at $f_s = 48$ kHz has been used as input signal and the maximum polynomial order has been set to 4. The frequency-domain adaptive filtering algorithm [12] has been applied, using a forgetting factor for the step-size normalization equal to 0.1, a fixed convergence speed $\mu = 0.01$ for the linear polynomial with an exponential decay for higher orders polynomials, a regularization constant $\epsilon = 10^{-5}$, and adaptive filters of length 1024 samples. Fig. 4 shows the MSE resulting from the loudspeaker model estimation considering the three different RMS input voltages, i.e., 3 V as reported in Fig. 4(a), 5 V as reported in Fig. 4(b), and 7 V as reported in Fig. 4(c). The nonlinear model identification has been performed using both the Legendre polynomials $L_i(x)$ and the Chebyshev polynomials $C_i(x)$ as orthogonal nonlinear functions $f_i(x)$. The obtained results show that, as expected, only negligible differences can be observed since the method is conceived to work with any orthogonal nonlinear functions.

Then, the common-pole set is obtained by the warped common-pole autoregressive method discussed in Section 2.2. with $\lambda = 0.95$ and filter order 40. The common poles are used as the denominators of the parallel filters. The numerator coefficients are obtained by minimizing the MSE between the impulse responses of the parallel filters and the responses estimated by the method of Section 2.1. The accuracy of the technique has been evaluated considering the response of the loudspeaker to a two-tone input signal with $f_1 = 300$ Hz and $f_2 = 500$ Hz with RMS input voltage equal to 3, 5, and 7 V. In particular, the measured output spectrum of the loudspeaker is displayed in curve (i), the estimated output spectrum using the Legendre polynomials and 1024 tap FIR filters is displayed in curve (ii), and the output spectrum modeled using the Legendre polynomials and 40th order IIR filters is displayed in curve (iii) of Fig. 5. Analogous curves are reported in Fig. 6 but using the Chebyshev polynomials for the estimation. As expected, the results are consistent with those obtained in Fig. 4 in terms of MSE. Indeed, the estimated impulse responses provide an adequate model of the real loudspeaker as it can be observed by comparing curves (i) and (ii) in Fig. 6. Moreover, no evident differences can be noted by comparing curves (ii) and (iii), proving that the common-pole filters provide a satisfying model of the estimated impulse responses.

Table 1: Total required filter order considering the FIR filters, the straightforward parallel filters and the common-pole parallel filters.

Filter structure	Filter order
FIR filters	1023
straightforward parallel filters	160
common-pole parallel filters	100

For the sake of completeness, the magnitude frequency responses of the estimated filters $H_i(z)$ and the parallel filters $\hat{H}_i(z)$ are reported in Fig. 7 using the Legendre polynomials and in Fig. 8 using the Chebyshev polynomials. It results that the 40th order common-pole filters model the estimated responses quite precisely. The figures are arranged as follows: the figures on the same row are related to the same polynomial order while the figures on the same column are related to the same input voltage. These results are consistent with those reported in Figs. 5 and 6 confirming that the efficient model implementation provides an acceptable model of real-world devices.

Finally, Table 1 summarizes the computational saving obtained using the common-pole implementation for the scenario assumed in the simulation. It results that the common-pole structure provides the reduction of the computational almost of a factor 2 with respect to the straightforward parallel filters implementation. Moreover, it is worth noting that the FIR filter length was set to 1024 in [1], thus proving the efficiency of the common-pole structure.

4. CONCLUSION

In this paper, the approach based on suitable orthogonal nonlinear functions and frequency-domain adaptive FIR filtering algorithm has been reviewed showing its independence from the orthogonal basis. In particular, the previous results obtained using the Legendre polynomials have been complemented with

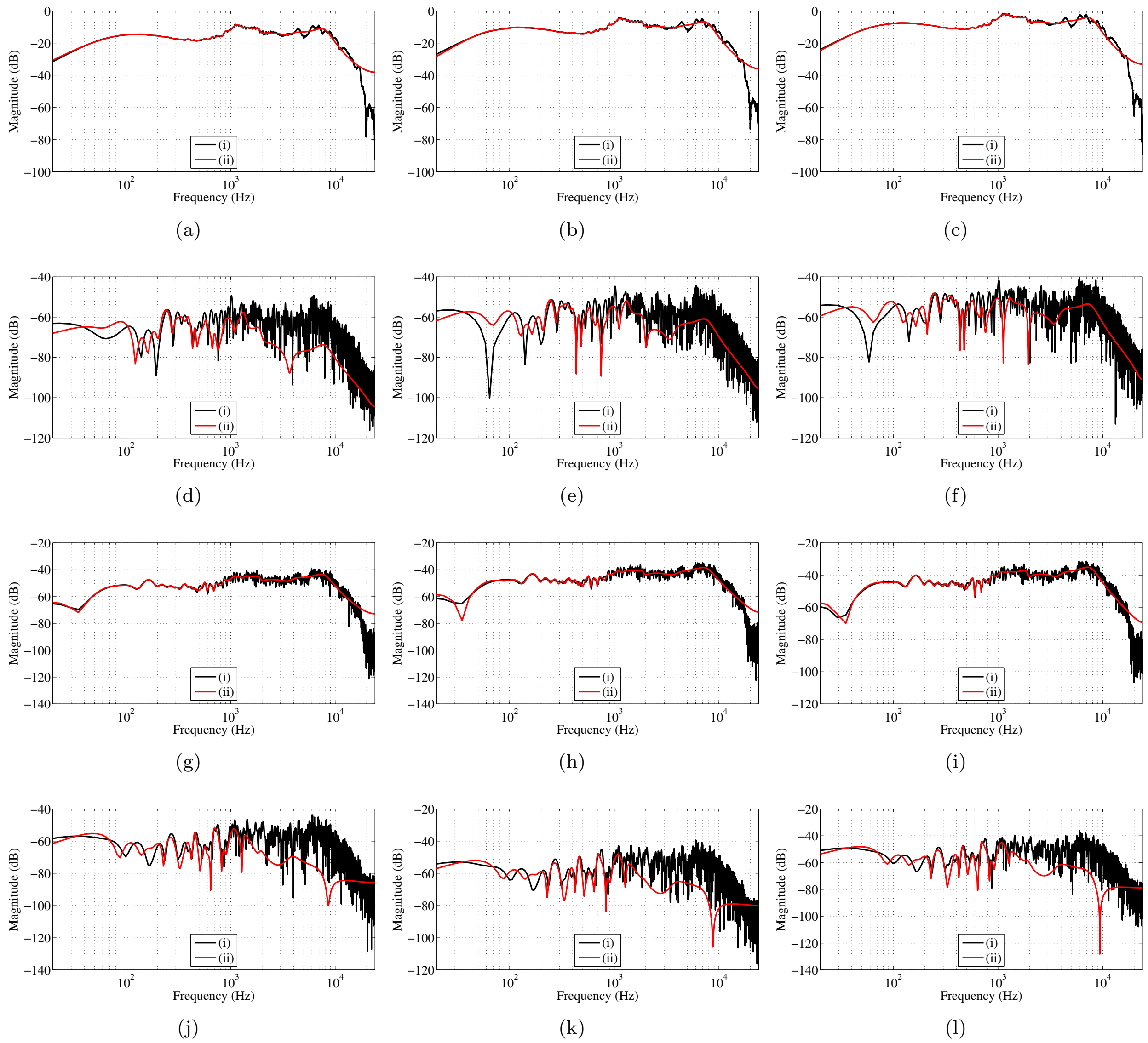

Fig. 7: Estimated (i) and modeled (ii) magnitude frequency responses using 40th order common-pole parallel filters. (a)-(c) Legendre polynomial $L_1(x)$ for RMS input voltage 3, 5, and 7 V. (d)-(f) Legendre polynomial $L_2(x)$ for RMS input voltage 3, 5, and 7 V. (g)-(i) Legendre polynomial $L_3(x)$ for RMS input voltage 3, 5, and 7 V. (j)-(l) Legendre polynomial $L_4(x)$ for RMS input voltage 3, 5, and 7 V. Note different scales on the y axis.

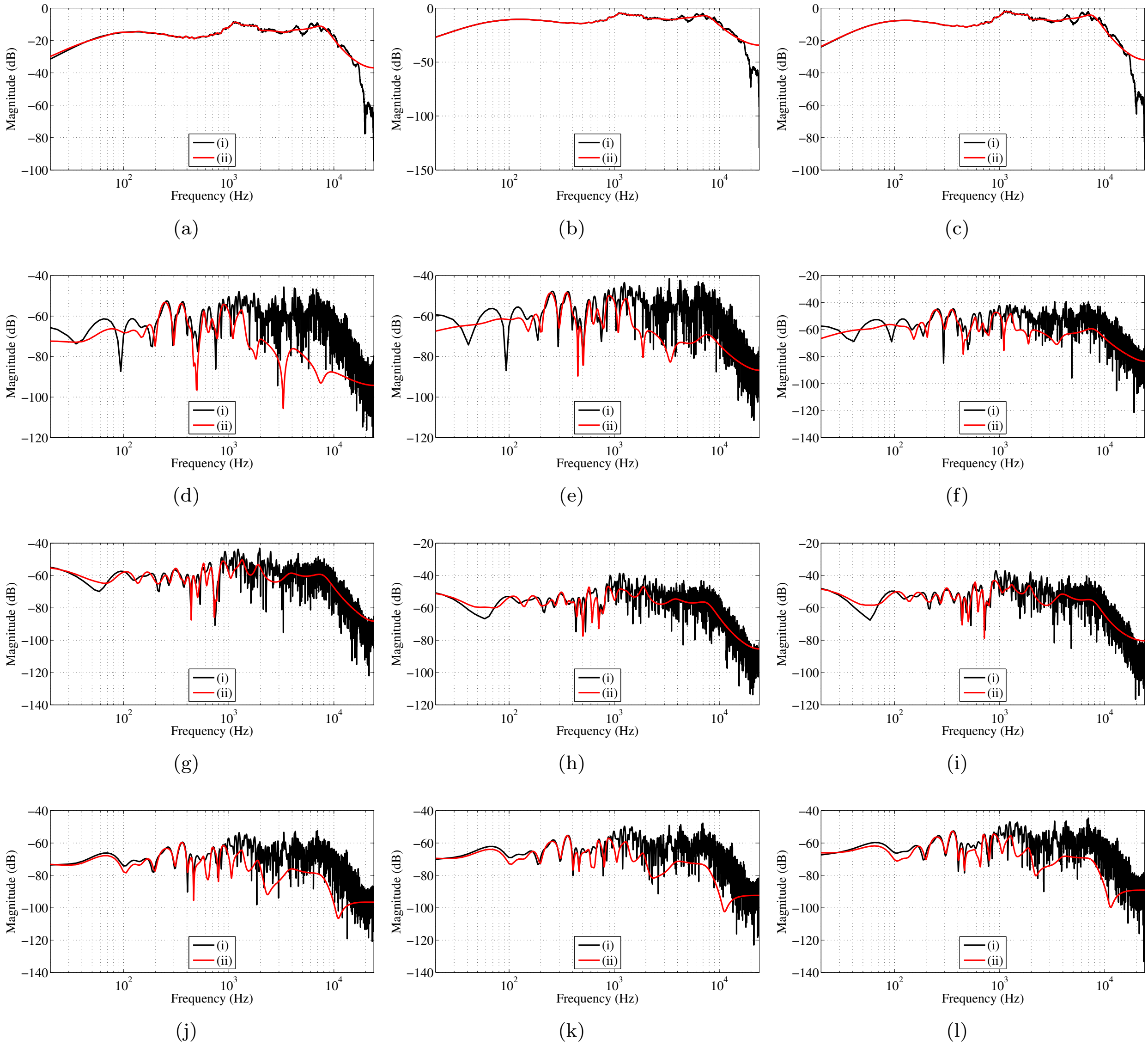

Fig. 8: Estimated (i) and modeled (ii) magnitude frequency responses using 40th order common-pole parallel filters. (a)-(c) Chebyshev polynomial $C_1(x)$ for RMS input voltage 3, 5, and 7 V. (d)-(f) Chebyshev polynomial $C_2(x)$ for RMS input voltage 3, 5, and 7 V. (g)-(i) Chebyshev polynomial $C_3(x)$ for RMS input voltage 3, 5, and 7 V. (j)-(l) Chebyshev polynomial $C_4(x)$ for RMS input voltage 3, 5, and 7 V. Note different scales on the y axis.

novel results obtained using the Chebyshev polynomials. Then, an efficient implementation of the technique has been derived exploiting a parallel filters structure. The efficiency comes from the introduction of parallel filters and a further improvement has been obtained using a common-pole model structure. Experimental results have proved the effectiveness of the approach in terms of MSE and accuracy of a real loudspeaker response to a two-tone sinusoidal input signal. Future work could be oriented towards a level independent nonlinear model derivation in order to obtain a technique working for any level of the input signal.

5. ACKNOWLEDGEMENT

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