EFFICIENT PHYSICS-BASED SOUND SYNTHESIS OF THE PIANO USING DSP METHODS

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ABSTRACT

In the recent years, digital waveguide modeling of musical instruments has proven to be an effective tool for sound synthesis purposes, but some practical questions still have remained unanswered. In this paper a new equivalent structure of the digital waveguide for string synthesis is presented. This structure can be used for highly efficient modeling of beating and two-stage decay, an important characteristic of the piano sound. The complexity of the traditional structure can be reduced by replacing one of the string models with a resonator bank.

1 INTRODUCTION

Physical modeling has gained more and more interest in the last decade. In contrast to traditional synthesis methods, it concentrates on the sound producing structure instead of modeling the resulting sound signal itself. This results in very efficient and realistic control of the model parameters, e.g., velocity of the piano key.

The piano can be decomposed into three functional parts. The first part is the excitation. It includes the hammer strike. The second is the string, which determines the frequencies of the harmonics and acts as a filter with a very low damping factor. The string signal is filtered through the radiator, which is the third part of the system and simulates the soundboard of the piano. The interaction between the string and the hammer is bidirectional, since the hammer force depends on the displacement of the string as well [1]. In this paper we focus on the modeling of string behavior.

Smith and Van Duyne used commuted waveguide synthesis for physical modeling of piano sound [2], [3]. Borin, Rocchesso, and Scalcon presented a digital waveguide model with nonlinear excitation [4]. A different approach was taken by Laroche and Meillier [5]. In their implementation a precomputed common excitation signal was filtered through second-order resonators.

This paper first describes the principles of digital waveguide modeling, a commonly used method for string simulation. After presenting the basic idea of beating and two-stage decay, a novel resonator-based structure for their simulation is proposed. The paper shows the abilities of the method with simulation examples. Summary and future plans conclude the paper.

2 MODELING THE STRING BEHAVIOR

A powerful approach for modeling the string and the acoustical tube was proposed by Smith [6]. The method called digital waveguide modeling is based on the time-domain solution of the one-dimensional wave equation. The solution can be presented as the sum of two waves traveling in opposite directions:

$$y(x,t) = f^{+}(x-ct) + f^{-}(x+ct) \quad (1)$$

where x denotes the spatial coordinate, t is the time, c is the propagation speed, and y refers to the transverse displacement of the string. In Eq. (1) f^{+} and f^{-} are the traveling wave components and they can be any twice differentiable functions [7].

Spatial and time-domain sampling of Eq. (1) results in a simple delay-line representation. In the case of an ideal string with perfectly rigid terminations, the two traveling wave components form two separate delay lines connected by multipliers (-1) realizing the inverting reflection at the boundary. Modeling a lossy, dispersive, and non-rigidly terminated string can be also easily achieved using a digital filter such as $H_r(z)$ in Fig. 1. If the linearity and time-invariance of the string is assumed, all the distributed losses and dispersion can be consolidated to one end of the digital waveguide [6]. In the case of one polarization of a piano string, the system takes the form shown in Fig. 1, where M is the length of the string in spatial sampling intervals and M_{in} denotes the position of the force input.

For a given input F_{in} the force F_{out} at the bridge (termination) can be computed as follows:

$$\frac{F_{out}}{F_{in}} = \frac{1}{1 - z^{-2M} H_r(z)} \left(1 - z^{-2M_{in}}\right) z^{-(M-M_{in})} \quad (2)$$

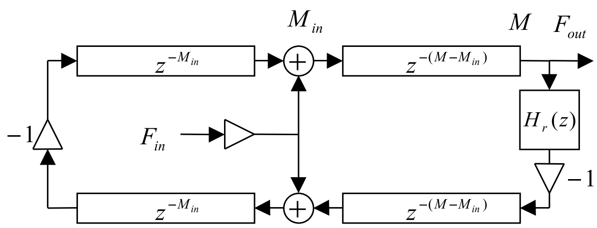

Figure 1: Digital waveguide model of a piano string with one polarization of vibration.

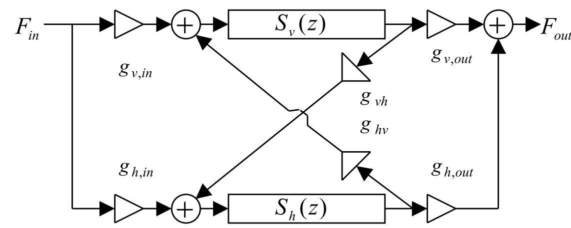

Figure 2: Parallel digital waveguide implementation of two-stage decay.

In this model, $H_r(z)$ is responsible for all the losses and dispersion. This structure can be viewed as a filter with a positive feedback loop, which emphasizes those frequencies where the denominator approaches zero.

Because of practical reasons, it is worthwhile to separate the loop filter $H_r(z)$ into two parts [6]. The part that is responsible for the dispersion can be realized as an allpass filter. The other part, introducing the losses, has an almost linear phase response. Methods for dispersion-filter design can be found in [8] and [9]. In the case of low piano strings, typically 20th order allpass filters are needed for good results, in contrast to the low-order loss filters.

3 BEATING AND TWO-STAGE DECAY

Since two or three slightly mistuned strings are sounding together when a single piano key is depressed (except for the lowest octave), a complicated modulation of the amplitudes is brought about. Weinreich [10] studied the behavior of two coupled strings and found that the normal modes of vibration are two exponentially decaying sinusoids for each harmonic frequency. All the frequency, initial amplitude, initial phase and decay time parameters can be different for these two modes. Depending on these differences, two-stage decay and beating can appear in the sound.

Two-stage decay means that in the early part of the tone the decay rate is higher than in the latter. Beating is an amplitude modulation, which is superimposed on the exponential decay. Another reason for the two-stage decay is the different decay rate of the horizontal and vertical polarizations of vibration [10].

Most piano keys correspond to three slightly mistuned strings, all with two polarizations, which would correspond to six normal modes. However, Weinreich's model [10] with two exponentially damping sinusoids was found to be a good approximation in describing the evolution of these complicated envelopes.

These effects can be taken into account by using two separate digital waveguides with different parameters [11], but it raises the computational complexity by a factor of two as well (see Fig. 2). Another problem with that approach is that the characteristic of the decay will be similar for all the harmonics, which is not found in real piano sounds.

4 THE NOVEL STRUCTURE

A new method is presented here which is based on the equivalent resonator structure of the digital waveguide in [12]. As shown there, Eq. (2) can be rewritten in the following way:

$$\frac{F_{out}}{F_{in}} = \frac{1}{N}\left\{\frac{a_1}{1-z^{-1}p_1} + \ldots + \frac{a_N}{1-z^{-1}p_N}\right\} z^{-M}$$
$$p_n = r_n e^{-j\vartheta_n} \quad (3)$$

where $N = 2M$, a_k are the complex amplitudes of the resonators forming conjugate pairs, ϑ_k refer to the pole frequencies, and r_k to the pole radii. The complex conjugate pole-pairs of Eq. (3) can be realized using second-order resonators. Their transfer function is of the form:

$$R(z) = \frac{2Re\{a_{2k}\} - 2Re\{a_{2k}\overline{p_{2k}}\}z^{-1}}{1 - 2Re\{p_{2k}\}z^{-1} + p_{2k}\overline{p_{2k}}z^{-2}}$$
$$p_{2k} = \overline{p_{2k+1}} \qquad a_{2k} = \overline{a_{2k+1}} \quad (4)$$

As the two-stage decay of the piano sound is mostly audible in the lowest harmonics, modeling only these using five or ten resonators yields a more efficient implementation than the parallel digital waveguide structure. The need for a high-order allpass filter can be avoided. The use of resonators gives a greater flexibility in controlling the amplitude envelopes, since it can be done for all the harmonics separately.

In Fig. 2, $S_v(z)$ and $S_h(z)$ refer to the digital waveguide string models (illustrated in Fig. 1) of the vertical and horizontal polarizations, respectively. This structure is similar to what can be found in [11]. The g parameters are real coefficients, which control the input and output amplitudes of the waveguides and determine the amount of coupling. In the new realization presented in Fig. 3 the second digital waveguide $S_h(z)$ is replaced with a set of resonators ($R_1(z) \ldots R_K(z)$) resulting in the reduction of the computational complexity.

Note that the need for implementing the delay z^{-M} of Eq. (3) can also be eliminated. In this case the output

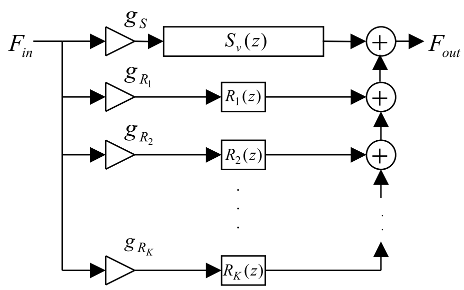

Figure 3: New realization of two-stage decay with one string model and a resonator bank.

F_{out} of the waveguide $S_v(z)$ is at the node M_{in} in Fig. 1, which is the force input position. The delay $z^{-M_{in}}$ of the waveguide cannot be further reduced. In order to compensate for this the initial phases of the resonators should be altered accordingly.

Table 1: Estimated implementation costs in terms of number of multiplications (MUL), additions (ADD), and unit delays (DELAY). WG stands for waveguide.

	MUL	ADD	DELAY
WG for note C_2 (65 Hz)	50	50	700
WG for note C_4 (262 Hz)	30	30	180
WG for note C_6 (1050 Hz)	10	10	50
5 resonators (any frequency)	20	24	15

As shown in Table 1, the replacement of the second digital waveguide with resonators is most beneficial for the lowest piano tones, although for the higher ones less resonators are needed. Savings are obtained in terms of reduced number of multiplications and additions and even more concerning the reduced need of fast memory.

5 PARAMETER ESTIMATION

The parameters of the resonators can be computed as follows: first the loss and dispersion filters of the digital waveguide have to be designed. The decay times of the harmonics can be measured by extracting the harmonics using heterodyne filtering and applying linear regression on the logarithm of the envelopes [13]. From these values the loss filter can be designed.

The measurement of beating and two-stage decay uses the same technique as a starting point. The linear regression curve is then subtracted from the envelope in the logarithmic amplitude scale. As a result, the deviation from the ideal exponential decay is obtained. Going back to the linear amplitude scale, our aim is to fit an exponentially decaying or growing sinusoid on this deviation, since that completely characterizes the two-mode model. Details of the analysis procedure will be published later.

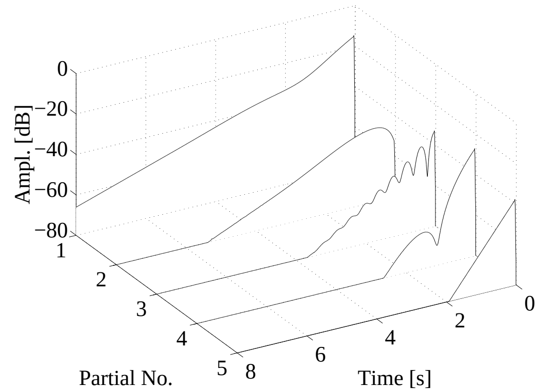

Figure 4: Different amplitude envelopes generated by the structure of Fig. 3.

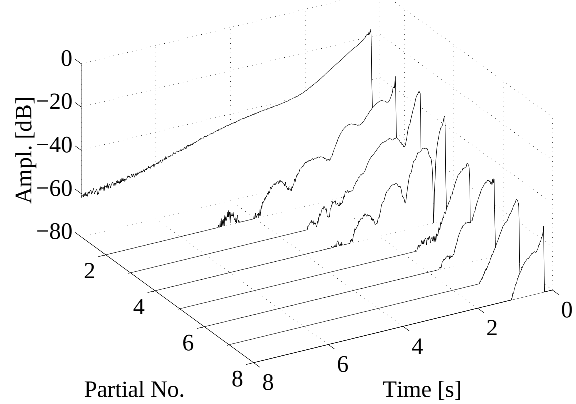

Figure 5: Amplitude envelopes of the original piano tone.

6 SIMULATION EXAMPLES

In Fig. 4 the envelopes of an imaginary piano tone are displayed. The parameters of the resonators were set to show the abilities of our model. Envelope No. 1 is simulating a two-stage decay, No. 3 is an envelope with beating, and No. 5 is a normal exponential decay. The envelopes No. 2 and 4 have interesting properties: No. 4 is an exponential decay but has almost zero amplitude at 1 sec, and No. 2 first grows before it decays. The latter can be used for the simulation of the generation of missing modes, often found in other string instruments [14].

Fig. 5 illustrates the amplitude envelopes of the note $A\#_4$ (466Hz), and Fig. 6 shows the envelopes of the synthetic signal. In the simulation, the structure of Fig. 3 with 5 resonators was used.

As it can be seen from the figures, the envelopes of the most prominent harmonics (1, 2, 3, 4, 5) are matched quite well, while the others have a simple exponential decay. Informal listening tests show that the precise

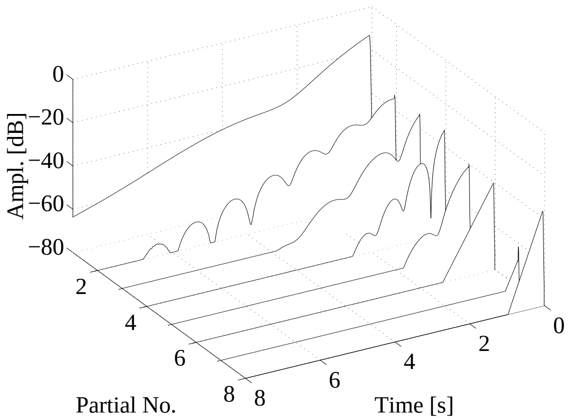

Figure 6: Amplitude envelopes of the synthesized tone using the structure of Fig. 3.

simulation of five or ten harmonics increases the quality of the synthesized tone significantly, while adding only a little computational load compared to the one-polarization, single delay-line case.

7 CONCLUSION AND FUTURE PLANS

A new method for the implementation of the beating and two-stage decay was presented facilitating effective yet high-quality synthesis of the piano sound. This approach, besides its benefits in the computational complexity, enables great flexibility in controlling the envelopes of individual harmonics, resulting in a more natural sound.

When the measurement of all the notes of the piano is possible, this method can give accurate simulation results, since the envelopes of the most prominent partials can be matched quite precisely. Furthermore, this method can take the advantage of nonlinear excitation and coupling between different notes, issuing in a dynamically varying timbre.

When the measurement of all piano notes is not possible, another solution could be the impedance measurement of the bridge. The normal modes can be calculated from the terminating impedance [10]. The measurement of bridge impedance at about five points could give adequate results, which would speed up the measurement and analysis process to a great extent.

Another open question is how the significance of the beating of different harmonics should be determined. In an efficient implementation, typically five or ten resonators are available, and thus it is necessary to deceide which partials should have their envelopes precisely matched. The best choice would be to use psychoacoustic criteria, but unfortunately they are not available in the literature.

It should be noted that the method presented here can be used not only for the simulation of piano sound, but of any other struck or plucked instrument.

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