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ADC Testing using a Resonator-Based Observer

Processing very long time records and/or testing systems with limited stability

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Abstract—**A common method to test AD converters is sine fitting. If a very large set of samples needs to be processed (long records), the iterative four-parameter sine fit algorithm is very sensitive to the initial estimate of the fundamental frequency. Even a slight error in the initial guess can spoil the fit because of the several local minima in the error surface. We propose a robust algorithm to fit the sine wave to the samples by using a resonator based state observer, and an Adaptive Fourier Analyzer (AFA). The method has the advantage that amplitude and frequency drift of the signal generator can be detected and compensated. Moreover, its recursivity allows the processing of many data points without requiring large amounts of memory. This method is also advantageous if the internal AD converter of a microcontroller or DSP needs to be tested in an embedded system, where the clock frequency cannot be arbitrarily tuned, and the absolute precision of the sampling frequency is also limited.**

Keywords – ADC testing, sine fit, resonator, Adaptive Fourier Analyzer, AFA, observer, frequency drift, embedded system

I. INTRODUCTION

Testing analog to digital converters (ADCs) is important for both the manufacturer and the consumer of the ADC. A common method to test its capabilities is the so-called sine fit test [1, 2, 11]. The ADC is excited with a pure sine wave, which can be relatively precisely generated and its quality (harmonic distortion) checked. The measured waveform is a sampled and quantized version of the original one slightly corrupted by linear and nonlinear distortions and noise. Four parameters of the sine wave (frequency, phase, amplitude, and in addition, the DC value) are fitted to the data series. The error of the AD converter is determined as the difference between the fitted sine wave and the sampled and quantized data series.

Different dynamic parameters can be extracted from the above test, like Signal to Noise and Distortion Ratio (SINAD), Signal to Noise Ratio (SNR), Effective Number of Bits (ENOB), Total Harmonic Distortion + Noise (THD+N) etc. These error parameters are defined in the standard [1].

The accuracy of the estimate of error parameters depends on the measurement noise and on the number of sampled points. A possible approach to increase the accuracy is to increase the measurement time; thus, using more data. Another reason for using long records is to detect drift and clock phase noise (see Section 5.4.4 in the standard [1]). However, if more than one million sampled points are processed, the sine fit will be very sensitive to the initial estimate of the frequency of the sine wave. The reason is that the error surface has many local minima, an obvious consequence of the periodicity of the sine.

In this paper the use of a resonator-based observer [3] is suggested, in order to estimate the parameters of the sine wave. The general idea of using an adaptive resonator for this purpose has been presented in [4]. The resonator-based structure observes the signal flow and builds up the conceptual signal model inside, which is a sine wave with harmonics. The advantage of the resonator-based structure is that the resonators can be set to any frequency, not only to those on the DFT grid, thus there is no leakage if the fundamental frequency is not matched to the sampling frequency (noncoherent sampling). The resonator positions are tuned adaptively with the aid of the Adaptive Fourier Analyzer (AFA, [5]), thus very rough information is enough about the fundamental frequency of the excitation signal. An additional advantage of the AFA is that fluctuation or slight drift of the frequency of the excitation signal can also be compensated by updating the resonator positions regularly. This feature enables us to consider time records of length of even tens of minutes. The Adaptive Fourier Analyzer will detect if the requirement of short-term stability of the signal generator (or clock source of the ADC) is not met, and this information can be used to warn the user, or to compensate for the error. Moreover, while an advanced user can check the measured signal, and observe instability of signal generator by breaking the long measurement record to smaller parts, investigating them and comparing the results of the sine fit of shorter records, the proposed approach provides the same information without any user interaction, thus the whole procedure can be automatized. If the user insists on using the 4-parameter sine fit as described in the standard, the proposed method is helpful in warning the user about instabilities in the sampling clock, in the excitation frequency, or in the amplitude. The Adaptive Fourier Analyzer can also be used to accurately measure the fundamental frequency, and to use that as initial parameter for the 4-parameter sine fit.

A resonator-based approach has already been proposed in another context in [6] by Simon et al., however, in the resonator-based structure they assumed the fundamental frequency to be known, and frequency tuning was carried out based on nonlinear minimization of the error of the complete time record. Our approach has the advantage that the frequency tuning is carried out continuously, thus there is no limit in the record length, and frequency and amplitude drift can be handled during the whole process.

The steps of the proposed method are the following:

1. Resonator based observer estimates the Fourier components of the signal as recursive Fourier Analyzer, and a reference waveform is reconstructed from the DC value and the Fourier component of fundamental frequency (Section III.).
2. Adaptive Fourier Analyzer estimates the fundamental frequency continuously, and provides this information for the resonator based observer (Section IV).

3. Resonator based observer updates resonator positions based on current estimate of fundamental frequency.
4. Special cases are handled, like cropped signals if the ADC is overdriven.
5. ADC parameters (SINAD, SNR, THD, THD+N, ENOB etc.) are calculated from the difference of the reference signal and the measured one.

II. MEASUREMENT EXAMPLE TO DEMONSTRATE THE PROBLEM

We investigated an 8-bit, successive approximation ADC (ADC0804LCN, specified for +/-1 LSB error including Full-Scale, zero error and non-linearity) in the acoustic frequency band, excited with a pure sine wave, having a fundamental frequency of around 110.385 Hz. The signal generator was a Stanford DS360 ultra low distortion function generator from Rohde&Schwarz (-100 dB THD, 25 ppm freq. accuracy, 1 mHz freq. resolution). The excitation signal was fed to the AD converter through a general purpose sample and hold amplifier (MAB398). The sampling frequency was chosen to 8110 Hz. Long time records have been collected (2 million samples, about 4 minutes measurement time). A usual sine wave generator can be tuned with mHz resolution. This means that we know the fundamental frequency with a relative accuracy of 10^{-5} (if only the quantization error of the frequency selection is considered). However, for very long records even 1 mHz change in the initial estimate of fundamental frequency spoils the convergence of the sine fit to the true minimum, as it can be seen in Table I, cases b and d. If the fundamental frequency is calculated from the DFT of the measured signal, and is defined as the location of the largest peak (or equivalently from counting the zero crossings after lowpass filtering the signal, and omitting counts too close to each other because of the noise) we cannot reach better initial guess. In our case the ration of the sampling- and the fundamental frequency was around 73, which means that the relative error of the estimate from the DFT is 2 mHz, which is worse then the a prior knowledge (Table I, cases a and e).

TABLE I. ERROR OF THE SINE FIT FOR DIFFERENT INITIAL ESTIMATES OF THE FREQUENCY (8 BIT ADC, 2 MILLION POINTS, MEASURED DATA)

cases	**Initial estimate of the fundamental frequency**	**Estimated fundamental frequency after 4 parameter sine fit**	**RMS error of the fit**
a	110.383 Hz	110.38629 Hz	155
b	110.384 Hz	110.38512 Hz	30.4
c	110.385 Hz	110.38494 Hz	0.57
d	110.386 Hz	110.38466 Hz	42.7
e	110.387 Hz	110.38335 Hz	163

The distance of the frequency estimate of the four-parameter sine fit algorithm from the true value is reduced by the fit (Case b and d in Table I.), but the error still remains too large compared to a good fit (case c), which is a clear indication of the existence of local minima in the error surface.

The error of the fit as a function of initial frequency estimate around the optimum (expected true value) is shown in Fig. 1. It should be noted that the error is expressed as the difference between the sine fit and the noisy quantized data (not the true input). Also note that the figure shows the dependence on the initial frequency estimate, not the error surface as a function of the frequency, DC value, amplitude and phase. In the case shown, given the initial frequency of the optimization, the parameters (including frequency) can be uniquely determined by an iterative least squares fit as shown in the figure.

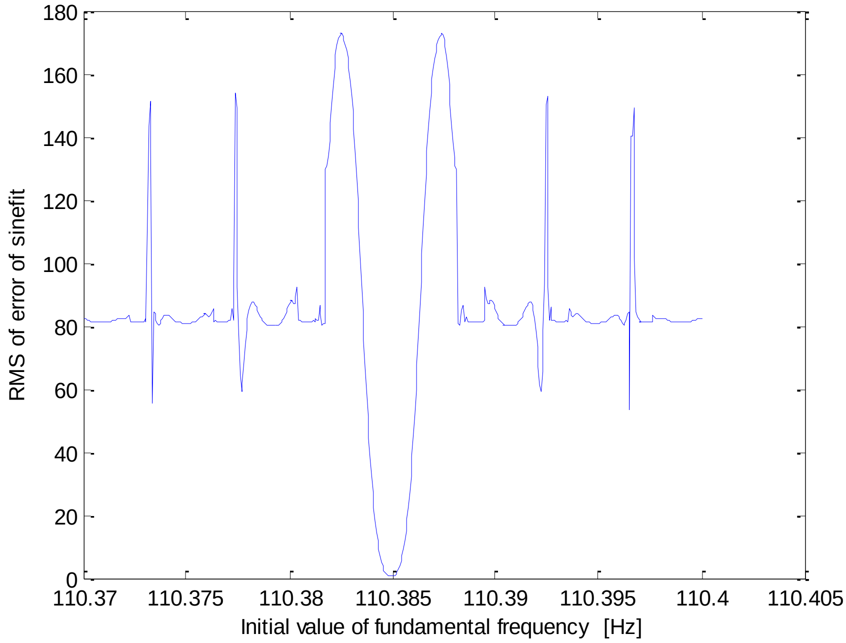

Figure 1. Error of the sine fit as a function of initial estimate of the frequency

In the following sections we will introduce a method that overcomes this limitation with the help of resonator-based state observer together with the adaptive Fourier analyzer.

III. Resonator structure as state observer

Gábor Péceli suggested a resonator based observer structure [3] which can build up a signal model from the time series being observed, and reconstruct it from the model (Fig. 2.).

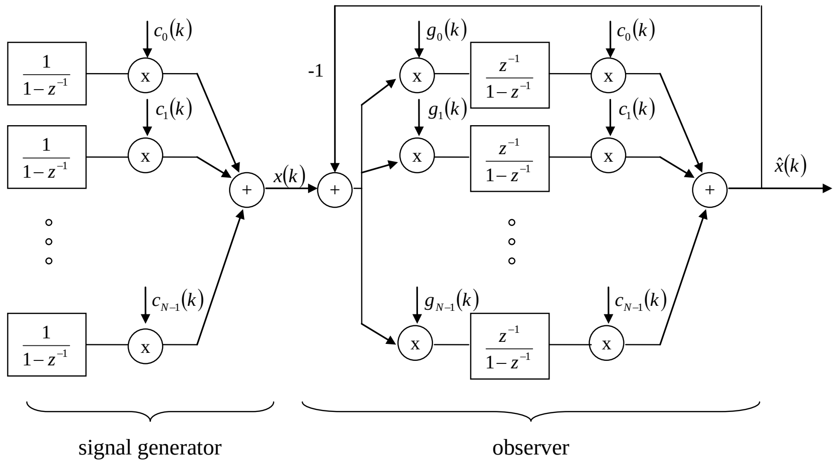

Figure 2. Conceptual signal model and the resonator based state observer

We will utilize the approach for the case of periodic signals where the conceptual signal model is built from different complex exponential signals. The signal generator and the observer have the following basis functions to represent the signal:

$$\left.\begin{aligned} c_m(k) &= e^{j\frac{2\pi}{N}mk} \\ g_m(k) &= \frac{1}{N}e^{-j\frac{2\pi}{N}mk} \end{aligned}\right\} \quad \begin{aligned} m &= 0, \pm 1, \pm 2, \ldots, \pm L, \\ N &= 2 * L + 1 \end{aligned} \tag{1}$$

The working principle is as follows [3]. The modulation in every channel of the observer shifts the corresponding frequency band of the error signal to DC, which is fed to the integrators. The signal is reconstructed from the integrator outputs by modulating them with complex exponentials, thus, shifting back the frequency band to its original location. This closed-loop controller reaches the steady state if the inputs of the integrators are zero, i.e. the reconstructed signal corresponds to the signal being observed. The integrator outputs correspond to the Discrete Fourier Transform (DFT) of the signal. Therefore, the observer with the above basis function can be used as a recursive Discrete Fourier Transformer, where the DFT is calculated for the last N samples. This structure with the given basis functions has a dead-beat nature, which means that after N-1 samples of transients zero error is set, and from this on the periodic signal is perfectly reconstructed, without any error.

The nice property of the resonator-based observer is that the resonator positions do not need to be on DFT grid (Fig. 3). If the sampling frequency and the fundamental frequency of the signal are not correctly matched, DFT faces the usual picket fence effect because of leakage, which can be avoided with the resonator based structure by moving the resonator positions to harmonics of the fundamental frequency:

$$\left. \begin{aligned} c_m(k) &= e^{j\omega_0 mk} \\ g_m(k) &= \frac{1}{N} e^{-j\omega_0 mk} \end{aligned} \right\} \begin{aligned} m &= 0, \pm 1, \pm 2, \ldots, \pm L, \\ N &= 2 * L + 1 \end{aligned} \tag{2}$$

where ω_0 is the relative fundamental angular frequency of the signal (ω_0 =$2\pi f_0 / f_s$, where f_s is the sampling frequency). This observation is very helpful if one is going to investigate the internal AD converter of a microcontroller or DSP in an embedded system, where the clock frequency of the processor is fixed, and the sampling frequency can be derived only from this clock source by a coarse pre- and postscaler. Theoretically it is enough if the fundamental frequency of the excitation signal can be tuned on a fine enough grid, if the sampling frequency is fixed and known. However, in embedded systems, the clock source may not be stable (long-term instability), thus one cannot perfectly match the ratio of the fundamental frequency to the sampling frequency. The ratio of the fundamental and sampling frequencies still needs to be known to make use of resonator based observer, which can be determined by means of an Adaptive Fourier Analyzer, detailed in the next section.

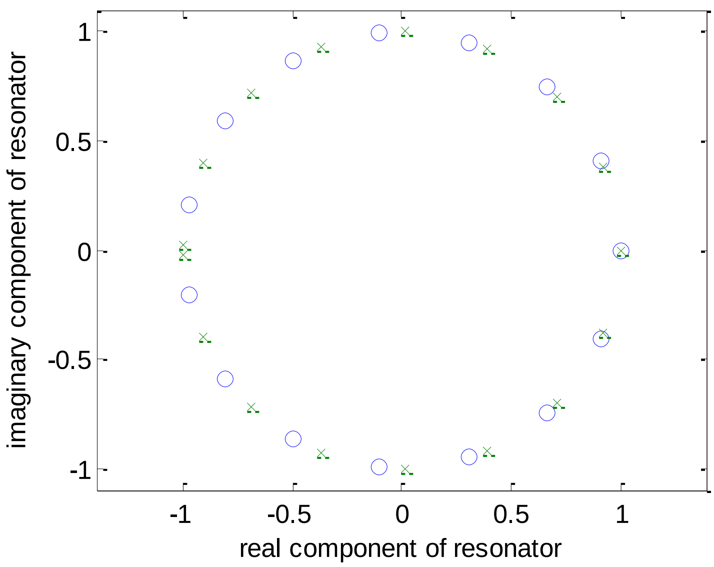

Figure 3. Resonator positions on DFT grid (o), and on multiples of arbitrary frequency (x) Here the DFT is calculated for an odd number of points.

IV. ADAPTIVE FOURIER ANALYZER

The resonator-based observer structure requires the correct tuning of resonator positions in order to avoid leakage. This can be handled with the aid of Adaptive Fourier Analyzer (AFA) proposed by Ferenc Nagy in [5]. The basic idea of the algorithm is that if the resonator positions of the observer and the fundamental frequency of the signal to be reconstructed are not in match, there will be a rotating error component in the state variables (outputs of the integrators). The rotational speed corresponds to the frequency mismatch, which can be utilized to correct for it. The frequency adaptation has the following form:

$$\omega_{1,n+1} = \omega_{1,n} + \frac{1}{N} angle\left(\hat{X}_{1,n+1}, \hat{X}_{1,n}\right) \tag{3}$$

where $\hat{X}_{1,n}$ denotes the state variable of the channel corresponding to the fundamental frequency, and *angle* means the phase angle between the two complex numbers interpreted as vectors in the complex plane. The new resonator positions can be modified by updating the basis functions in the following way:

$$\begin{aligned} c_{m,n+1} &= c_{m,n} e^{j\omega_{1,n+1} m} \\ g_{m,n+1} &= \frac{1}{N} c^{*}_{m,n+1} \end{aligned} \tag{4}$$

where superscript * denotes complex conjugate. The noise influence on the AFA can be decreased if the angle mismatch in (3) is distributed among more sample points:

$$\omega_{1,n+1} = \omega_{1,n} + \frac{1}{N \cdot Q_{damp}} angle\left(\hat{X}_{1,n+1}, \hat{X}_{1,n}\right) \tag{5}$$

where Q_{damp} is the damping factor. Special care should be taken if the fundamental frequency changes much. In that case either new resonators need to be initialized, if the frequency is decreased, or old ones need to be deleted if the frequency is increased. Gy. Simon and G. Péceli provided a thorough analysis of the convergence properties the AFA for special circumstances [7]. The general convergence of the above algorithm is not yet mathematically proved; however, applications in wide range (as e.g. active noise cancellation) confirmed its usability.

There are different methods proposed in the literature to increase the robustness of the estimate of the fundamental frequency. One possibility is to compare the Fourier components of the fundamental frequency at P samples apart from each other, instead of comparing two consecutive estimates (Block AFA).

$$\Delta\hat{\omega}_{1,n+1} = \frac{1}{P} angle\left(\hat{X}_{1,n}, \hat{X}_{1,n-P}\right) \tag{6}$$

Another alternative is to average the Fourier component (just the fundamental channel) in a sliding window of length B [8].

$$\Delta\hat{\omega}_{1,n+1} = \frac{1}{P} angle\left(\hat{X}_{aver(1,n)}, \hat{X}_{aver(1,n-P)}\right) \qquad \hat{X}_{aver(1,n)} = \frac{1}{B}\sum_{b=1}^{B} \hat{X}_{1,n+1-b} \tag{7}$$

V. DEALING WITH CROPPED TEST SIGNALS

Thorough testing of the AD converter requires excitation of every bin, even those close to +/- full scale. (Although this is a challenge for AD converters having very high resolution (e.g. 24 bits), we are going to be close to the ideal.) Thus, the amplitude of the excitation signal is reasonable to be chosen higher then fitting in the ADC input range. In [9] it has been shown that e.g. repeatability of the ENOB measurement is poor if the AD converter is not overdriven, and the number of ADC bins is low. The sampled signal is therefore a cropped version of the original one (overdrive), which is not a pure sine anymore. The signal is still periodic, having a discrete spectrum. Unfortunately, the fundamental Fourier component is also modified with the overload.

A common method to overcome this problem through the sine fit process is to omit those samples from the fit which are outside of the ADC input range. Kollár and Blair suggested a histogram based test to evaluate and select the samples for the fit in order to better fulfill the assumptions of the nonlinear least-squares fit [10].

In the case of the resonator-based observer we suggest to skip the adaptation of the feedback loop during ADC overdrive, and artificially feed zero match error instead. This modification is perfect if there is no noise, since in steady state the error is zero anyhow, which is reached after the first N steps. However, if the signal to be observed is noisy (as it is in our case), the adaptation is continuous, and the state variables are only noisy estimates of the true ones. Switching off the adaptation means that the noise propagates as long as the feedback is broken. This effect is proportional to the noise level. 8-bit quantizers possess significant quantization noise, thus the estimate at the peaks, where the feedback is switched off, is distorted. However, the observer accommodates quickly as the feedback is turned back (Fig. 4.). If the ADC is not overdriven, we do not face this problem, and the fit will be better.

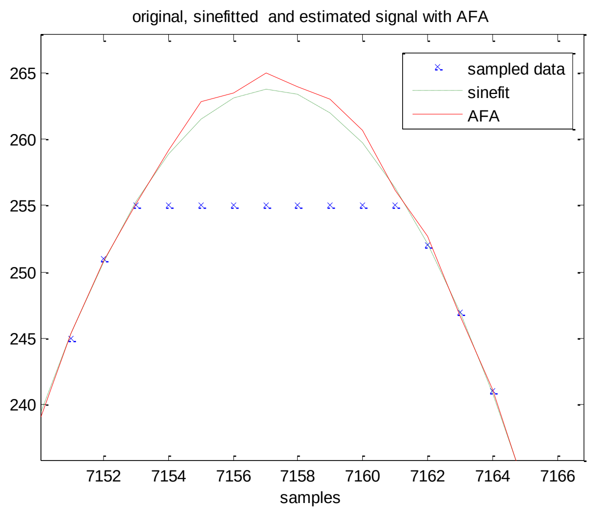

Figure 4. Effect of noise if the feedback of the observer is switched off around the cropped peak.

VI. PARAMETER ESTIMATION USING THE RESONATOR STRUCTURE

A. *Reference signal is generated with the resonator structure*

We propose the following algorithm to handle very long time records for sine fit. First we need a rough initial value about the fundamental frequency of the excitation signal. 10% precision is enough, however, the better the initial guess, the shorter the convergence of frequency estimate will be. We start the resonator-based state observer to estimate the Fourier components of the signal. The resonator positions are set to the multiples of the initial value of the fundamental frequency, which does not need to be on DFT grid. The Adaptive Fourier Analyzer runs in parallel. It estimates the fundamental frequency, and rearranges the resonator positions in every single step, if a mismatch is detected. The feedback of the error is eliminated in the peak region of the sine, where the data is cropped because of overdriving the input range of the ADC. The resonator based observer reconstructs the measured signal. However, because of quantization and other noise sources, harmonics will also appear in the spectrum. We need only the fundamental frequency and DC components, which correspond to the first and second channels of the resonator (and certainly to the symmetric negative frequencies, too). The signal reconstructed from these two channels will be our reference waveform. The parameters of the sine can be extracted from the resonator structure, if needed.

Both the resonator structure and the AFA require certain time for settling. In the case of the resonator-based observer, the settling time equals the number of resonator positions, multiplied with the sampling time (dead-beat observer). The AFA requires several periods to settle, if the initial estimate of the frequency was incorrect, thus this front part should be eliminated from the final result. If it is undesirable, we can let the resonator and AFA run twice on the front part of the signal. First, we let it settle, second, we can start the structure with the correct initial state, where the resonator state variables (amplitude and phase of the given Fourier component at the beginning of time record) are also set correctly. The convergence of the algorithm can be increased if the front part is adapted to the fitted sine wave, where the sine fit is accomplished to the first couple of thousand AD samples only.

The remaining part of the proposed algorithm is described next, for two different scenarios.

1) *Fundamental frequency of the excitation signal or timing of sampling circuitry is unstable*

If the signal generator or the sampling clock has short term instability, i.e. there is a slight drift in the fundamental frequency during the long measurement, AFA will lock to the change like a Phase Locked Loop (PLL) and follow it. The fundamental frequency of the reference signal coming from the resonator will follow this change. If this is unwanted, this measurement cannot be used for testing, and AFA is utilized to warn the user for omitting the result.

2) Fundamental frequency of the excitation signal and timing of sampling circuitry are stable (short term stability)

To suppress the noise influence on the estimate of the fundamental frequency (small fluctuation) after the settling, we propose to rerun the resonator structure with fixed resonator positions, where the fundamental frequency is defined as the mean value of frequency estimate coming from the AFA (after settling). Certainly, resonator positions might be fixed only if no frequency drift has been observed by the AFA, i.e. neither the signal generator, nor the sampling circuitry shows timing inaccuracy.

Instability of the generator regarding drift of amplitude of the excitation signal or the DC value can be detected be the resonator structure. Since the adaptation of the resonators is continuous, the reference signal will follow the change of the excitation signal. Similarly, if the reference voltage of the AD converter is unstable, it can be detected on the same way.

B. *Reference signal is generated by sine fitting*

If the adaptation of the AFA and the resonator structure shows very stable excitation signal and sampling circutry, we might generate the reference signal by 4 parameter sine fitting, however, starting the iteration of the sine fit from the fundamental frequency detected by the AFA (averaging of the tail part is required). In this case AFA together with the resonator based observer is utilized to check the stability of the generator, and to provide a very precise estimate for the fundamental freq. of the excitation signal. Without this precise estimate the 4 parameter sine fit is unstable as it was shown in Section II.

VII. Measurement results

The proposed approach has been tested on acquired data (8 bit ADC, 8110 Hz sampling frequency, 110.385 Hz sine wave as excitation signal). For the sine fit method three different frequencies have been set as initial values: 110.383, 110.385 and 110.387 Hz. The AFA has been started from a rough initial frequency (110.0 Hz). 73 resonators were placed uniformly on the unit circle. Fig. 5. shows how the Adaptive Fourier Analyzer locks to the fundamental frequency after a short transient. Q_{damp} damping factor controls the amount of feedback from the phase error (see. eq. 5). Larger value means smooth estimate, smaller value means faster reaction. This damping factor has to be chosen according to the a prior knowledge about possible freq. drift and noise level. If no information is available, Q_{damp}=5 is a good compromise in ADC testing environment. A small part of the reconstructed signal generated with the resonator structure is shown in Fig. 6, together with the fitted sine estimated from the optimal initial frequency (the two estimates are very close to each other, they are the same within the graphics resolution). Here, Q_{damp}=5 was chosen. It should be noted that in this case the initial parameters of the sine fit were very carefully chosen. If there is a slight error in the initial frequency value, the proposed algorithm is robust for it, however, sine fit is spoiled (Table II.)

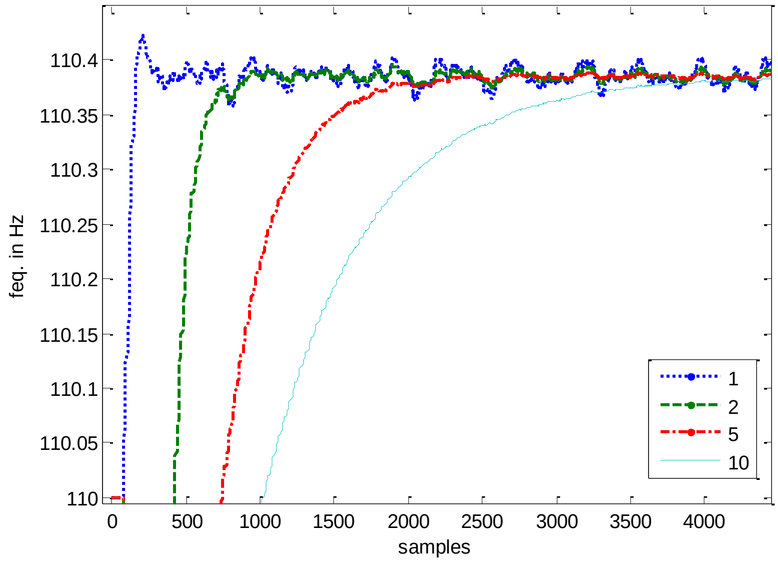

Figure 5. Adaptation of the AFA at the front part of the signal, whith different damping factors Q_{damp} (f_{start}=110.0 Hz, measurement result)

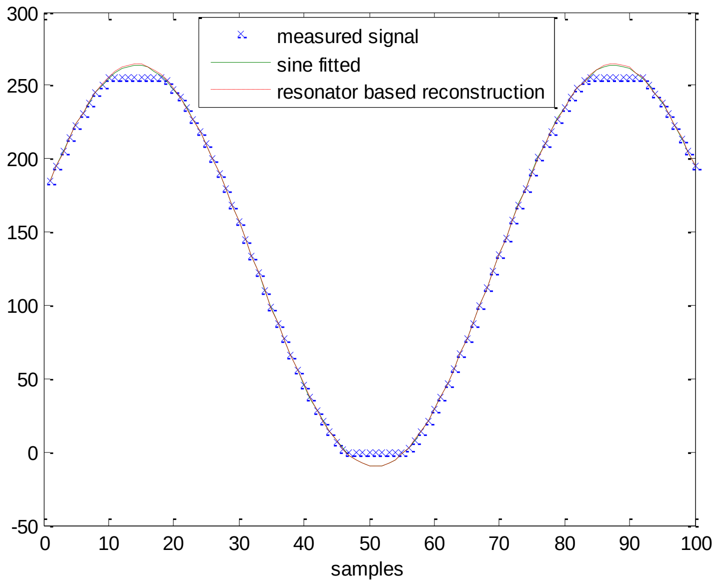

Figure 6. Small part of the reconstructed signal

(original sampled signal, sine fitted, and reconstructed with our proposed approach).

The two fits are the same within the graphic resolution (measurement result).

TABLE II. ERROR OF FIT (8 BIT ADC, 10^6 POINTS, MEASURED DATA).

THE INITIAL FREQUENCY OF THE OPTIMIZATION IN THE CASE OF SINE FIT IS IN PARANTHESIS IN THE FIRST COLUMN

fit method (init. freq. [Hz])	Estimated DC value [LSBs]	Estimated amplitude [LSBs]	Estimated frequency [Hz]	RMS error of the fit [LSBs]
4-par sine fit (110.383 Hz)	126.95	159.10	110.385	34.1
4-par sine fit (110.387 Hz)	126.95	160.75	110.384	38.4
4-par sine fit (110.385 Hz)	126.94	136.85	110.3849	0.35
resonator (110.000 Hz)	127.01	136.72	110.3849	0.34

VIII. CHECKING THE ROBOUSTNESS (SIMULATION RESULTS)

A. *Noise sensitivity (frequency is stable)*

First let us investigate the resonator based observer with respect to noise sensitivity. If there is no noise, resonator based observer can estimate the periodic signal without any error after the Nth step. In a noisy environment the recursive Fourier transformation is an estimate of spectrum calculated from the last N samples of the noisy signal. Thus, the estimate of the DC value, amplitude and phase of the sine wave will be continuously adapted not only to the signal, but also to the noise. Since the resonator based observer runs continuously, the Fourier components are also updated from sample to sample.

Let us demonstrate this on a simulation example. Gaussian noise is added to the sine wave (SNR=55 dB), then it is quantized with an 8 bit quantizer. The final SNR after quantization remains nearly the same. The Adaptive Fourier Analyzer is not yet utilized, the fundamental frequency is assumed to be known, and only the adaptation of the resonator structure with respect to noise is investigated. The amplitude estimate throughout the adaptation of the observer is shown in Fig. 7. The Fourier component does not settle because of the noise, after the transient at the beginning of the record it oscillates around the true value with amplitude of around 0.1% of the true value, and a bias of about 0.01% (transient part is neglected). Certainly, the sine fit provides a constant estimate for the whole record (dashed line in the figure), which is still biased, with the same amount as the resonator based observer (0.01%). The same is true for the phase estimate (Fig. 8). The observer estimates the phase with an oscillation (caused again by the noise) having an amplitude of about 0.03 degrees, and a mean value of $3\cdot10^{-4}$ while the sine fit has a constant $2\cdot10^{-3}$ degrees bias. (Estimate of DC value has the same nature, see Fig. 9.) The slight oscillation of the parameter estimates of resonator based observer do not cause much difference in the reconstructed signal compared to the sine fitting, since the observer keeps the estimate always close to the input waveform (by adjusting both the amplitude and phase).

The RMS error of the sine fit is 0.286, while that of the resonator based estimate is 0.291, which means that the proposed method provides the same performance as the sine fit, assuming that the sine fitting algorithm found the global optimum. In this case the advantage of the proposed method is its stability (see Section II., Table I.).

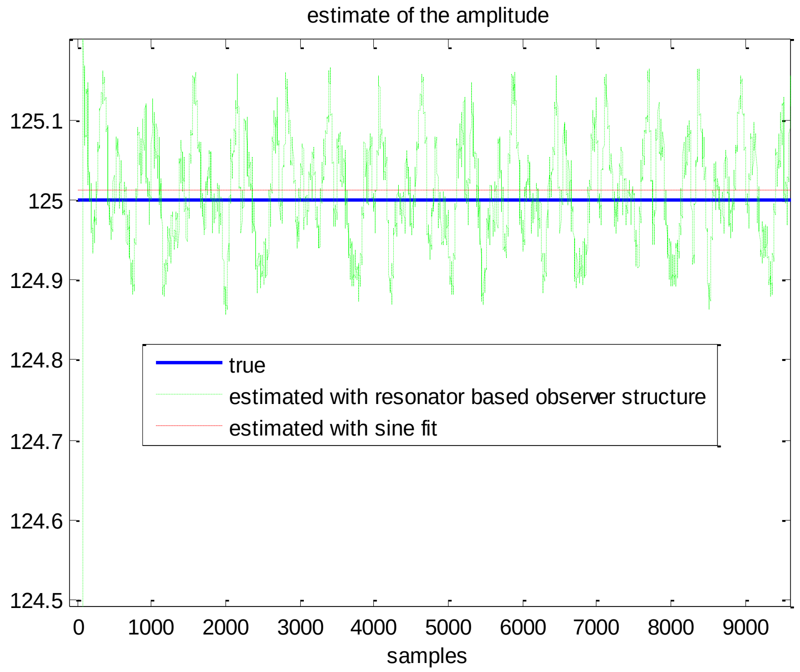

Figure 7. Front part of the adaptation of amplitude estimate with the resonator based structure, and estimate with the sine fit (SNR=55 dB).

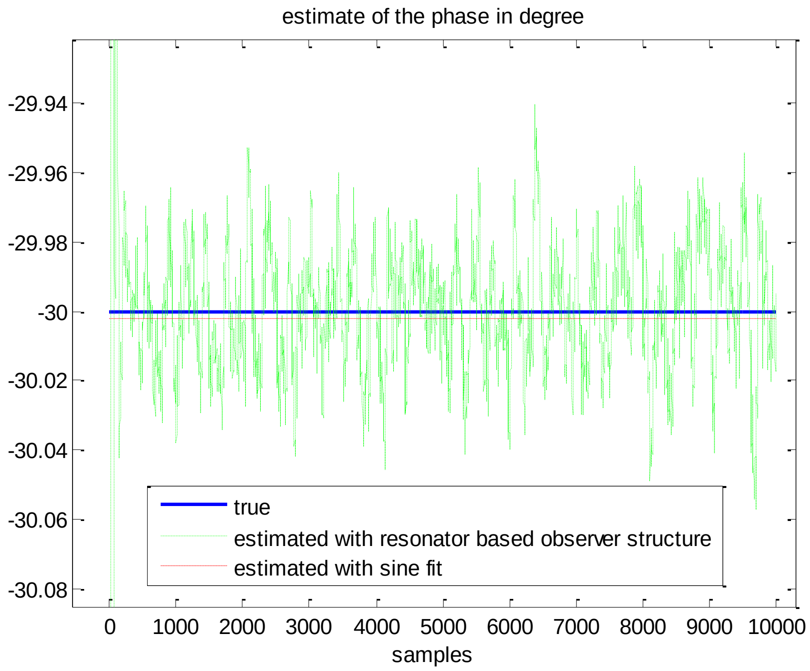

Figure 8. Front part of the adaptation of phase estimate with the resonator based structure, and estimate with the sine fit (SNR=55 dB).

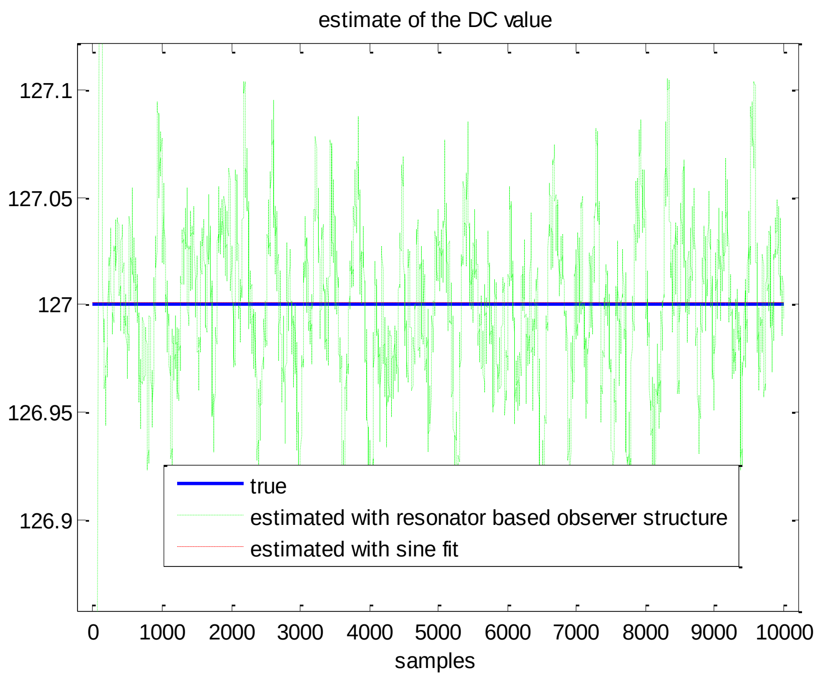

Figure 9. Front part of the adaptation of the estimate of DC value with the resonator based structure, and estimate with the sine fit (SNR=55 dB).

B. *Unstable signal generator*

The benefit of our approach is that it can detect and tolerate frequency or amplitude drift during the measurement. This is demonstrated again in a simulated environment, where the amplitude has a 1% drift over the time record. -55 dB Gaussian noise was added to the simulated signal. Fig. 10 shows the adaptation of the amplitude estimate of the resonator based structure. The estimate follows the change well. The variation around the true value is caused by the measurement noise. For comparison, the amplitude estimate of the sine fit is also shown. The RMS error of the sine fit is 0.46 (assuming that sine fitting procedure found the global optimum), while that of the resonator based estimate is 0.30, which means that the proposed algorithm outperforms the sine fit even if the sine fitting algorithm converges to the global optimum.

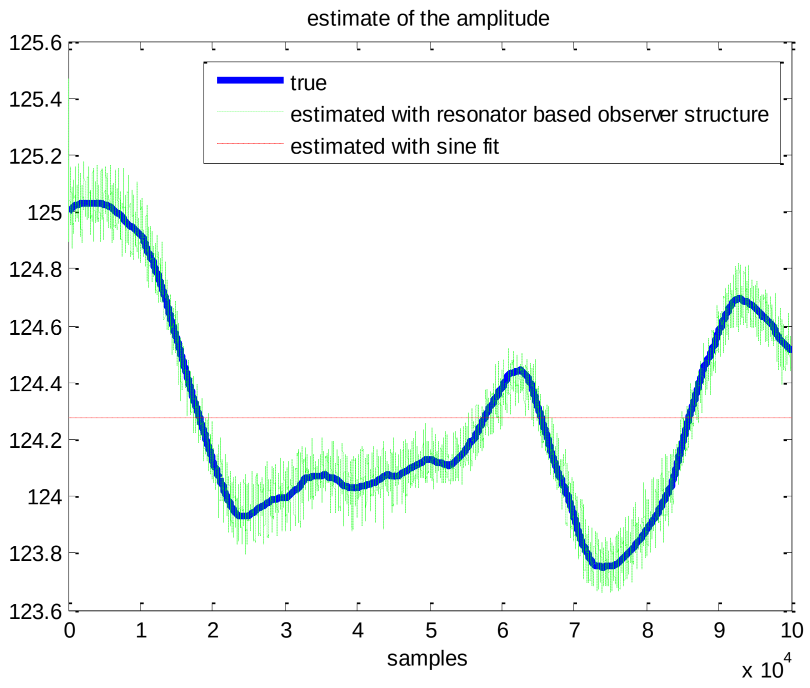

Figure 10. Front part of the adaptation of amplitude estimate with the resonator based structure, if there is a drift in the amplitude (SNR=55 dB).
The horizontal line is the estimate with sine fit (simulated signal)

Finally, let us check how the observer behaves if the frequency of the generator becomes slightly unstable. A random phase drift was added to the sine wave. The instantaneous frequency is interpreted as the derivative of the phase. We switch on the frequency adaptation with the Adaptive Fourier Analyzer, and continuously update the resonator positions according to the estimated fundamental frequency. Fig. 11 shows the adaptation of the AFA, which follows quite well the drift. The RMS error of the fit with resonator based observer remains nearly the same as without phase drift (0.3), however, that of the sine fitting becomes much worse (10.2). Adaptive Fourier Analizer is very robust with respect to noise. It will provide a reasonable estimate for the fundamental frequency up to 10 dB SNR. The limit of convergence is at around 0 dB. At large noise levels AFA might provide noisy estimate for the fundamental frequency, however, the resonator based observer can compensate for slight errors by continuously adapting the phase of the signal to the frequency mismatch.

Our approach handles the case of a distorted sine wave, i.e. the signal generator produces harmonics. In that case the first couple of harmonics estimated by the observer structure should be added to the reference signal also. This is an obvious consequence of the Fourier Analyzer, and will not be investigated in more detail. Certainly, if the generator is good enough with respect to harmonic distortion, it is better to include only the DC and fundamental components to the reference signal.

The error of the fit is summarized for the different cases in Table III.

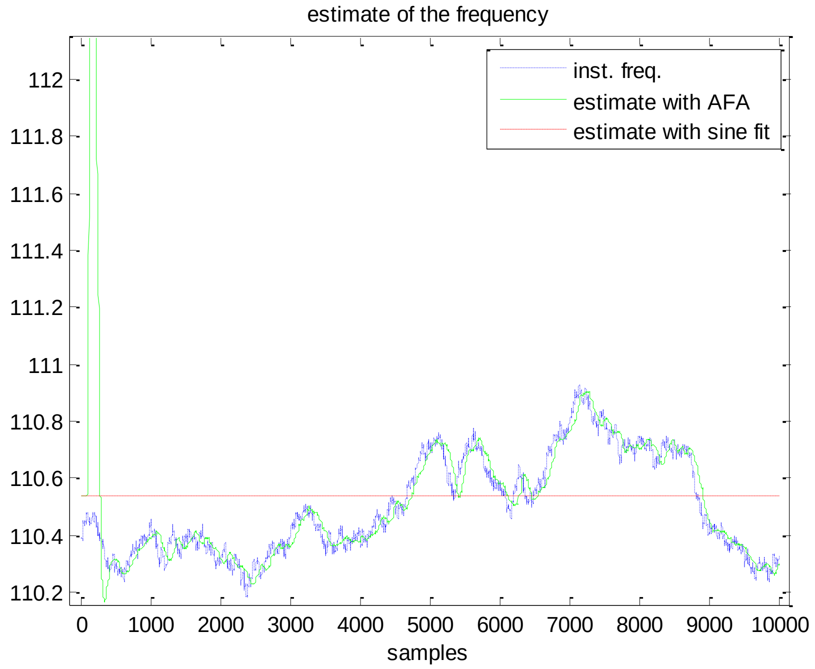

Figure 11. Front part of the adaptation of frequency estimate with the resonator based structure, if there is a drift in the phase (SNR=55 dB).
The horizontal line is the estimate with sine fit (simulated signal)

TABLE III. RMS ERROR OF FIT IN LSBS (8 BIT ADC, 10^6 POINTS, SIMULATED DATA).
SINE FIT IS CALCULATED FOR TWO DIFFERENT INITIAL FREQUENCY ESTIMATES: TRUE ONE (GLOBAL OPTIMUM), AND WITH 1 MHZ ERROR (LOCAL OPTIMUM)

type of instability	**sine fit (local minimum)** **RMS error of the fit [LSBs]**	**sine fit (global minimum)** **RMS error of the fit [LSBs]**	**proposed method** **RMS error of the fit [LSBs]**
stable system	4.5	0.286	0.291
amplitude drift	4.5	0.41	0.30
phase drift	27	10	0.29

We can conclude that if the record length is very long, Adaptive Fourier Analyzer together with resonator based state observer is much robust for the fundamental frequency estimate. If there is a chance that during the long record signal generator is not stable enough (DC, amplitude, phase or frequency drift), the above method is helpful again to warn the user, and also to follow the signal. The same is true, if not the generator, but the sampling clock of the ADC has limited accuracy, as it is the case many times in embedded systems (e.g. internal AD of a microcontroller) where the clock source has limited precision regarding both short and long time stability.

IX. Conclusions

A new approach has been proposed to test AD converters with sine fit method, if the record length is extremely long, or the signal generator is not stable enough throughout the measurment. Instead of least squares or maximum likelihood sine fit, we propose a resonator-based observer structure which is analog to a recursive Fourier Analyzer with the benefit that frequency of the signal does not need to be on DFT grid. Our signal model can also cope with slowly varying fundamental frequency, which cannot be handled with least squares or Maximum Likelihood sine fitting. The frequency of the excitation signal is estimated with a nonlinear observer, namely the Adaptive Fourier Analyzer. The proposed approach is very robust, and can handle frequency or amplitude drift during the long measurement. The algorithm is recursive, which allows its implementation in embedded environment with limited memory.

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