Reconstruction of Nonlinearly Distorted Signals With Regularized Inverse Characteristics

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***Abstract*—A signal distorted by a system having static, invertible, nonlinear characteristics can be exactly restored in the absence of noise. In this case, the inverse of the characteristics can be used. When noise is superimposed to the distorted signal, the inverse characteristics may not be proper because the noise is strongly amplified. This noise has to be suppressed in the reconstructed signal, which can be accomplished only at the price of bias. This article presents a method to compensate the effect of static nonlinearities in the presence of noise. This method is based on Tikhonov's regularization operators and provides a compromise between noisy and biased estimates.**

***Index Terms*—Nonlinear distortion, regularization, signal reconstruction.**

I. Introduction

NONLINEARITY of measurement systems or communication channels distort the measured or transmitted signal. If the distortion is unacceptable, the detected signal should be post processed to reconstruct the original one. The nonlinearly distorted signal is usually corrupted by noise. In this case, the inverse of the nonlinearity may not be optimal for reconstruction, because the noise is amplified during the reconstruction process. Such an optimal inverse characteristic is needed, which suppresses the noise.

A. Preliminaries

Several works deal with compensation of nonlinear systems. These works usually use Volterra-kernels to describe the nonlinearity of the system. The Volterra-series method can handle a wide range of nonlinearities [1]–[4].

A simpler method is used for nonlinear compensation in [5]. In this work a static, nonlinear system (a cathode ray tube) is described. The static nonlinearity and the compensation function are approximated by polynomials.

In [4], [6], and [7], the proposed algorithms are made specifically for sound-restoration. In [6], the histogram equalization technique is used to estimate a memoryless nonlinear transfer function. In [7], the nonlinear function is assumed to be given and an iterative technique is used for restoration of the signal. In [4], a statistical model-based reconstruction process is described.

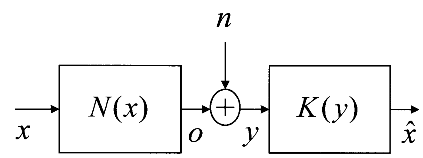

Fig. 1. Signal model of the reconstruction process.

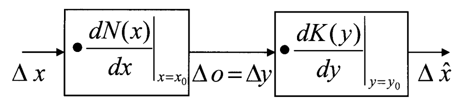

Fig. 2. Signal model of the reconstruction process for small perturbations.

B. Novelties

As most of these previous works state, the effects of noise in the proposed algorithms are not clearly described. Further work is needed to establish the effect of noise in [1] and the analysis in [2] is also a purely deterministic approach. In [6], the algorithm is quite sensitive to noise whose distribution is markedly different from that of speech. In [3] and in [4], a prefiltering technique was used, where the effects of noise are smaller; therefore, the effects of noise were not handled. However, sometimes only the post-filtering technique can be used to reconstruct the detected, noisy and nonlinearly distorted signals, where the noise has strong effects. The algorithms in [7] and [8] are applicable to handle the effect of noise, but they are iterative algorithms and they require high computation time.

The proposed method in our paper is a post-processing technique that works on static nonlinearities (the word "static" means that the examined nonlinear device does not contain linear distortion before or after the nonlinear distortion function; therefore, the output does not depend on the previous input or output samples). We take the effect of the noise into account. The emphasis in this article is to find the inverse characteristics of a known static and invertible nonlinear function, which handles the propagation of the noise through the nonlinear inversion. The signal reconstruction itself is a one-step process, so it does not require intensive computation time. In Section II, we define the model of the nonlinear distortion and the reconstruction process. In Section III, the problems of the reconstruction will be discussed. Next, in Section IV, the proposed method with regularization operators will be shown. Section V includes a simulation example demonstrating the result, which can be achieved by regularized inverse characteristics. In Section VI, a practical application will be discussed. Conclusions are given in Section VII.

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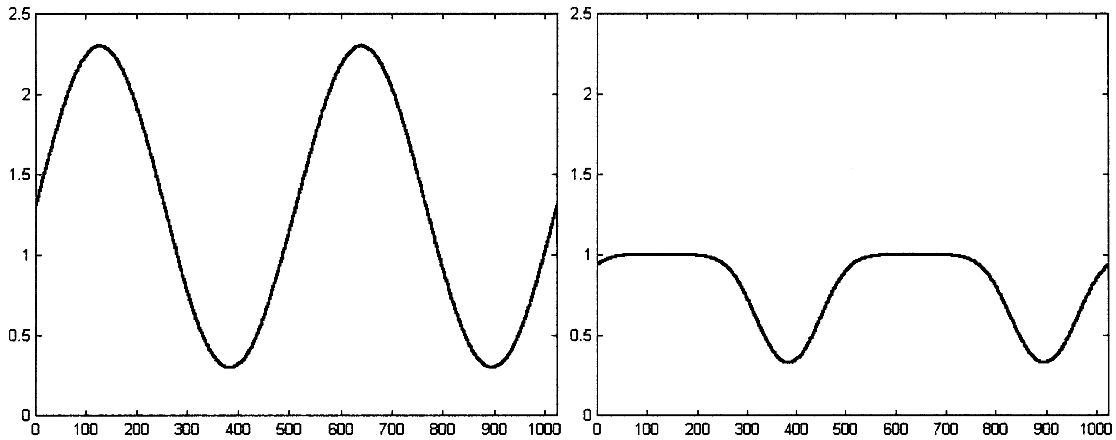

Fig. 3. Input signal (left) and the nonlinearly distorted; noisy signal (right).

II. Modeling the Distortion

The signal model of the reconstruction process can be seen in Fig. 1, where $N(x)$ denotes the nonlinear function of the measurement system, x denotes the input, and o denotes the distorted output of the system, where $o = N(x)$.

The observation, o, is disturbed by the measurement noise, n, that is modeled as an additive one. $K(y)$ is the inverse nonlinear function and $\hat{x}$ is the estimation about the input signal.

The mathematical analysis of this model is difficult because in general, the nonlinear equations cannot be analytically solved. In a given working point, at small changes of x, we can approximate o with the first elements of the Taylor polynomial, which will be a linear approximation

$$o = o_0 + \Delta o \approx N(x_0) + \left.\frac{dN(x)}{dx}\right|_{x=x_0} \cdot \Delta x. \tag{1}$$

So the perturbation of o is

$$\Delta o \approx \frac{dN(x)}{dx} \cdot \Delta x. \tag{2}$$

The reconstruction process for the perturbation is shown in Fig. 2.

This model is already linear and we can apply this model in each point of the original characteristics. The noise in this model is not additive, but affects the working point, hence the amplification of the second box.

III. Difficulties at Reconstruction

To optimize the model, first we need to define a measure for the quality of the estimate. We define the best solution as the minimum of the following equation:

$$Cost = \min_{\Delta\hat{x}} \left(\|\Delta x - \Delta\hat{x}\|\right) \tag{3}$$

where $\|a\|$ is a norm of a. This is an input error criterion. Unfortunately, the solution of this error criterion is not applicable, because it requires the knowledge of Δx. If we choose $\|\Delta y - \Delta\hat{y}\|$ for the error criterion (output error criterion), it leads to the solution, where $K(y)$ is the inverse of $N(x)$. The problem of this solution is that the noise is amplified at $\hat{x}$ during reconstruction. The noise amplification can be seen in the case of small noise amplitude, if the estimation is rewritten into the following form:

$$\hat{x}_0 = N^{-1}(y_0) = N^{-1}(o_0 + n) \approx x_0 + \frac{1}{\left.\frac{dN(o)}{do}\right|_{o=o_0}} \cdot n. \tag{4}$$

The noise will be amplified, where the derivative of the nonlinear function of the system is small.

IV. Regularized Compensation

The solution of ill-posed equations was originally proposed by Tikhonov [8], who created a method to solve ill-posed integral equations with regularization operators. The error criterion used in Tikhonov's method is an extension of the output error criterion. One possible form is

$$Cost = \min_{\Delta\hat{x}} \left(\|\Delta y - \Delta\hat{y}\| + \lambda \|\Delta\hat{x}\|\right). \tag{5}$$

In practice, the l_2 norm is used, because minimizing the l_2 norm of the error minimizes the energy of it. In the case of sampled signals we can write

$$Cost = \min_{\Delta\hat{x}_i} \left(\sum_i (\Delta y_i - \Delta\hat{y}_i)^2 + \lambda \cdot \sum_i \Delta\hat{x}_i^2\right). \tag{6}$$

The solution of this equation is found, where the derivative $\partial/\partial\Delta\hat{x}_i$ equals 0. The solution expressed for $\Delta\hat{x}/\Delta\hat{y}$ is

$$\frac{\Delta\hat{x}}{\Delta y} = \left.\frac{dK(y)}{dy}\right|_{y=y_0} = \frac{\left.\frac{dN(\hat{x})}{d\hat{x}}\right|_{\hat{x}=\hat{x}_0}}{\left(\left.\frac{dN(\hat{x})}{d\hat{x}}\right|_{\hat{x}=\hat{x}_0}\right)^2 + \lambda}. \tag{7}$$

We can create the regularized $K(y)$ inverse filter of the nonlinear system by integrating (7), but we need a condition to determine the integration constant. If $K(y) = F(y) + C$, we can determine the value of C by minimizing $\|y - \hat{y}\|$ in the function of C. If the norm is the Euclidean-norm, the solution is the solution of the following equation:

$$E\{N(F(y) + C)\} - E\{y\} = 0. \tag{8}$$

The regularized inverse characteristics of the nonlinear system can be computed by numerical integration from (7) and (8). The

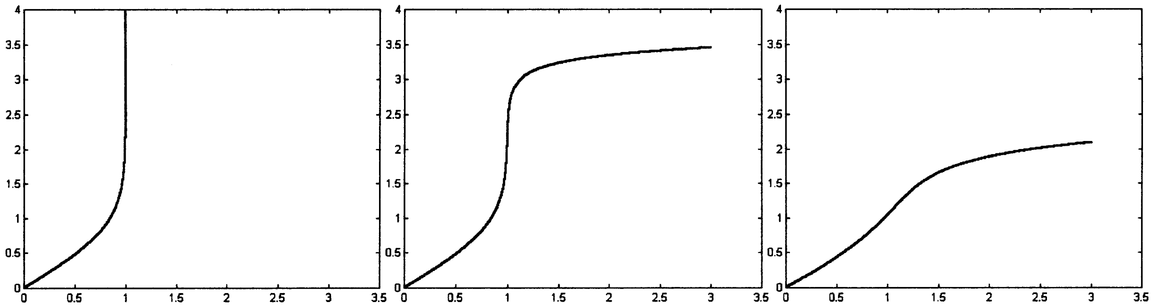

Fig. 4. (Left) Unregularized inverse characteristics, (middle) regularized inverse characteristics for $\lambda = 0.01$, and (right) $\lambda = 0.1$.

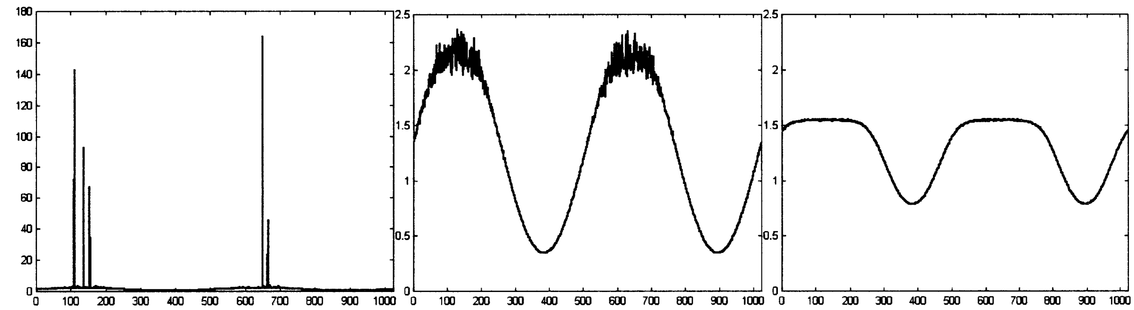

Fig. 5. (Left) Reconstructed signals: unregularized inverse characteristics, (middle) regularized inverse characteristics for $\lambda = 0.01$, and (right) $\lambda = 0.1$. Note that the scale of the figure of the unregularized characteristic is higher because of the high noise amplitude.

resulting characteristics can be used as $K(y)$, without any further iteration; thus the reconstruction itself is a one-step process.

V. Simulation Example

To illustrate the proposed algorithm, we solve a simple example using a sine input signal $x = \sin(2pi/1024 \cdot t) + 1.3$ and an error-function nonlinearity $o = erf(x)$. We added a Gaussian noise to o with $\approx$ 50 dB signal-to-noise ratio. The original and the distorted, noisy signal can be seen in Fig. 3. The inverse of the nonlinear function and the regularized inverses for $\lambda = 0.01$ and $\lambda = 0.1$ can be seen in Fig. 4.

Fig. 5 shows the estimations. In the first figure, the estimation reached by the inverse of the nonlinear function can be seen. The maximum amplitude of the noise is about 100 times higher than the original signal. In the second figure, the estimation reached by regularized inverse for $\lambda = 0.01$ can be seen and in the third figure that for $\lambda = 0.1$ can be seen.

The oscillation of the signal, caused by the noise, is small at $\lambda = 0.1$, but the distortion caused by the regularized inverse is quite high. The unregularized inverse has no bias from regularization, but the aberration caused by the noise is extremely high. At $\lambda = 0.01$ the distortion and the noise are also small. Presently we find this trade-off between noise and bias manually.

VI. Practical Application

An example for practical application is the reconstruction of the optically recorded sound of old movies, in which the movie-film has static nonlinear intensity characteristics. This nonlinearity can be extremely strong, especially when the film was badly developed, which causes a distorted sound.

On the professional film, the sound is optically recorded. Today, the transversal recording technique is used, where the sound information is carried by the width of the sound-stripe. This is a very safe method, because the development and the strength of the recording light cannot harm the linearity of the sound characteristics. However, until the 1950s, the variable density recording was used. In this method, the sound information is carried by the darkness (density) of the sound stripe. At a high volume level of the sound, or a wrong working point of the development, the changing of the density can come out from the linear domain of the characteristics and the recorded sound can be strongly distorted and incomprehensible. In addition, a strong noise is added to the old film produced by the chemical decay of the film. The signal-to-noise ratio could be worse than 30 dB. The sound cannot be properly restored by the inverse of the density characteristics due to the strong noise.

The algorithm was tested on these distorted sound signals of real movie-films and gave good results. Experiments show that human hearing is less sensitive for small distortions than for the amplified noise.

VII. Conclusion

The reconstruction of nonlinearly distorted signals was examined, where the signals were distorted by a strong static nonlinearity and the distorted signals were corrupted by noise. The effects of the noise were taken into account. The noisy signal cannot be restored directly with the inverse of the nonlinear characteristics, because the noise will be extremely amplified. A method was shown, which is a point-by-point correction of the recorded data for the nonlinearity. The method based on Tikhonov's regularization operators can provide a trade-off between noisy and biased reconstruction. The performance of the method was shown on simulated signals.

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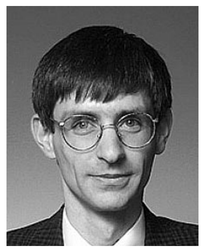

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