IEEE Instrumentation and Measurement
Technology Conference
Budapest, Hungary, May 21-23, 2001.

Reconstruction of Nonlinearly Distorted Signals With Regularized Inverse Characteristics

Tamás B. Bakó and Tamás Dabóczi
Department of Measurement and Information Systems
Budapest University of Technology and Economics, H-1521 Budapest, Hungary
Phone: +36 1 463-2057, Fax: +36 1 463-4112, email: {bako,daboczi}@mit.bme.hu

Abstract ***– A signal distorted by a static, invertable, nonlinear characteristics can be exactly restored in the absence of noise. In this case the inverse of the characteristics can be used. When noise is superimposed to the distorted signal, the inverse characteristics may not be proper, because the noise is strongly amplified. This noise has to be suppressed in the reconstructed signal, which can be accomplished only at the price of bias. This article presents a method to compensate the effect of static nonlinearities in the presence of noise. The method is based on Tikhonov's regularization operators and provides a compromise between noisy and biased estimates. The proposed algorithm has been succesfully applied to reconstruct the sound of old movie-films.***

Keywords – Nonlinear distortion, signal reconstruction, regularization, static nonlinearity.

I. INTRODUCTION

Nonlinearity of measurement systems or communication channels distorts the measured or transmitted signal. If the distortion is unacceptable the detected signal should be post processed to reconstruct the original one. An example is the optically recorded sound of old movie-films. The movie-film has a static nonlinear intensity-characteristics. This nonlinearity can be extremely strong, especially when the film was badly developed, which causes a distorted sound. This distortion can be (partly) compensated, assuming that the nonlinear function is known.

The nonlinearly distorted signal is usually corrupted by noise. In this case the inverse of the nonlinearity may not be optimal for reconstruction, because the noise is amplified during the reconstruction process. An inverse characteristics is needed, which suppresses the noise.

Preliminaries

A lot of work deals with identification of nonlinear systems, which is important for nonlinear control or for signal reconstruction. Some work deals with compensation of nonlinear systems. These works usually use Volterra-kernels to describe the nonlinearity of the system, because this method can handle a wide range of nonlinearities. The work in [1] describes a least squares approach for identification and equalization of a very generalized class of nonlinear and IIR systems using Volterra-filters. The aim of this work is to find a Pth-order Volterra-filter by using signal patterns, which makes optimal equalization on a communication channel. A fixed point approach of nonlinear equalization is described in [2]. The aim was here to find stable filter systems to equalize nonlinear communication channels. A Volterra-kernel based compensation filter is described in [3]. This filter reduces the strong static and dynamic nonlinear distortions of horn loudspeakers.

A bit different method is used for nonlinear compensation in [4]. In this work a static, nonlinear system (a cathod ray tube) is described. The static nonlinearity is approximated by a polinom. The compensation of the distortion is implemented by the inverse of this polinom.

In most of the situations, noise can cause problems. At [4] the noise level was relatively small, therefore eliminating the effects of noise was not necessary. At [3] a prefiltering process solved the problem. However, in most cases, we do not have access to the input of the nonlinearity, due to the physical structure of the system. In other cases we have only distorted and noisy recordings, which are not repeatable. In these cases only post-processing techniques can be used and we have to handle the effects of noise.

Novelties

As these previous works state, the effects of noise in the proposed algorithms are not clearly described. Further work is needed to establish the effect of noise in [1], and the analysis of the solution in [2] is also a purely deterministic approach. In [3] a prefiltering technique was used, and in [4] the effects of noise was not handled.

The proposed method in our paper works on static nonlinearities and takes the effect of the noise into account. The emphasis is to find an inverse characteristics of a known static and invertable nonlinear function, which handles the propagation of the noise through the nonlinear inversion. In Section II, we define the model of the nonlinear distortion and the reconstruction process. In Section III, the problems of the reconstruction will be discussed. Next, in Section IV, the proposed method with regularization operators will be shown. Section V includes a simulation example, demonstrating the result, which can be achieved by regularized inverse characteristics. In Section VI, a practical application will be discussed. Conclusions are given in Section VII.

II. MODELING THE DISTORTION

The signal model of the reconstruction process can be seen in Figure 1, where $N(x)$ denotes the nonlinear function of the measurement system, x denotes the input and o denotes the distorted output of the system where $o = N(x)$.

The observation o is disturbed by the measurement noise, n, that is modeled as an additive one. $K(y)$ is the inverse nonlinear function and $\hat{x}$ is the estimation about the input signal.

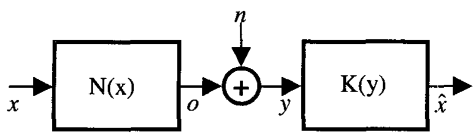

Fig. 1 Signal model of the reconstruction process

The mathematical analysis of this model is difficult, because superposition cannot be applied in the nonlinear environment, and, in general, the nonlinear equations cannot be analytically solved. However, in a given working-point, at small x alterations we can approximate o with the first elements of the Taylor polynomial, which will be a linear approximation:

$$o = o_0 + \Delta o \approx N(x_0) + \left.\frac{dN(x)}{dx}\right|_{x=x_0} \cdot \Delta x . \tag{1}$$

So the alteration of o is:

$$\Delta o \approx \left.\frac{dN(x)}{dx}\right|_{x=x_0} \cdot \Delta x . \tag{2}$$

The reconstruction process for the alteration is sown in Figure 2.

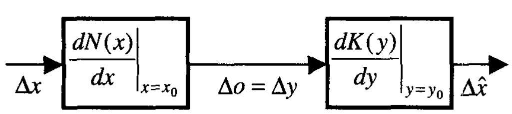

Fig. 2 Signal model of the reconstruction process for small alterations

This model is already linear and we can apply this model in each point of the original characteristics. The noise in this model is not additive, but affects the working-point. If the amplification of the second box is not the reciprocal of the first one because of the noise, it causes differences between Δx and $\Delta\hat{x}$.

III. DIFFICULTIES OF RECONSTRUCTION

Let us define the best solution as the minimum of the norm of the difference of Δx and $\Delta\hat{x}$ in the linear model:

$$Cost = \min_{\Delta\hat{x}} \left(\|\Delta x - \Delta\hat{x}\| \right), \tag{4}$$

where $\|a\|$ is the norm of a. The solution of this error criterion is not applicable because it requires the knowledge of Δx.

If we choose $\|\Delta y - \Delta\hat{y}\|$ for error criterion (output error criterion), it leads to the following solution if the norm is the l_2 (Euclidean) norm:

$$\left.\frac{\Delta\hat{x}}{\Delta y}\right|_{y=N(x_0)} = \frac{dK(y)}{dy} = \left.\frac{1}{\frac{dN(\hat{x})}{d\hat{x}}}\right|_{\hat{x}=\hat{x}_0} \tag{5}$$

This solution means that $K(y)$ is the inverse of $N(x)$ in the original reconstruction process. The problem of this solution is that the noise is amplified at $\hat{x}$ during reconstruction. If the noise level is small, the estimation can be rewritten into the following form:

$$\hat{x}_0 = N^{-1}(y_0) = N^{-1}(o_0 + n) \approx x_0 + \frac{1}{\left.\frac{dN(o)}{do}\right|_{o=o_0}} \cdot n \tag{6}$$

As we can see, the noise will be amplified, where the derivative of the nonlinear function of the system is small. The criterion of the best solution needs to be redefined. This will be introduced in the next section.

IV. REGULARIZED COMPENSATION

Our problem at the derivative system will produce an ill-posed integral-equation at the original system. Solution of ill-posed equations was originally proposed by Tikhonov [5], who created a method to solve ill-posed integral equations with regularization operators. The error criterion used in Tikhonov's method is an extension of the output error criterion. One possible form is:

$$Cost = \min_{\Delta\hat{x}} \left(\|\Delta y - \Delta\hat{y}\| + \lambda \|\Delta\hat{x}\| \right) \tag{7}$$

In practice the l_2 norm is used, because minimizing the l_2 norm of the error minimizes the energy of it. In the case of sampled signals we can write:

$$Cost = \min_{\Delta\hat{x}_i}\left(\sum_i (\Delta y_i - \Delta\hat{y}_i)^2 + \lambda\cdot\sum_i \Delta\hat{x}_i^2\right) \quad (8)$$

The minimum of this equation is at the point where $\frac{\partial}{\partial\Delta\hat{x}_i} = 0$:

$$\sum_i 2\left(\left.\frac{dN(\hat{x})}{d\hat{x}}\right|_{\hat{x}=\hat{x}_0}\right)^2 \cdot \Delta\hat{x}_i - 2\left.\frac{dN(\hat{x})}{d\hat{x}}\right|_{\hat{x}=\hat{x}_0} \cdot \Delta y_i + 2\lambda\hat{x}_i = 0 \quad (9)$$

If we assume that all terms in this sum are equal to zero, the solution for $\Delta\hat{x}/\Delta y$ is:

$$\frac{\Delta\hat{x}}{\Delta y} = \left.\frac{dK(y)}{dy}\right|_{y=y_0} = \frac{\left.\frac{dN(\hat{x})}{d\hat{x}}\right|_{\hat{x}=\hat{x}_0}}{\left(\left.\frac{dN(\hat{x})}{d\hat{x}}\right|_{\hat{x}=\hat{x}_0}\right)^2 + \lambda} \quad (10)$$

We can create the regularized $K(y)$ inverse filter of the nonlinear system with the integration of (10), but we need a condition to determine the integration constant. If $K(y) = F(y) + C$, we can determine the value of C by minimizing $\|y - \hat{y}\|$ in the function of C. If the norm is the Euclidean-norm, the constraint is the solution of the following equation:

$$E\{N(F(y)+C)\} - E\{y\} = 0 . \quad (11)$$

The regularized inverse characteristics of the nonlinear system can be computed by numerical integration from (10) and (11). The resulted characteristics can be used as $K(y)$, without any further iteration, thus the reconstruction itself is a one-step process.

V. SIMULATION EXAMPLE

To illustrate the proposed algorithm, we solve a simple example using a sinusoid input signal, $x = \sin(2pi/1024\cdot t) + 1.3$, and an error-function nonlinearity, $o = erf(x)$. We add a Gaussian noise to o with ≈ 50 dB signal-to-noise ratio. The original and the distorted, noisy signal can be seen in Figure 3. The inverse of the nonlinear function and the regularized inverses for $\lambda = 0.01$ and $\lambda = 0.1$ can be seen in Figure 4.

The unregularized inverse characteristics has steep parts. This will cause high oscillations in the reconstructed signal. In the regularized inverse characteristics, the rise of the curves, hence the oscillations are limited. At $\lambda = 0.01$, the steep parts are distorted, but the flat parts have very low distortion. At $\lambda = 0.1$, the whole characteristic is strongly distorted already. The optimum is somewhere between the extremes.

Figure 5 shows the estimations. In the first figure, the estimation reached by the inverse of the nonlinear function can be seen. The maximum amplitude of the noise is about 100 times higher than that of the original signal. In the middle figure, the estimation reached by regularized inverse for $\lambda = 0.01$ can be seen and in the right figure for $\lambda = 0.1$.

The oscillation of the signal, caused by the noise, is small at $\lambda = 0.1$, but the distortion caused by the regularized inverse is quite high. The unregularized inverse has no bias from regularization, but the oscillation caused by the noise is extremely high. We can find a trade-off between noise and bias by choosing a proper λ value.

VI. A PRACTICAL APPLICATION

The proposed technique can be effectively used in the sound-restoration of old movie-films. On the professional (35 mm wide) film, the sound is optically recorded. Nowadays the transversal recording technique is used, where the sound information is carried by the width of the sound-stripe (left of Figure 6). This is a very safe method, because the development and the strength of the recording light cannot harm the linearity of the sound characteristics. However, until the 1950's, the intensity recording was used (right of Figure 6). In this method, the sound-information is carried by the darkness (density) of the sound stripe. The density-characteristics of the film is a static nonlinear function of the intensity (Figure 7). At a high volume level of the sound, or a wrong working-point of the development, the changing of the intensity can come out from the linear domain of the density characteristics and the recorded sound can be strongly distorted and incomprehensible. In addition, a strong noise is superimposed on the old film produced by the chemical decay of the film. The decay causes optical imperfections in the film, hence in the optical sound-band, and these imperfections produce also a strong wide-band noise during playing. The signal-to-noise ratio could be worse than 30 dB. The sound cannot be properly restored by the inverse of the density characteristics due to the strong noise.

If we use the inverse of the density-characteristics, the noise will be extremely amplified. The strong noise and modulation disturbs the artistic enjoyment of the film. The proposed algorithm reduces these disturbing effects as the price of some bias.

The algorithm was tested on distorted sound-signals of real movie-films and gave good results. Experiments shows that human hearing is less sensitive for small distortions, than for strong and modulated noise.

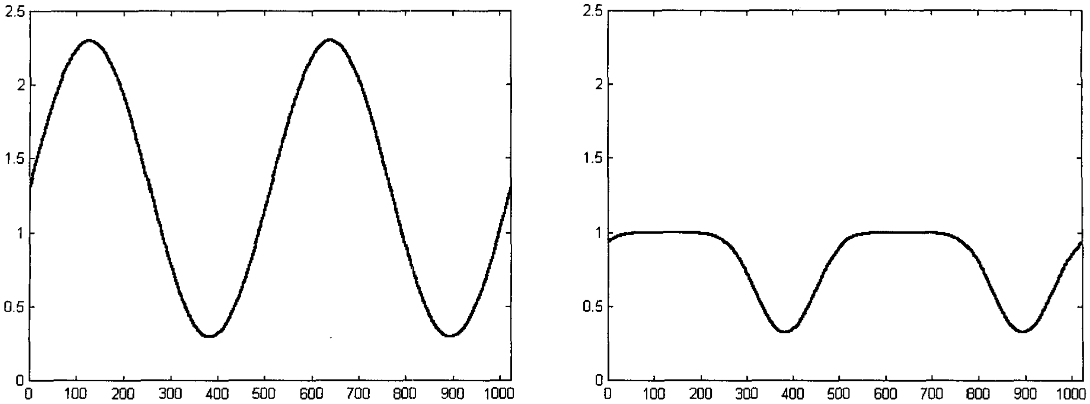

Fig. 3 Input signal (left) and the nonlinearly distorted, noisy signal (right)

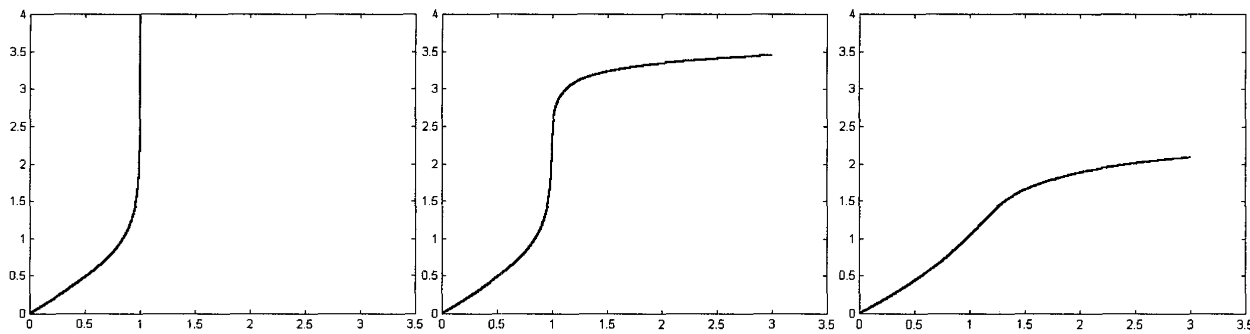

Fig. 4 Unregularized inverse characteristics (left), regularized inverse characteristics for $\lambda = 0.01$ (middle) and $\lambda = 0.1$ (right)

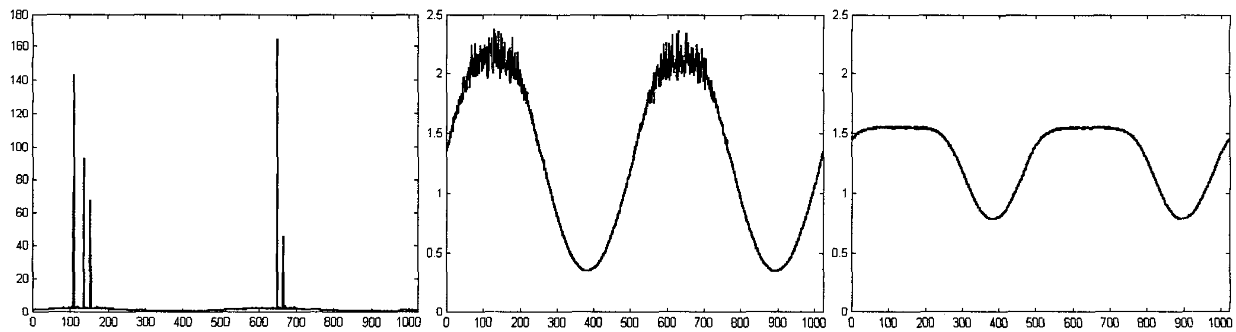

Fig. 5 Reconstructed signals: with unregularized inverse characteristics (left), with regularized inverse characteristics for $\lambda = 0.01$ (middle) and $\lambda = 0.1$ (right). Note that the scale of the figure of the unregularized characteristic is larger because of the high noise amplitude

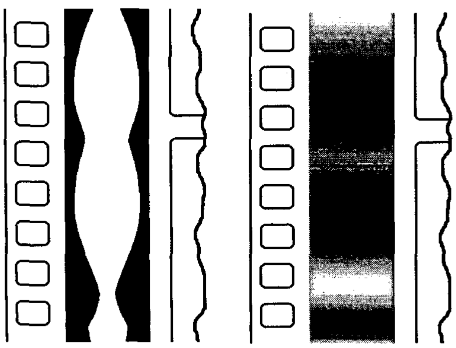

Fig. 6 The shape of the sound band on movie-films created with transversal (left) and intensity (right) recording method

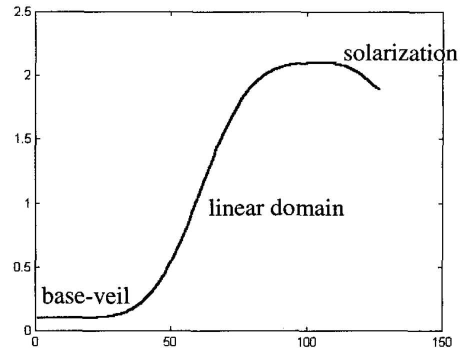

Fig. 7 The density characteristics of the film

VII. CONCLUSIONS

Reconstruction of nonlinearly distorted signals were examined, where the signals were distorted by a strong static nonlinearity and the distorted signals were corrupted by noise. The effects of the noise were taken into account. The noisy signal cannot be restored directly with the inverse of the nonlinear characteristics, because the noise would be extremely amplified. A method was shown, based on Tikhonov's regularization operators that can provide a trade-off between noisy and biased reconstruction. The performance of the method was shown on simulated signals. The proposed algorithm has been succesfully applied to reconstruct the sound of old movie-films.

ACKNOWLEDGEMENT

We would like to thank the help of the Hungarian National Film Archive. The support of the Hungarian Scientific Research Fund (No. OTKA F-026136 and OTKA F-034900) and the Research and Developement Fund for Higher Education (Ref.:FKFP-0098/2001) are also acknowledged.

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