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Nonparametric Identification and Signal Reconstruction as two Consecutive Deconvolution Steps

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Abstract *– Inverse filtering of time domain signals is investigated. Inverse filtering requires the knowledge of the transfer function of the measurement system, which can be estimated on the base of measurements (system identification). The quality of the system identification influences the quality of the signal reconstruction. We investigate the influence of the identification on the signal reconstruction in the case of ill-posed problems. It is shown that overfiltering the noise in the identification stage introduces bias in the pass- and transition region of the transfer function, which causes trouble in the signal reconstruction stage. Underfiltering the noise in the identification stage also causes bias, but its effect is mostly in the stopband, which can be suppressed in the signal reconstruction stage.*

Keywords *– signal reconstruction, inverse filtering, deconvolution, signal processing, identification, ill-posed problem.*

I. INTRODUCTION

The quality of the measurement of time domain waveforms is often limited by the finite bandwidth of the measurement system. This means that the signal to be measured is distorted, sharp edges are smoothed, peak locations are delayed etc. If the level of the distortion is not acceptable, a better (higher bandwidth) measurement system has to be used. If it is available, this is the best way to improve the quality. However, many times the bandwidth cannot be increased more, because we already reached the technical limits (e.g. calibration instruments). Another constraint might be if the improvement of the measurement system is uneconomic. In both cases the solution might be the digital compensation of the distortion by postprocessing the measured data. This procedure is called inverse filtering or deconvolution.

Inverse filtering is an ill-posed problem. This means that small changes in the measurement data caused by the noise leads to large changes in the reconstructed data [1]. The amplified noise has to be suppressed, which can be done only at the price of introducing bias to the estimate. Many algorithms are proposed in the literature to reduce the noise and keep a balance between bias and variance [1]-[4].

Inverse filtering requires the knowledge of the distortion, i.e. the knowledge of the transfer function of the measurement system. The transfer function is estimated on the base of measurements. This procedure is called system identification. If the system is linear and time-shift invariant, nonparametric system identification is also a deconvolution problem, as in the case of signal reconstruction. The difference is that the impulse response is estimated instead of the excitation signal. System identification is also an ill-posed problem, i.e. measurement noise strongly influences the estimate of the impulse response.

Restoration of such distorted signals requires two steps one after the other: first an identification step and then a signal restoration phase. Obviously the quality of the second deconvolution depends on the quality of the first one.

In this study we investigated the influence of the noise reduction of the system identification stage on the quality of the signal reconstruction.

II. VISUALIZATION OF THE PROBLEM

Let us consider the following linear and time-shift invariant system, and the method of restoration, shown in Fig. 1. We can write in the frequency domain:

$$\begin{aligned} Z(f) &= X(f)H(f) + N(f), \\ \hat{X}(f) &= Z(f)K\left(\hat{H}(f), f\right) \\ &= X(f)H(f)K\left(\hat{H}(f), f\right) + N(f)K\left(\hat{H}(f), f\right), \end{aligned} \tag{1}$$

where capital letters correspond to the Fourier transform of the signals. Note that the inverse filter depends on the estimate of the transfer function of the measurement system instead of the true transfer function. The best way to follow the noise reduction is to split the inverse filter into a cascade system; the first stage compensates the measurement system in the least squares sense (minimizing the prediction error), and the second one reduces the noise while keeping balance between bias and variance. This second part is often called regularization filter, because it is responsible to regularize (suppress) the amplified noise (Fig. 2). Similarly, the deconvolution filter of the system identification can also be separated (Fig. 3).

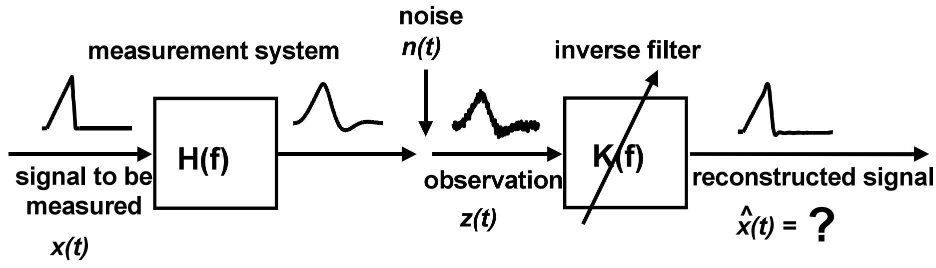

Fig. 1. Setup of the measurement and reconstruction system

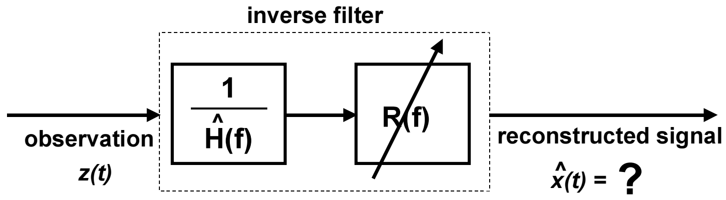

Fig. 2. Deconvolution in the case of signal reconstruction

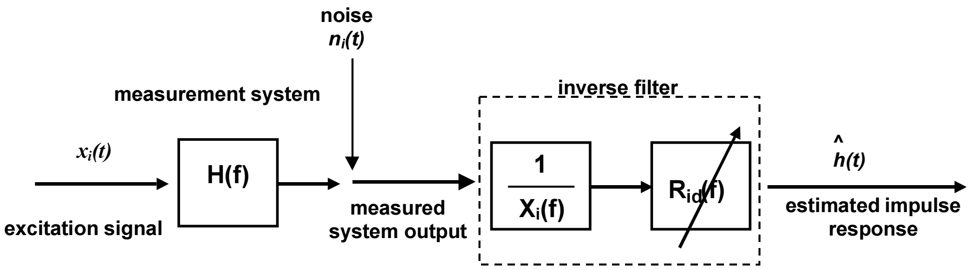

Fig. 3. Deconvolution in the case of system identification

From the above separation it is obvious that the stronger we suppress the noise in the system identification phase, the stronger we will amplify the noise in the signal reconstruction phase, because we divide by $\hat{H}(f)$:

$$\hat{X}(f) = \frac{Z(f)}{\hat{H}(f)} R(f), \tag{2}$$

where *Z(f)* is the Fourier transform of the measured signal.

Large noise suppression in the system identification phase increases the ill-posedness of the signal reconstruction task, even if the noise is suppressed in the stopband, where not much information can be extracted from the identification measurement. There is an optimal regularization level for the system identification, which leads to the best signal reconstruction.

The question is, however, if we have noisy identification measurements and the level of good regularization can be set only with an uncertainty, which is the safe side? To suppress the noise a little bit more than the optimal (which is not known), and receive a smooth estimate for the impulse response, or underregularize the estimate? In the next section we will answer this question.

III. ERROR ANALYSIS OF THE SIGNAL RECONSTRUCTION

Let us consider a measurement system which can be well modeled with a second order lowpass filter. We will excite the system with a rectangle signal to identify its transfer function (Fig. 4 and 5).

For simplicity let us use the Tikhonov's regularization method to suppress the noise in both the system identification and signal reconstruction steps [5, 6]. The optimal estimate of the transfer function with this type of regularization filter is depicted in Fig. 6. (Optimum is defined from the point of view of the reconstructed signal, and not the identification itself. The optimum is the transfer function, which leads to the minimal deviation of the reconstructed signal from the true one in least squares sense.)

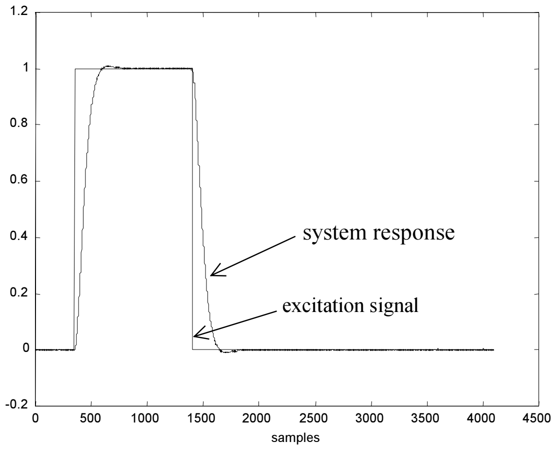

Fig. 4. Excitation signal and system response for the identification

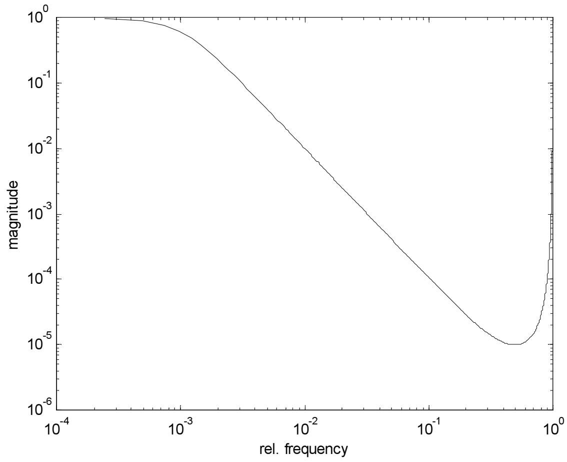

Fig. 5. Transfer function of the system

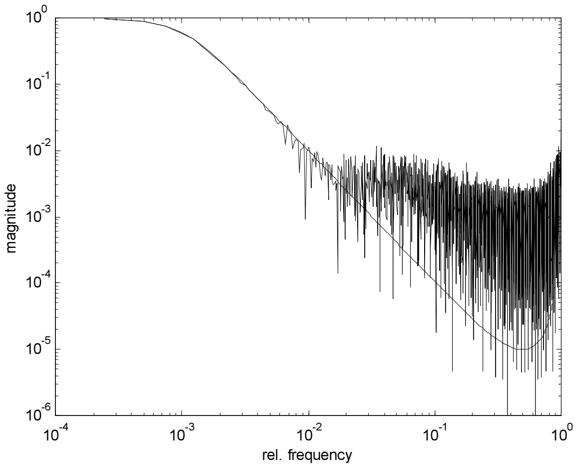

Fig. 6. Optimal estimate of the transfer function.
True transfer function (smooth line) and the estimate (noisy line).

Using the above estimate for the transfer function let's try to reconstruct two Gaussian shaped pulses distorted with the second order system. Fig. 7 shows the signal to be reconstructed and the simulated noisy system response (noise level is small, hard to observe on the picture). Fig. 8 shows the optimal reconstruction using the Tikhonov's regularization.

The level of noise suppression (regularization) can be set either manually or automatically. If it is set manually, it needs expertise, and introduces subjectivity. It can also be set automatically, based on heuristics or signal models ([1, 4, 7]). In both cases there is a chance that the chosen noise suppression differs from the optimum.

Let us investigate what happens if the system identification is over- or underregularized. In Fig. 9 and 10 the transfer function estimate is under- and overregularized with the same amount. (Same amount is meant by providing the same error in least squares sense. The error is defined as the deviation of the impulse response estimate from the true one.) Underregularization consists more noise at the stop band, less bias in the transition region, while overregularization introduces more bias at the transition region and less noise at the stopband.

Using the under- and overregularized estimates the best possible reconstructions with the given regularization filter is depicted in Fig. 11 and 12. The deviations of the estimates from the true input signal are the following (sum of squared error): cost_optimum=0.47, cost_underregularized=0.59, cost_overregularized=4.42.

As it can bee seen from the simulation example, the amplified noise in the transfer function estimate is concentrated at the stopband. In the signal reconstruction stage, stopband information is lost more or less because the noise is highly amplified (ill-posed problem). This region will be suppressed anyhow, thus it is less important to reconstruct the transfer function in this region than to reconstruct it in the pass- and transition bands. Moreover, the amplified noise in the stopband acts as regularization, since in the signal reconstruction stage the ill-posedness of the problem is decreased ($1/\hat{H}(f)$ will be smaller).

The bias of the transfer function estimate in the pass- and transition bands, however, misleads the signal reconstruction. In these regions the bias introduced in the identification stage causes bias in the signal reconstruction, which cannot be corrected.

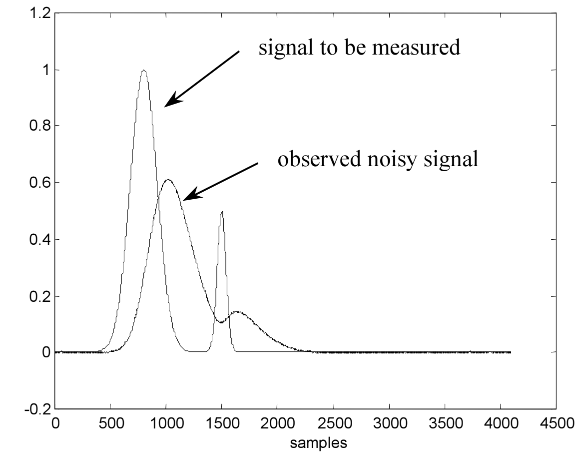

Fig. 7. Signal to be measured and observed signal

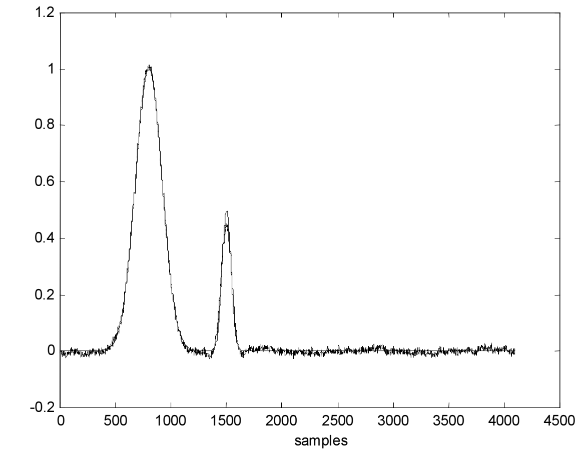

Fig. 8. Optimal reconstruction. Reconstructed signal (noisy curve) and signal to be measured (smooth curve).

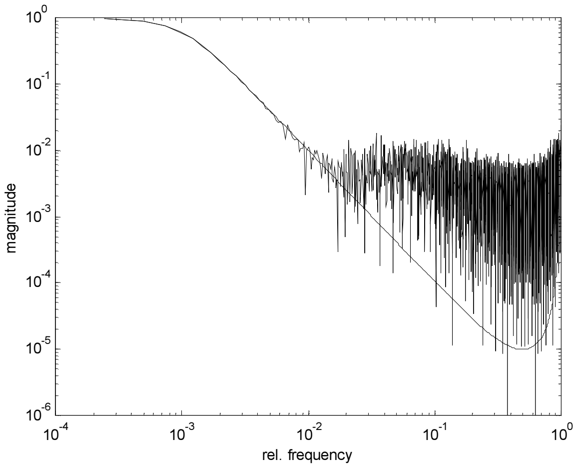

Fig. 9. Underregularized estimate of the transfer function. True transfer function (smooth line) and the estimate (noisy line).

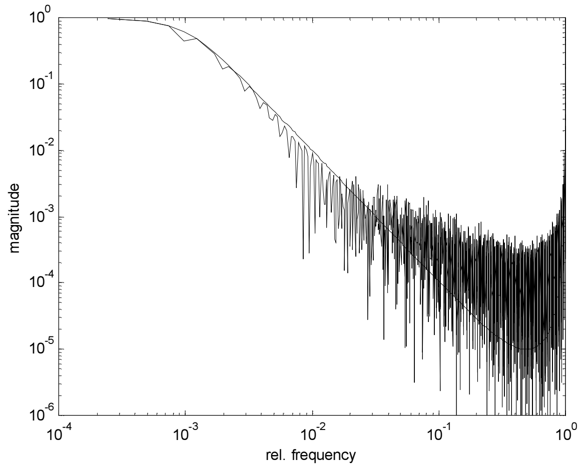

Fig. 10. Overregularized estimate of the transfer function. True transfer function (smooth line) and the estimate (noisy line).

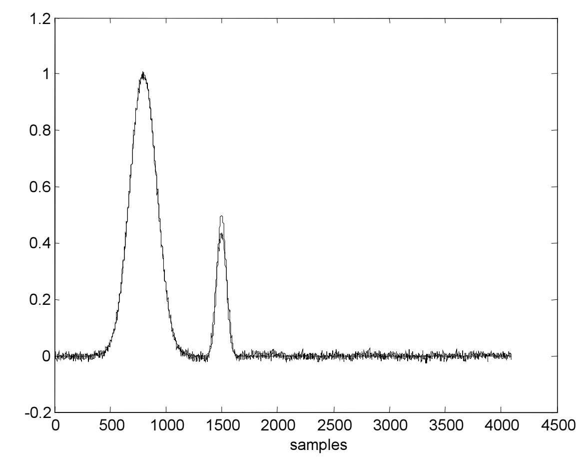

Fig. 11. Signal reconstruction using underregularized transfer function estimate. Reconstructed signal (noisy curve) and signal to be measured (smooth curve).

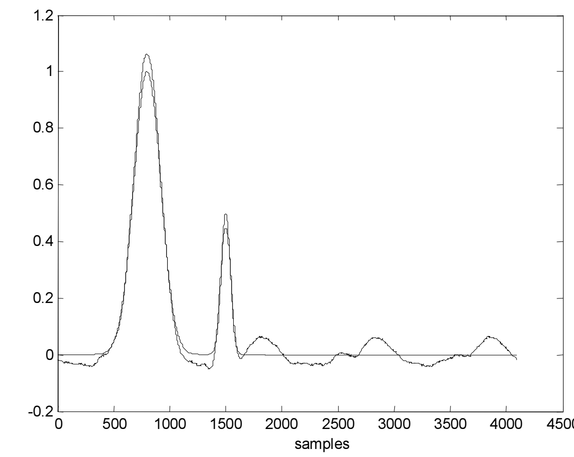

Fig. 12. Signal reconstruction using overregularized transfer function estimate. Reconstructed signal (oscillating curve) and signal to be measured (smooth curve).

Let us have a look at the error surface of the signal reconstruction as a function of regularizations in the system identification and signal reconstruction phases. In Fig. 13 lambda means the regularization parameter on logatihmic scale. Larger number means more regularization, i.e. larger noise suppression.

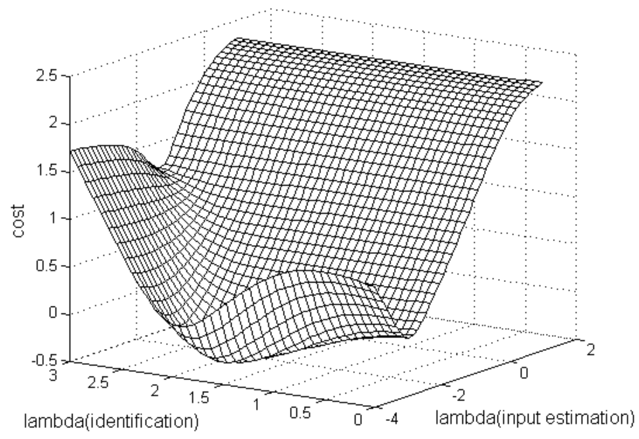

Fig. 13. Error surface of the signal reconstruction as a function of the regularization amount in both the system identification stage (lambda(identifiication)), and the signal reconstruction stage (lambda(input estimation)).

It can be observed that the slope of the surface becomes larger if we move to the larger regularization direction in the system identification axis, than if we move to the moderate regularization direction. This also visualizes that a slight underregularization is less harmful than overregularization in the system identification step.

The problem is the same, if we use parametric identification (i.e. providing a model for the system). The question is then how to choose the model. And the answer is: in the stopband it is better to model a moderate suppression instead of a high one, because a high suppression increases the ill-posedness of the signal reconstruction problem (see again Fig. 11 and 12).

IV. CONCLUSION

Inverse filtering of time domain signals has been investigated. Inverse filtering requires the knowledge of the transfer function of the measurement system, which can be estimated on the base of measurements (system identification). The quality of the system identification influences the quality of the signal reconstruction. It has been shown that overfiltering the noise in the identification stage introduces bias in the pass- and transition region of the transfer function, which is disadvantageous.

Our recommendation: provide the best identification you can. But if you are uncertain, stay on the safe side. And the safe side is to allow more noise amplification in the identification step (not to overregularize in the identification step), instead of introducing bias in critical regions by overfiltering the estimate of the transfer function.

V. REFERENCES

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