Inverse Filtering of Optical Images

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***Abstract*—The quality of images is limited by the performance of the optical system used. The imperfections of the optical system cause distortion of the image. If the distortion is known it can be (partly) compensated. This procedure is called inverse filtering. The problem is, however, ill-posed, which means that the measurement noise is amplified by the inverse filtering process. Suppression of the noise causes bias in the reconstruction. A tradeoff has to be found between the noisy and biased estimates. In this paper, the reconstruction of images will be investigated, assuming that the distortion of the optical system is known. An algorithm will be introduced to estimate the optimal level of noise suppression of the two-dimensional inverse filter.**

***Index Terms*—Deconvolution, defocus, image reconstruction, inverse filtering.**

I. Introduction

IMAGES are recorded by optical systems. However, the performance of these systems is limited. There are different types of errors in such a system. Certain errors exist even in optical systems consisting of perfect spherical lenses. Spherical aberration, coma, chromatic aberration, pincushion distortion, etc., belong to this kind of distortion [1]. Other errors are caused by the imperfect manufacturing or the normal wear of the optical system.

Modern lens systems are compensated for many types of errors. However, they cannot be compensated for all of them. In many cases a certain error can be compensated only for a limited range of usage (e.g., spherical aberration can be compensated for certain object and image distances). The imperfection of manufacturing always remains a limitation. A good example is the zoom lens of a photo or video camera. The focal length is changed by moving different lenses in different ways together. A small hysteresis of the moving mechanism of the lenses causes blurred (out of focus) images.

Even the user of the optical system can cause distortion by adjusting the system imperfectly. Consider an example from photography, where the photographer has to adjust the object distance of the lens. If the object distance is set automatically by an "autofocus" circuitry, the system can set an unimportant object to be sharp. A typical example is that of two people standing next to each other who are out of focus while the tree far behind them is sharp.

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A. Modeling the Distortion

An ideal imaging system establishes a point-by-point correspondence between the object and the image. This means that the image of a point is a point in the image plane. In real optical systems, however, the intensity of the light is dispersed over an extended area [2]. The distribution of the intensity is called a "point spread function." This effect causes a blurred image. If the spread of the light is space-shift-invariant, and linear, this distortion can be described by a two-dimensional (2–D) convolution, i.e., filtering with the point spread function. A reasonable assumption for the measurement noise is that it is additive at the output of the system

$$i(x,y) = o(x,y) * \mathrm{psf}(x,y) + n(x,y) \tag{1}$$

where

$o(x,y)$	intensity of the object;
$i(x,y)$	intensity of the image;
$\mathrm{psf}(x,y)$	space invariant point spread function;
$n(x,y)$	noise;
$*$	convolution.

The above relationship also can be described in the Fourier domain

$$I(f_x,f_y) = O(f_x,f_y)\mathrm{PSF}(f_x,f_y) + N(f_x,f_y) \tag{2}$$

where capital letters stand for the Fourier transform of the corresponding signals. Convolution in the space domain becomes multiplication in the Fourier domain.

B. Reconstruction

The image can be (partly) compensated for the distortion, assuming that the point spread function is known and the optical system is linear and space-shift invariant. This procedure is called inverse filtering or deconvolution.

Inverse filtering is usually an ill-posed problem [3], [4], i.e., measurement noise is amplified to a great extent. The amplified noise has to be suppressed at the price of bias in the estimate. A tradeoff has to be found between the biased and noisy reconstruction. The inverse filter has to compensate for the effect of the measurement system in the pass and attenuation bands, but in the stopband, it has to suppress the noise. The Fourier transform of the estimated intensity of the object $O_{\mathrm{est}}(f_x,f_y)$ is given by

$$O_{\mathrm{est}}(f_x,f_y) = I(f_x,f_y)K(f_x,f_y) \tag{3}$$

where $K(f_x,f_y)$ is the transfer function of the inverse filter. The noise reduction is usually controlled by only one parameter (e.g., cutoff frequency of a lowpass filter, regularization parameter, number of maximal iterations, etc.).

In order to assure repeatability and eliminate subjectivity, the level of noise reduction must be set automatically. In this paper, we will describe a model-based algorithm that automatically sets the optimal level of noise reduction for 2-D images. The method is based on a previous work [6], developed for one-dimensional (1–D), time-domain transient measurements [7]. The algorithm minimizes the approximate error of the estimate in a least squares sense. The technique utilizes rough Fourier domain models of the signals. The models are built automatically from the measurement, and no human interaction is required.

In Section II, the automatic inverse filtering algorithm will be recalled, and the 1-D algorithm will be extended to 2-D signals. In Section III, the algorithm will be checked on images, and it will be shown that the proposed optimization technique is useful for 2-D signals, as well.

II. Inverse Filtering

A. Automatic Deconvolution

The optimal reconstruction is defined for which the sum of the squared error is minimal. The error function can be written both in the time and frequency domains, utilizing Parseval's theorem

$$\begin{aligned}\mathrm{EE} &= \sum_{x=0}^{N_x-1}\sum_{y=0}^{N_y-1}(o(x,y)-o_{\mathrm{est}}(x,y))^2\\ &= \frac{1}{N_xN_y}\sum_{f_x=0}^{N_x-1}\sum_{f_y=0}^{N_y-1}|O(f_x,f_y)-O_{\mathrm{est}}(f_x,f_y)|^2 \end{aligned}\tag{4}$$

where EE denotes the energy of the error, and N_x and N_y are the dimensions of the sampled image. The core of the last sum in (4) can be expanded by substituting (3) and (2) into $O_{\mathrm{est}}(f_x,f_y)$

$$\begin{aligned}&|O(f_x,f_y)-O_{\mathrm{est}}(f_x,f_y)|^2\\ &\quad=|O(f_x,f_y)-O(f_x,f_y)\mathrm{PSF}(f_x,f_y)K(f_x,f_y,p)\\ &\qquad-N(f_x,f_y)K(f_x,f_y,p)|^2\\ &\quad=|O(f_x,f_y)(1-\mathrm{PSF}(f_x,f_y)K(f_x,f_y,p))|^2\\ &\qquad+|N(f_x,f_y)K(f_x,f_y,p)|^2\\ &\qquad-2|O(f_x,f_y)(1-\mathrm{PSF}(f_x,f_y)K(f_x,f_y,p))|\\ &\qquad\cdot|N(f_x,f_y)K(f_x,f_y,p)|\cos(\varphi(f_x,f_y))\end{aligned}\tag{5}$$

where $\varphi(f_x,f_y)$ denotes the phase angle of the two absolute valued terms in the last sum and $K(f_x,f_y,p)$ denotes the inverse filter having parameter(s) p to control the level of noise reduction. The core of the cost function is split into three terms

$$\mathrm{EE}=\mathrm{EE}_{\mathrm{bias}}+\mathrm{EE}_{\mathrm{noise}}+\mathrm{EE}_{\mathrm{bias,noise}}\tag{6}$$

where subscript *bias* stands for the bias of the estimate, subscript *noise* for the noise while *bias, noise* denotes their cross connection. The following approximations will be used to compute the cost function:

- the $\mathrm{EE}_{\mathrm{bias,noise}}$ term will be neglected;
- instead of the absolute values of the signal and noise spectra, an approximate spectral model will be substituted into $O(f_x,f_y)$ and $N(f_x,f_y)$.

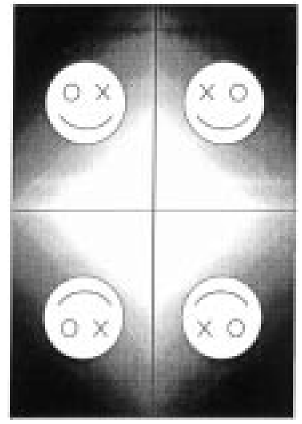

Fig. 1. Extension of images to reduce the effect of intensity step on opposite edges.

The cost function is then the following:

$$\begin{aligned}\mathrm{cost}=&\frac{1}{N_xN_y}\sum_{f_x=0}^{N_x-1}\sum_{f_y=0}^{N_y-1}|O_{\mathrm{mod}}(f_x,f_y)|^2\\ &\cdot|1-\mathrm{PSF}(f_x,f_y)K(f_x,f_y,p)|^2\\ &+\frac{1}{N_xN_y}\sum_{f_x=0}^{N_x-1}\sum_{f_y=0}^{N_y-1}|N_{\mathrm{mod}}(f_x,f_y)|^2|K(f_x,f_y,p)|^2\end{aligned}\tag{7}$$

where subscript mod denotes a frequency-domain model of the absolute value of the spectra of corresponding signals. The cost function has to be minimized with respect to parameter(s) p in the inverse filter. The new cost function does not require the knowledge of the intensity of the object. Only an approximate model has to be provided for the absolute value of its spectrum.

B. Fourier Transform of the Signals

The cost function (7) requires the calculation of frequency-domain data. Convolution in the frequency domain corresponds to multiplication in the spacial domain; however, convolution becomes circular, since signals are assumed to be periodic. For time-domain transient data, this effect is usually reduced by padding zeros to the back of the record before computing the Fourier transform. Images, however, are not transient signals; two points on opposite edges have different intensities. To reduce the effect originated from the intensity step along opposite edges, we extended the images with their flipped versions (see Fig. 1).

C. Modeling of Signals

The signal models are built automatically in the same way as was proposed for 1-D transient signals [6]. White noise will be assumed for the noise. Its level can be either *a priori*, measured or extracted from the high-frequency part of the spectrum of the noisy image. The absolute value of the spectrum of the undistorted image will be modeled iteratively, starting from a rough model (noisy and distorted measurement), and improving it in several steps by substituting the result of the estimated image into the model.

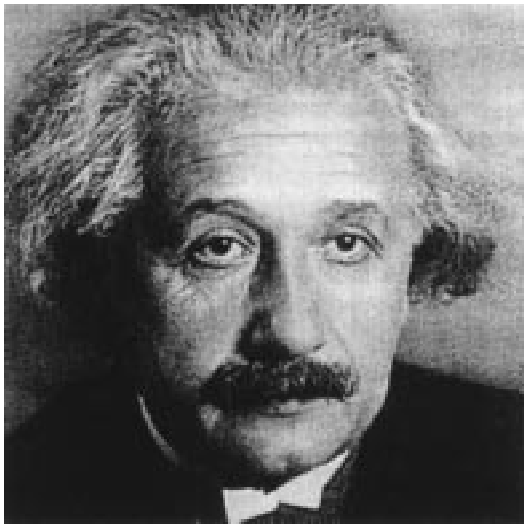

Fig. 2 Original picture.

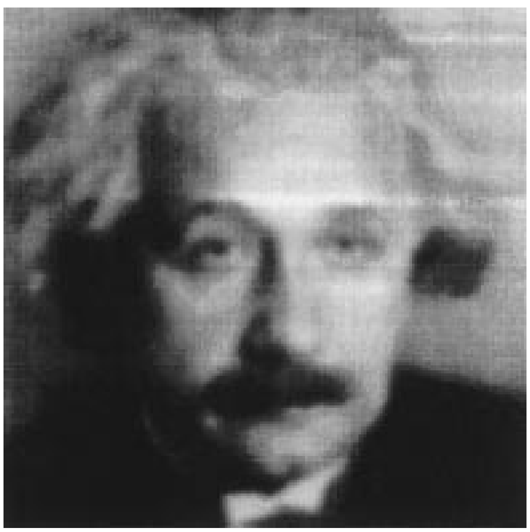

Fig. 3. Distorted and noisy image.

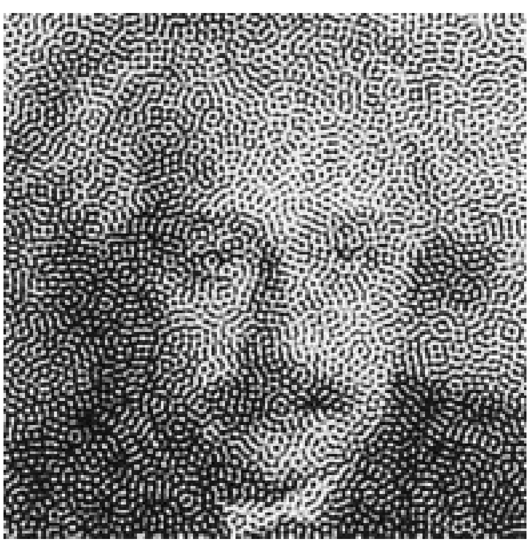

Fig. 4. Under regularized reconstruction.

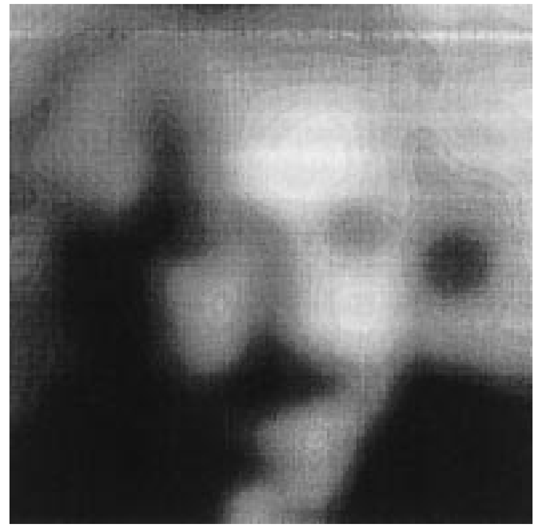

Fig. 5. Over regularized reconstruction.

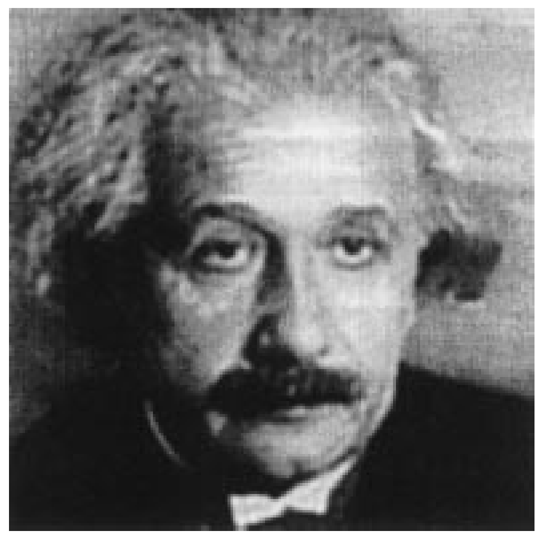

Fig. 6. Reconstruction with the proposed algorithm.

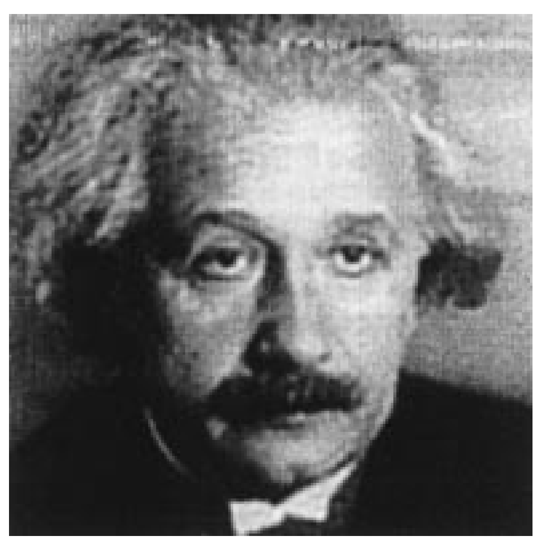

Fig. 7. Best reconstruction which can be achieved with the given inverse filter.

III. Simulation Example

The picture of Einstein has been scanned (Fig. 2) in gray scale. The size of the picture, 512×512 pixels, and the grayscale intensity are quantized in 8 bits. A 2-D Gaussian distribution with standard deviation of 5 pixels was chosen to model the point spread function of the optical system. Uniformly distributed noise simulates the measurement uncertainty. The width of the noise is 1 LSB of the quantizer (1 out of 8 bits). The distorted and noisy image is shown in Fig. 3.

An often-used technique of signal reconstruction is the regularization of the transfer function [4], [5], which takes the following form for 2-D signals. We used this algorithm to show the capabilities of the proposed optimization technique

$$O_{\mathrm{est}}(f_x, f_y, \gamma) = \frac{I(f_x, f_y)\mathrm{PSF}(f_x, f_y)^*}{|\mathrm{PSF}(f_x, f_y)|^2 + \gamma |D(f_x, f_y)|^2} + \frac{N(f_x, f_y)\mathrm{PSF}(f_x, f_y)^*}{|\mathrm{PSF}(f_x, f_y)|^2 + \gamma |D(f_x, f_y)|^2} \quad (8)$$

where γ is the regularization parameter that controls the level of noise suppression. If the noise is not suppressed enough in the inverse filtering process [γ is small in (8)], the reconstruction will be noisy (Fig. 4). Suppressing the noise too much [γ is large in (8)] results in a smooth, but distorted, image (Fig. 5). The optimal level of noise reduction has been calculated with the proposed algorithm. The obtained reconstruction is shown in Fig. 6.

For simulated signals, we can check the performance of the result. We calculated also the best reconstruction that can be achieved with the chosen inverse filter. It is calculated by minimizing (4) directly

$$\mathrm{cost}(\gamma) = \sum_{f_x=0}^{N_x-1} \sum_{f_y=0}^{N_y-1} |O(f_x, f_y) - I(f_x, f_y)K(f_x, f_y, \gamma)|^2. \quad (9)$$

Of course, it cannot be calculated in general, only for simulated data, since it requires the knowledge of the original undistorted image. The above expression provides the reconstruction shown in Fig. 7. The estimated optimum and the true optimum are very close to each other, which validates the usefulness of the proposed algorithm.

IV. Conclusion

Inverse filtering of optical images was investigated. A model-based optimization technique was shown, which has been adapted from the environment of 1-D transient signals to 2-D optical images. The performance of the method was shown on a simulated signal. We showed the behavior of the deconvolution for a special inverse filter (regularization). However, the optimization method is not limited to this application. It can be used with any optical inverse filter, which has a limited number of parameters to optimize.

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