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Unbiased Reconstruction of Nonlinear Distortions

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Abstract* – When a signal goes through a system having a static, nonlinear transfer function, the output signal will be distorted. In addition to it, the detected output signal is usually contaminated by noise, due to the noisy environment. If we compensate the nonlinearity with its inverse, the expected value of the output signal will be different from it's original one, due to the nonlinearly distorted noise. If this difference is not acceptable, we have to use a different compensation function. In this paper we will show an iterative method, which can produce a unbiased compensation function in the knowledge of the original nonlinear distortion and the noise distribution. The resulted output signal has the same expected value as the original one.

Keywords – *Nonlinear distortion, nonlinear compensation, unbiased reconstruction*

I. INTRODUCTION

Preliminaries

When a distortion caused by a nonlinear device is not acceptable, we have to compensate it. Several works occupy with nonlinear compensation. An orthogonal polynomial representation can be seen in [1]. Practical examples from the area of industry can be seen in [2]-[4]. Audio related examples are in [5]-[10].

The compensation can have different aims. The aim is usually to minimize the difference between the original and the estimated signal in least square sense. Most of these examples above use least square approach. In this case the noise on the estimated signal will be small, but the estimate will be biased. None of the works above deals with unbiased reconstruction, however, in certain cases the unbiased estimate may be an important aim.

In the case of audio related examples, the least squares approach is obvious, because we have to minimize the energy of the disturbing error signal. However, when we make precise measurements about a system that contains a nonlinearity, we may need unbiased expected values about the analyzed parameters.

Novelty

Our aim was to make an unbiased estimate from the nonlinearly distorted signal. In this paper we will show that this problem is ill-posed. We will also show that an improper solution can produce high noise amplification. A novel iterative method will be introduced, which is able to find an unbiased solution in a given signal interval. The method provides small noise intensification.

II. MODEL OF THE SYSTEM

The model of the compensation can be seen in Figure 1:

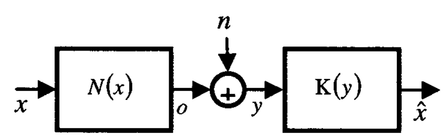

Figure 1. Model of signal compensation.

Here x and $\hat{x}$ refer to the original and estimated signals, n to the noise, o and y to the output and observed signals, respectively. $N()$ is the static, nonlinear transfer characteristics of the distorting device, and $K()$ is our compensation function. In this case, the expected value of the estimate will be

$$E\{\hat{x}\}\big|_{x=x_0} = \int_{-\infty}^{\infty} f_{\hat{x}}(\hat{x}) \cdot \hat{x} d\hat{x} = \int_{-\infty}^{\infty} f_y(y) \cdot K(y) dy = \\ = \int_{-\infty}^{\infty} f_n(y - N(x_0)) \cdot K(y) dy \tag{1}$$

where the probability density-function of a variable a is denoted by $f_a(a)$. This equation shows a correlation function between $f_n(n)$ and $K(y)$. We have to solve this equation in a given $[x_1, x_2]$ interval, to find the correct value of $K(y)$. This equation is ill-posed, because in a given interval infinite number of solutions exist. We can see this ill-posedness, if we write (1) for sampled signals:

$$E\left\{\begin{bmatrix} \hat{x}_i \\ \vdots \\ \hat{x}_{i+N} \end{bmatrix}\right\} = \begin{bmatrix} f_n(y_{i-\frac{M}{2}} - N(x_i)) & \cdots & f_n(y_{i+\frac{M}{2}} - N(x_i)) \\ \vdots & \ddots & \vdots \\ f_n(y_{i-\frac{M}{2}+N} - N(x_{i+N})) & \cdots & f_n(y_{i+\frac{M}{2}+N} - N(x_{i+N})) \end{bmatrix} \cdot \begin{bmatrix} K(y_{i-\frac{M}{2}}) \\ \vdots \\ K(y_{i+\frac{M}{2}}) \end{bmatrix} \tag{2}$$

In this matrix-equation, due to the noise, $K(y)$ should be mapped in a wider range than $\hat{x}$. In this case, the number of equations will be less then the number of unknown parameters. This set of equations becomes underdetermined, which means that infinite number of solutions exist, i.e. a subspace is the solution. We have to find a proper solution from this subspace based on further considerations.

Since correlation transforms to multiplication after a Fourier-transformation, it gives the idea to find the solution with the Fourier-transforms:

$$F\{K(y)\} = \frac{F\{N(x)\}}{\overline{F\{f_n(n)\}}} \tag{3}$$

where $F\{\ \}$ means Fourier-transform and $\overline{F\{f_n(n)\}}$ is the complex conjugate of $F\{f_n(n)\}$. This method gives an unbiased compensation function, but usually this function will oscillate. In those points, where $abs(F\{f_n(n)\})$ is close to zero, but $abs(F\{N(x)\})$ is not, $F\{K(y)\}$ will contain a peak, which after an inverse Fourier-transformation transforms to an oscillation. This kind of compensation function is unusable, due to the oscillation. The variance of the estimate will be unacceptably high. We have to find a solution, which provides good noise suppression, so a better method should be used that provides a "smooth" compensation function.

III. A SOLUTION BASED ON GOLD-ITERATION

We developed a method, which is based on Gold-iteration [11]. As the first step, $K(y)$ will be $N^{-1}(y)$. Then

$$\kappa(o) = \int_{-\infty}^{\infty} K_0(y) \cdot f_n(y-o) dy \tag{4}$$

where $\kappa(o)$ is the resulted characteristics that can be computed as

$$\kappa(o)\Big|_{x=x_0} = \frac{E\{\hat{x}\}\big|_{x=x_0}}{o\big|_{x=x_0}} \tag{5}$$

The next step is:

$$K_i(y) = K_{i-1}(y) \cdot \left(\frac{N^{-1}(y)}{\kappa(y)} \right)^{\mu} \tag{6}$$

where the exponent, μ, is an arbitrary number to make the iteration convergent. In most cases $\mu = 1$ is appropriate.

In our experiments, we have found that the convergence of this method is extremely fast. Usually only one iteration is enough to get a proper characteristics.

At the numerical computation, we have to take care by choosing the beginning interval of $N^{-1}(y)$. If the range of interest is $[y_1, y_2]$, $f_n(n)$ is sampled in P points, Q iteration is made, and the spacing distance is d, the beginning interval should be $\left[y_1 - \frac{PQ}{2}d, y_2 + \frac{PQ}{2}d \right]$, because in each iteration, the computed $K_i(y)$ will be distorted at the ends and the usable part will be smaller.

IV. SIMULATION

To show the capabilities of the proposed algorithm, a sine-wave signal (Fig. 2) was distorted with a Gaussian error-function (Fig. 3) and contaminated with white-noise. The noise is equally distributed in the range of $[-0.05, 0.05]$. The signal-to-noise ratio is 25 dB. The distorted and noisy signal can be seen in Fig. 4.

Fig. 5 shows the estimation produced by the exact inverse of the original nonlinearity. Fig. 6 shows the estimation produced by our proposed method. In this case only one iteration was used to calculate the proper shape of the characteristics. The differences between the estimate signals and the original one can be seen in Fig. 7 and Fig. 8. The noise in the case of the exact inverse is strongly distorted and causes a biased estimation, however in the case of our method the noise is unbiased, hence the estimate will be also unbiased.

V. CONCLUSIONS

In this paper, unbiased reconstruction of static, nonlinear distortions was shown. A novel method is proposed, which is able to design a nonlinear compensation function that produces an unbiased estimate in the case of noisy observed signals. The shape of the compensation function is not obvious and an improper function can produce high noise intensification. An iterative method was shown, which is able to find a "smooth" solution and provides small noise intensification. The algorithm converges fast and usually only one iteration is enough to get an appropriate compensation function. The results of the algorithm were shown by simulation examples.

ACKNOWLEDGEMENT

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REFERENCES

[1] John Tsimbios and Kenneth V. Lever, "Nonlinear System Compensation Based on Orthogonal Polynomial Inverses," IEEE Trans. On Circuits and Systems: Fundamental Theory of Applications, vol. 48, No. 4, April 2001, pp. 406-417.

[2] Sergio Bittanti, Francesco Alessandro Cuzzola, Fabrizio Lorito and Guido Poncia, "Compensation of Nonlinearities in a Current Transformer for the Reconstruction of the Primary Current," IEEE Trans. On Control Systems Technology, vol. 9, No. 4. July 2001, pp. 565-573.

[3] Minoru Okada, Hideki Nishijima and Shozo Komaki, "A Maximum Likelihood Decision Based Nonlinear Distortion Compensator for Multi-Carrier Modulated Signals," IEICE Trans. On Communications, vol. E81 B, No. 4, April 1998, pp. 737-744.

[4] S. Kerry Wilson and Patrick Delay, "A Method to Improve Cathode ray Oscilloscope Accuracy," IEEE Trans. Instrumentation and Measurement, vol. 43 n 3, June 1994, pp.483-486.

[5] Paul T. Troughton and Simon J. Godsill, "Restoration of Nonlinearly Distorted Audio using Markov Chain Monte Carlo Methods," Presented at the 104th Convention of the Audio Engineering Society, preprint 4679, Amsterdam, May 1998.

[6] Wolfgang Klippel, "Compensation for Nonlinear Distortion of Horn Loudspeakers by Digital Signal Processing," AES: Journal of the Audio Engineering Society, vol. 44 n 6, June 1996, pp. 470-480.

[7] D. Preis and H. Polchlopek, "Restoration of Nonlinearly Distorted Magnetic Recordings," J. Audio Eng. Soc., vol. 32, No 1/2, January/February 1984, pp. 26-30.

[8] Stanley A. White, "Restoration of Nonlinearly Distorted Audio by Histogram Equalization," J. Audio Eng. Soc., vol. 30, No 11, November 1982, pp. 828-832.

[9] Tamás B. Bakó, Tamás Dabóczi, "Restoration of Nonlinearly Distorted Signals with Regularized Inverse Characteristics", IMTC 2001, Budapest, May 21-23, 2001, pp. 1565-1570.

[10] Tamás B. Bakó, Balázs Bank, Tamás Dabóczi, "Restoration of Nonlinearly Distorted Audio with the Application to Old Motion Pictures", AES 20th International Conference on Archiving, Restoration and New Methods of Recording, Budapest, Hungary, Oct 5-7, 2001, pp. 191-198.

[11] P. E. Siska, "Iterative unfolding of intensity data, with application to molecular beam scattering," The Journal of Chemical Physics, vol. 59, No 11, December 1973, pp. 6052-6059.

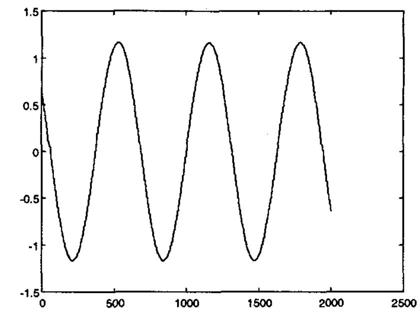

Figure 2. The original, undistorted signal.

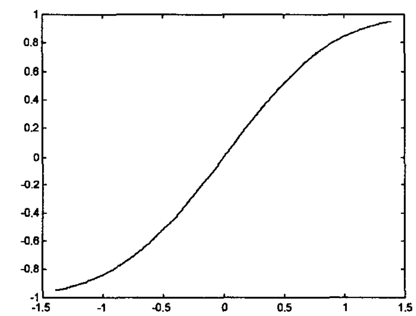

Figure 3. Nonlinear characteristics.

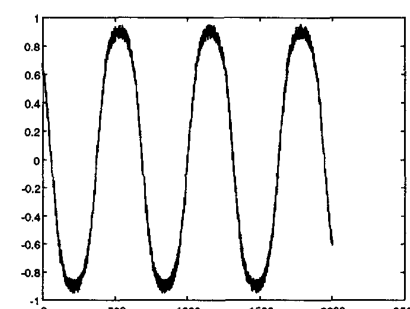

Figure 4. Distorted, noisy signal.

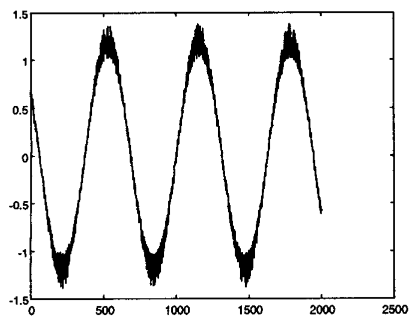

Figure 5. Estimate produced by the exact inverse.

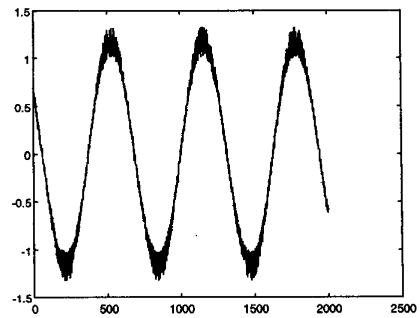

Figure 6. Unbiased estimate.

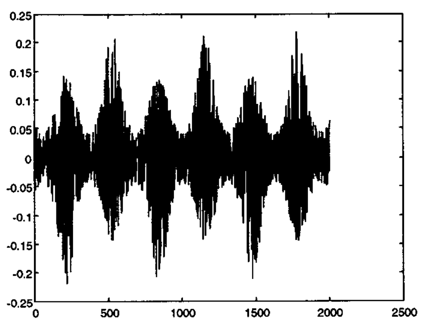

Figure 7. Difference between the original signal and the estimate one produced by the exact inverse.

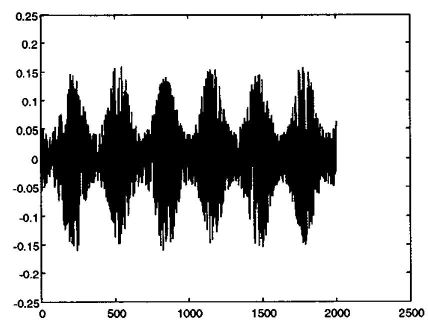

Figure 8. Difference between the original signal and the estimate one produced by the proposed method.