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Time domain uncertainty bound of signal reconstruction in the case of jittery and noisy measurements

Tamás Dabóczi
Department of Measurement and Information Systems
Technical University of Budapest, H-1521 Budapest, Hungary
Fax: +36 1 463-4112.
Email: daboczi@mmt.bme.hu

Abstract – ***Time domain measurements are distorted by the measurement system if the bandwidth of the system is not sufficiently high compared to that of the signal to be measured. If the distortion is known the measured signal can be compensated for it (inverse filtering or deconvolution). Since the measurement is always corrupted by noise the reconstruction is an estimation task.***

Our aim is to investigate the errors related to the signal reconstruction, and provide an error bound around the reconstructed waveform. Based on their nature we can distinguish between two types of errors, bias and variance. In this paper we investigate the stochastic type errors and suggest a method to calculate the uncertainty (variance) of the reconstruction.

We developed the method for the calibration of high speed sampling systems. Beside the stationary additive noise of the measurement system (quantization, electromagnetic interferences etc.) the waveforms are distorted also due to the uncertainty of the time base (jitter). The effect of the jitter can be described as a nonstationary additive noise. This noise causes both variance, and bias, since the mean value of the noise is not zero. We will take both stationary and jitter related nonstationary noise into account and provide an uncertainty bound around the reconstructed signal. Because of the nonstationary nature of the jitter the uncertainty bound is a function of time. The complete error analysis should consist also the investigation of the bias. This will be carried out in the future.

Keywords – ***Deconvolution, inverse problems, jitter, ill-posed problem, calibration.***

I. INTRODUCTION

Measurement of time-domain signals becomes difficult if the bandwidth of the measurement system is not high enough compared to that of the signal to be measured. In this case the measurement system distorts the waveform. Assuming a linear and time-shift invariant model for the measurement system the relation between the input and the output of the system can be described by convolution of the excitation signal with the impulse response of the system. If the distortion is known (impulse response or transfer function is known) the measured signal can be compensated for it. This operation is called deconvolution or inverse filtering.

Deconvolution is usually an ill-posed problem, i.e. small changes in the measured signal due to the noise cause large deviations in the reconstruction [1], [2]. To make the problem well posed one has to define additional constraints to put a limit to the noise amplification. Several approaches are proposed in the literature ([3]-[9]). The inverse filter attenuates usually at those frequency bands where the noise dominates and lets the signal through where the useful signal dominates. In practical cases this means that the high frequency noise is suppressed on different ways depending on the inverse filtering method.

Since the measurement is corrupted by noise the reconstruction is an estimation task. The estimated input signal of the measurement system consists two types of errors: bias and variance.

Bias is due to the filtering of the (noiseless) input signal by the transfer function of the measurement system and the inverse filter. The inverse filter is not the true inverse of the measurement system, because the noise has to be suppressed in the stopband of the measurement system. This filtering distorts the input signal and causes the bias of the estimate.

Variance is due to the fact that the reconstructed signal is a filtered version of the noisy measurement. The reconstructed signal can change from measurement to measurement even using the same inverse filter, since the noise records are different. We will call it uncertainty of the reconstruction, and we will distinguish the uncertainty from the bias. With this approach the uncertainty is the fluctuation of the estimate due to the difference in successive noise records.

Our aim is to provide an error bound around the reconstructed signal, which shows the limitations of the reconstruction. This paper describes a method which provides uncertainty bounds. Bias related errors will be developed and reported in the future. The specialty of the proposed method is that it takes both stationary and nonstationary noises into account.

Our special interest is to provide uncertainty analysis for calibration of high speed sampling systems. These fast pulse oscilloscope systems work in equivalent time sampling mode with an equivalent sampling frequency of around 500 GHz. The uncertainty of the time base generator causes a jitter, which can be modeled as a nonstationary noise. Quantization error and other disturbances can be modeled as stationary noises.

II. UNCERTAINTY ANALYSIS OF THE DECONVOLUTION RESULT

Fast pulse sampling systems have two different type of noise sources. The first one is the jitter noise, which is due to the uncertainty of the time base [11], [12]. The mean value of the jitter noise is not zero, even if the probability density function (pdf) of the uncertainty of the time base is symmetric. The effect of the mean value of the jitter noise can be modeled as a filtering of the signal with the pdf of the time axis uncertainty of the jitter. This causes a bias in the estimate. After removing the mean value the jitter noise can be modeled as a nonstationary zero mean additive white noise. This zero mean noise causes fluctuation in the measurement and its effect in the reconstruction will be considered as jitter related uncertainty. All other noise sources can be modeled as stationary additive output noises (Fig. 1).

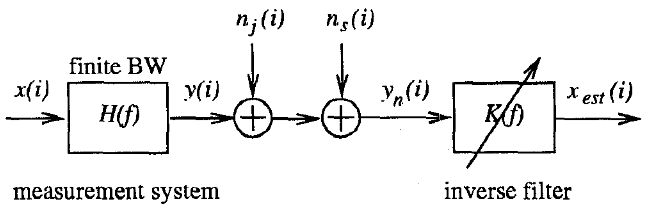

Fig. 1. Model of the measurement and inverse filtering process

The estimate of the signal can be written as the sum of the filtered input signal and filtered noises:

$$\begin{aligned} X_{est}(k) &= X(k)\underbrace{H(k)K(k)}_{bias} \\ &+ \underbrace{N_j(k)K(k) + N_s(k)K(k)}_{noise} \end{aligned} \quad (1)$$

where $X(k)$ is the DFT of the input signal, $H(k)$ is the discrete transfer function of the measurement system, $K(k)$ is the transfer function of the inverse filter, $N_j(k)$ and $N_s(k)$ are the DFT of the records of the jitter and stationary noises and $X_{est}(k)$ is the DFT of the estimated (reconstructed) input signal. The difference between the input signal and its estimate is:

$$\begin{aligned} X(k) - X_{est}(k) &= X(k) - X(k)H(k)K(k) - N_j(k)K(k) \\ &\quad -N_s(k)K(k) \\ &= X(k)(1 - H(k)K(k)) - N_j(k)K(k) \\ &\quad -N_s(k)K(k). \end{aligned} \quad (2)$$

The correct value of the input signal expressed with the estimated value is than:

$$\begin{aligned} X(k) &= X_{est}(k) + \underbrace{X(k)(1 - H(k)K(k))}_{bias} \\ &\quad \underbrace{-N_j(k)K(k) - N_s(k)K(k)}_{noise}. \end{aligned} \quad (3)$$

The effect of the jitter is usually reduced by means of signal enhancement, which reduces the variance of the jitter noise. However, jitter causes a bias, since the mean value of the jitter noise is not zero. This bias can be modeled as a linear filtering of the input signal with the probability density function (pdf) of the timeshift of the jitter.

$$\begin{aligned} X(k) &= X_{est}(k) + \underbrace{X(k)(1 - H_j(k)H(k)K(k))}_{bias} \\ &\quad \underbrace{-N'_j(k)K(k) - N'_s(k)K(k)}_{noise}, \end{aligned} \quad (4)$$

where $H_j(k)$ is the DFT of the jitter pdf. The DFT of the output noise and zero mean jitter after signal enhancement are denoted by $N'_j(k)$ and $N'_s(k)$, respectively.

The error analysis provides the uncertainty bound for the stochastic part of the estimate. After inverse filtering the distribution of the noises will converge to Gaussian one. First the standard deviation will be computed for every time instant of the estimate. Based on the required confidence level different bounds can be provided for the uncertainty (e.g. 2 σ, 3 σ confidence intervals).

A. Measurement of the noise levels

The uncertainty analysis requires an estimate for the variance of the noise of the reconstructed signal. This noise has two sources, the jitter related and an output noise, which are independent on each other. Both sources are filtered by the inverse filter.

The variance of the output noise is constant along the time record. The filtered output noise will also have a constant variance. The jitter related noise is nonstationary. Its nonstationarity can be well modeled by relating the variance to the derivative of the output signal. It will be assumed that the noisy observation is close enough to the noiseless output signal ($y(i)$ in Fig. 1), and the derivative of the output signal will be estimated by the derivative of the noisy observation.

It is proposed that the level of the noises be measured during the acquisition of the data on the following way. Several records are collected and averaged to reduce the effect of the jitter (signal enhancement). Two separate records are proposed to collect. They should contain samples of the record at two different time instants from consecutive measurements. One record is being collected at steady topline, the other one at the sharp edge of the step like waveform. The standard deviation of the first record will be an estimate of the level of the stationary part of the output noise, since this part of the signal does not contain jitter noise.

$$std\_n_s = \text{std}\left\{y_{n,k}(t_{topline})\right\}, \tag{5}$$

where $y_{n,k}(i)$ denotes the k^{th} record of subsequent measurements of y_n-s, and $t_{topline}$ denotes a time instant at the flat part of the topline.

The second record, collected at the sharp edge, contains both jitter related noise and output noise.

$$std\_n_o(t_{edge}) = \text{std}\left\{y_{n,k}(t_{edge})\right\}, \tag{6}$$

where $std\_n_o(i)$ denotes the standard deviation of the total output noise at time instant i, and t_{edge} denotes a time instant at the sharp edge. The total output noise is the sum of the stationary and jitter related noises. The variance of the jitter related noise can be obtained by subtracting the variance of the stationary noise from the variance of this record, since the two noise sources are independent on each other.

$$std\_n_{jit}(t_{edge}) = \sqrt{std\_n_o(t_{edge})^2 - std\_n_s^2} \tag{7}$$

If the effect of the jitter is little compared to the stationary noise it can happen that the level of the total output noise is measured lower than the level of the stationary noise, since the measurement of the noise levels have uncertainty, either. In this case the effect of the jitter will not be taken into account.

$$std\_n_{jit}(t_{edge}) = 0 \quad \text{if } std\_n_o(t_{edge}) < std\_n_s \tag{8}$$

These noises are filtered by the inverse filter, which modifies their variances. The standard deviation of the filtered stationary noise becomes:

$$std\_n_{s,invfilt} = std\_n_s \sqrt{\frac{1}{Nf}\sum_{k=0}^{Nf-1}|K(k)|^2} \tag{9}$$

The standard deviation of the jitter noise is assumed to be proportional to the derivative of the output signal:

$$std\_n_{jit}(i) \sim \text{diff}\{y(i)\}, \tag{10}$$

where $std\_n_{jit}(i)$ denotes the jitter noise, diff$\{y(i)\}$ denotes the derivative of the output signal.

The level of the estimate of the standard deviation of the jitter related noise is adjusted to the measured noise level at the edge of the signal.

$$std\_n_{jit}(i) = \text{diff}\left\{y(i)\right\}\frac{std\_n_{jit}(t_{edge})}{[\text{diff}\{y(i)\}]_{t_{edge}}} \tag{11}$$

where $std\_n_{jit}(i)$ denotes the standard deviation of the jitter noise. The central difference is calculated by averaging the forward and backward differences. Since the noiseless output signal is not known, its derivative in (11) will be approximated by derivative of the measured noisy output signal.

$$std\_n_{jit}(i) \simeq \text{diff}\left\{y_n(i)\right\}\frac{std\_n_{jit}(t_{edge})}{[\text{diff}\{y_n(i)\}]_{t_{edge}}} \tag{12}$$

The jitter noise is filtered by the inverse filter. Its variance along the time axis is redistributed. Let us denote the filtered jitter noise by $n_{jit,invfilt}$.

$$n_{jit,invfilt}(i) = \sum_{l=0}^{N-1} k(l)n_{jit}(i-l), \tag{13}$$

where $k(i)$ denotes the discrete impulse response of the inverse filter. The convolution becomes circular if computed by DFT. The distribution of the variance can be calculated based on the following relations.

$$\text{var}\{a\eta + b\} = a^2\text{var}\{\eta\} \tag{14}$$

$$\text{E}\{a\eta + b\} = a\text{E}\{\eta\} + b \tag{15}$$

If ζ and η are independent random variables:

$$\text{var}\{\zeta + \eta\} = \text{var}\{\zeta\} + \text{var}\{\eta\} \tag{16}$$

The variance of (13) is the convolution of the variance of the jitter noise and the squared impulse response of the inverse filter:

$$\begin{aligned} \text{var}\{n_{jit,invfilt}(i)\} &= \text{var}\left(\sum_{l=0}^{N-1} k(l)n_{jit}(i-l)\right) \\ &= \sum_{l=0}^{N-1}\text{var}\{k(l)n_{jit}(i-l)\} = \sum_{l=0}^{N-1}k^2(l)\text{var}\{n_{jit}(i-l)\} \\ &= \text{conv}\left(k^2(i), \text{var}\{n_{jit}(i)\}\right), \end{aligned} \tag{17}$$

where *conv* denotes convolution. This convolution is circular again if the estimated input signal is calculated by means of DFT. It should be emphasized that the variance of the jitter noise is the function of time, since this noise is nonstationary. The standard deviation of the inverse filtered jitter noise is the square root of the variance:

$$std\_n_{jit,invfilt}(i) = \sqrt{\mathrm{var}\{n_{jit,invfilt}(i)\}} \quad (18)$$

The uncertainty envelope will be based on the estimated standard deviations of the inverse filtered noises.

$$\begin{aligned} uncert\_n_{s,invfilt}(i) &= a\ std\_n_{s,invfilt} \\ uncert\_n_{jit,invfilt}(i) &= a\ std\_n_{jit,invfilt}(i) \quad (19) \end{aligned}$$

where a is a multiplication factor, expressing the level of the confidence of the uncertainty. The confidence level can be calculated from Gaussian probability distribution:

$$\Phi(a) = \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} e^{-\frac{t^2}{2}} dt \quad (20)$$

$$confidence\_level = \Phi(a) \longrightarrow a.$$

Since the two noise sources in (19) are independent on each other their variances will be summed, thus the standard deviation is:

$$uncert\_x_{est}(i) = \quad (21)$$
$$\sqrt{uncert\_n_{s,invfilt}(i)^2 + uncert\_n_{jit,invfilt}(i)^2}$$

III. SIMULATIONS

The sampling heads of fast digital oscilloscopes are calibrated with step like signals. We will check the results on simulated step like waveforms. The signals are generated from measured data by filtering it to reduce the noise. Thus the dominant noise will be that added by the simulation. Nahman-Guillaume technique will be used to mirror the signal and force it to be duration limited. This signal was distorted by the estimated impulse response of the sampler. Jitter was simulated by distorting the waveform with the pdf of the jitter as an impulse response and adding noise to the output. Gaussian noise with standard deviation of 1 was multiplied point by point by the derivative of the (noiseless) output signal to simulate the nonstationary jitter noise. Stationary Gaussian output noise was added to the output, either, to simulate all other noise sources. The standard deviation of the jitter pdf is 3 Δt, where Δt is the sampling interval. The signal to noise ratio of the stationary noise is 55 dB. The signal to be measured and the distorted, noisy one are depicted in Fig. 2 and Fig. 3.

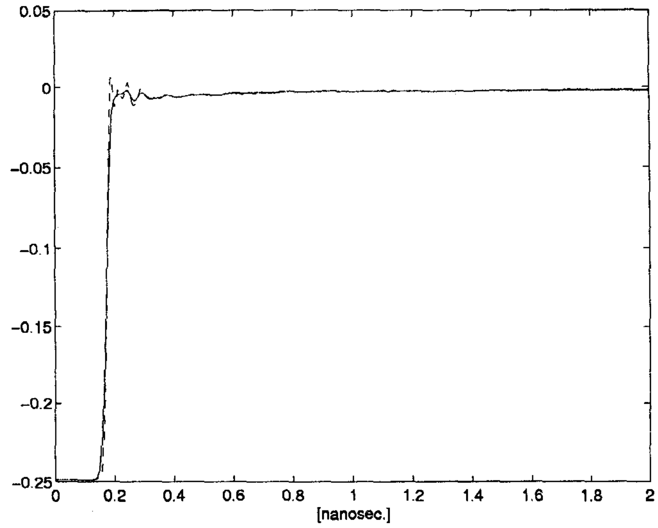

Fig. 2. Simulated steplike waveform. Jitter free input signal: dashed line, noisy output signal: solid line. SNRs=55 dB, $std_{jitter} = 3\Delta t$

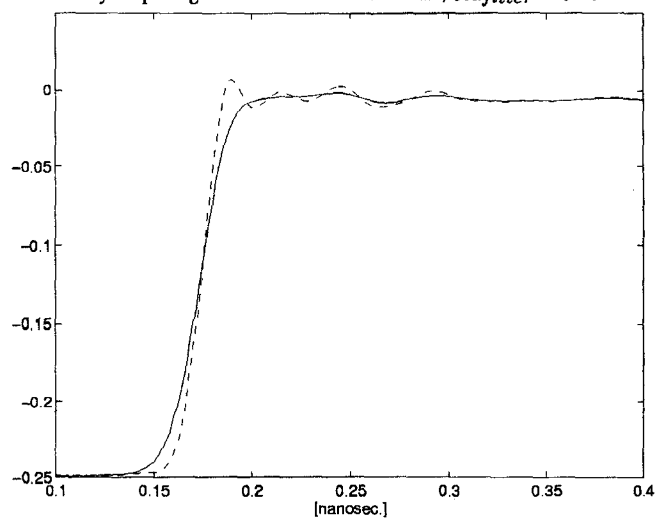

Fig. 3. Simulated steplike waveform. Front part enlarged.

We used the regularization method to reconstruct the input waveform [9], [10]. The estimated input signal is shown in Fig. 4. The uncertainty of the reconstruction is calculated based on the equations described in Section II. The resulting bounds (2 σ bound) and the simulated noises are compared in Fig. 5. The total uncertainty is obtained by adding the effects of jitter related and stationary noises (Fig. 6). The uncertainty envelope around the reconstructed signal is shown in Fig. 7.

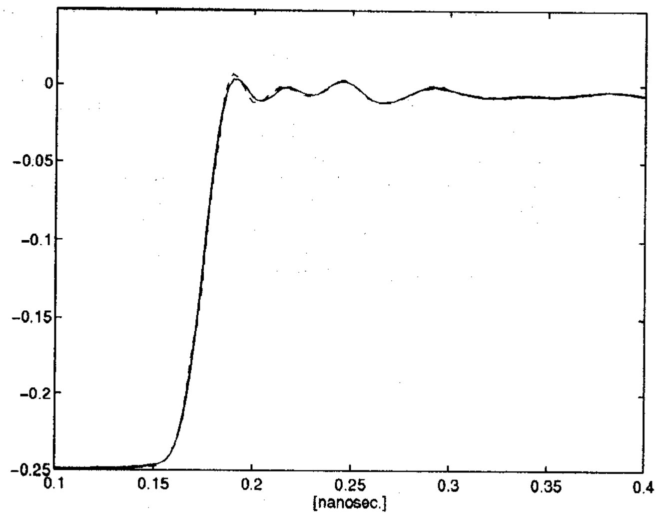

Fig. 4. Input signal (dashed line) and the estimated input signal (solid line)

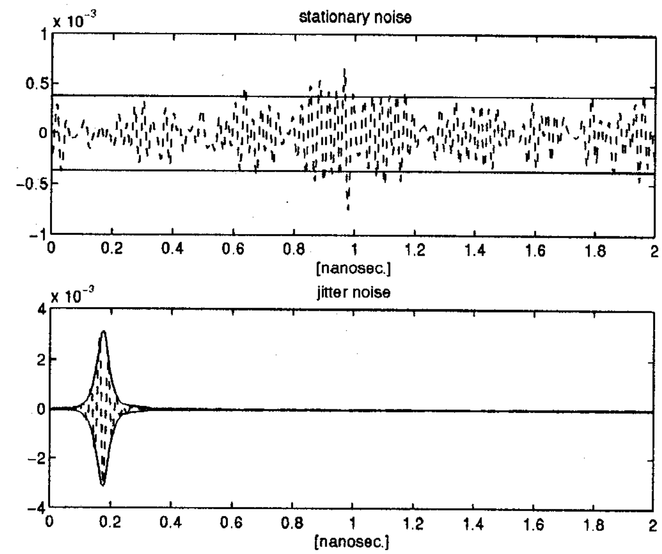

Fig. 5. Filtered noises (dashed) and the estimated uncertainty bounds (solid). (2σ confidence interval) SNR=55 dB, $std_{jitter} = 3\Delta t$

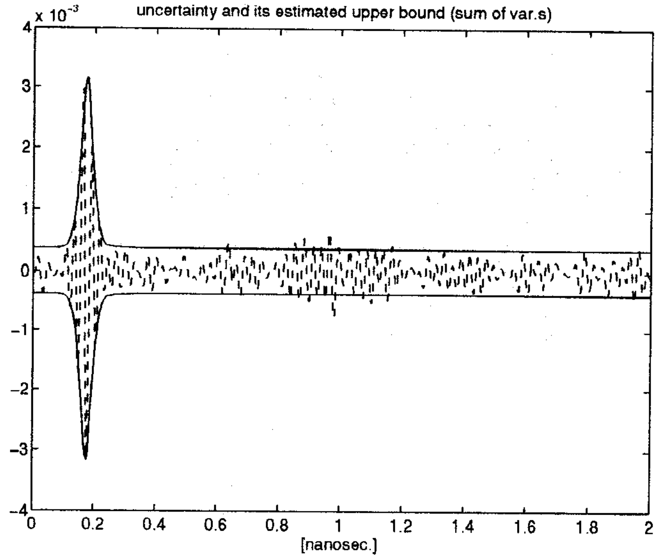

Fig. 6. Estimated total uncertainty bound (solid) and the level of filtered noises (dashed). SNR=55 dB, $std_{jitter} = 3\Delta t$

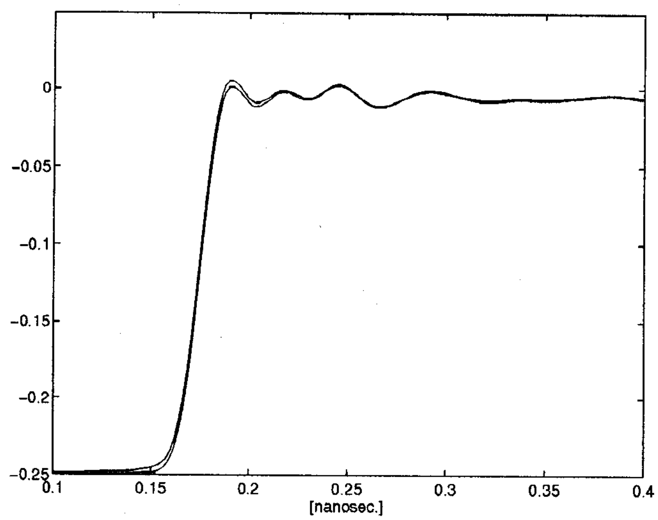

Fig. 7. Uncertainty envelope around the reconstructed signal.

IV. CONCLUSIONS

Uncertainty analysis of signal reconstruction was carried out. We modeled jitter related nonstationary noise and stationary noises from different sources (e.g. quantization, electromagnetic interferences). We developed a method to calculate uncertainty bound around the reconstructed waveform, based on the required confidence level. The uncertainty bound is a function of time, because of the nonstationary nature of the jitter. We aim at developing bias related bounds in the future to extend the error analysis.

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