Improved ADMM based TV minimized Image Deblurring Without Boundary Artifacts

Dániel Hadházi, Áron Horváth, Gábor Horváth

Department of Measurement and Information Systems

Budapest University of Technology and Economics

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Contents of the presentation

- Basics of the image deblurring
  - Motivation of the TV minimization
- Introduce the proposed method
- Conclusions
  - Take some interesting conclusions of the deblurring
  - With visual examples

Image deblurring

- Main goal: invert degradation of acquisition
  - Linear shift invariant system + additive noise
  - PSF of the system and type of the noise are known
- Can be considered as a MAP estimation
  - $\arg\min_{\mathbf{x}} \left\{ -\log\left(\Pr\left(\mathbf{x}|\mathbf{y}\right)\right)\right\} = \arg\min_{\mathbf{x}} \left\{ \varphi\left(\mathbf{x},\mathbf{y}\right) + \kappa\left(\mathbf{x}\right)\right\}$
  - $\varphi\left(\mathbf{x},\mathbf{y}\right)$: loss term – penalizing the inconsistency between the deblurred and the input image
  - $\kappa\left(\mathbf{x}\right)$: regularizer term – ensures stability and defines some constraints

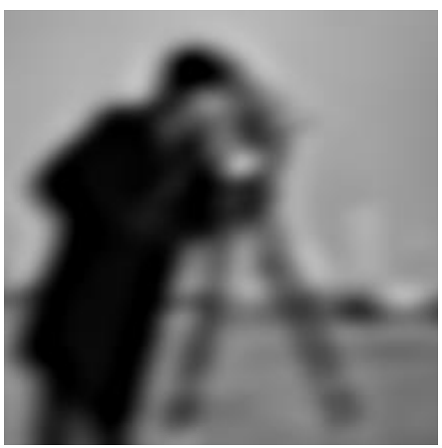
Image deblurring

- Loss term ($\varphi(\mathbf{x}, \mathbf{y})$):
  - Typically quadratic functions of the error
    - Assumption of additive observation Gaussian noise
  - Examined the Huber and the Sum of Absolute Error as well
    - With Gaussian observation noise
- Regularizer term ($\kappa(\mathbf{x})$):
  - PSF typically is a kind of low-pass filter
  - High noise sensitivity of the deblurring in high frequencies
  - Therefore the size of the gradient penalized with this term
  - Introduced domain constraints of the intensities

The size of the gradient

<u>Square of L2 norm:</u>

- Kind of ***Thikhonov*** regularization
- Similar to a low-pass post filtering
- The fine details of the picture are lost



<u>L2 norm:</u>

- ***Total Variation minimization*** based regularization
- Assumes that the gradient image is sparse
- Preserves edges and fine details

The proposed method

- The optimization is based on the Alternating Direction Method of Multipliers algorithm.
- Is an improved Split Bregman method:
  - Non negativity constraint is introduced on the intensities of the deblurred image.
  - The applied optimization method is modified in order to increase the rate of the convergence.
  - Defined a more general cost function that enables using different kind of loss functions.
  - Weighted loss terms are introduced that enables modelling different amount of noise for every pixel.

Formalization of the problem

- The formula of the primal optimization problem after applying the variable splitting:

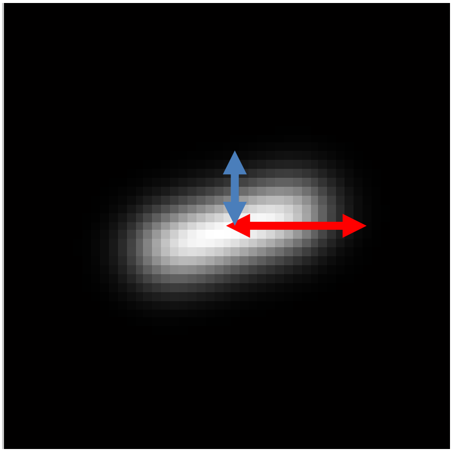
$$\min_{\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{z},\mathbf{q}}. \quad \varphi_{\mathrm{w}}(\mathbf{u}) + \lambda \cdot \sum \left\| \left[\mathbf{v}_{(i,j)}, \mathbf{z}_{(i,j)} \right] \right\|_2 + I\left(\mathbf{q}_{(i,j)}\right)$$

$$\text{s.t.} \quad R\mathbf{x} = \mathbf{v},\ G\mathbf{x} = \mathbf{z};\ \mathbf{y} - H\mathbf{x} = \mathbf{u};\ \mathbf{q} = \mathbf{x}$$

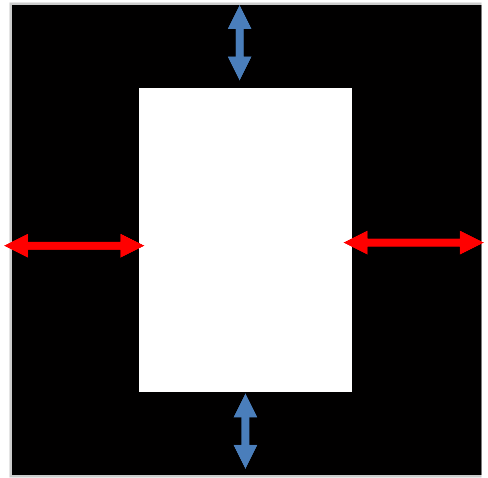
  - $\varphi_{\mathrm{w}}(\mathbf{u})$: is the loss function – penalizing the inconsistency
  - λ: is the weight term of the TV prior
  - $I(\cdot)$: is the domain constraint term – its value is infinite if the intensities of the deblurred picture are not nonnegative

Formalization of the problem

- The operators replaced by its circular counterparts:
  - The computations of the iterations can be done effectively
  - Cost of every iteration ~ 4× 2D FFT
  - If the weight matrix of the image error defined adequately than this change not introduce Boundary artifacts:



PSF

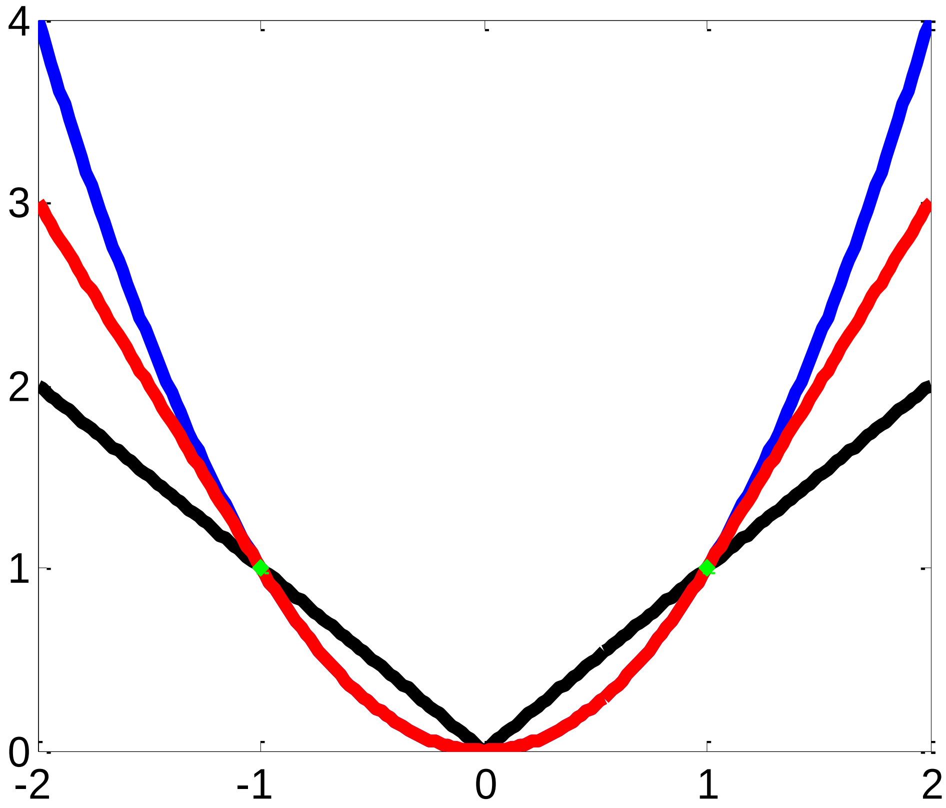


Weight matrix

Definition of the Loss term

- Weighted sum of square errors (SSE):
  - Zero mean Gaussian additive observation noise model
  - Moderate quality if there are high frequency textures
- Weighted sum of absolute errors (SAE):
  - Assumes that the additive noise is heavy-tailed
  - Theoretically corresponds to i.i.d Laplace noise
  - Better quantitative results, but high frequency artifacts
- Weighted Huber loss function (HLF):
  - Practically the mixture of the SAE and SSE

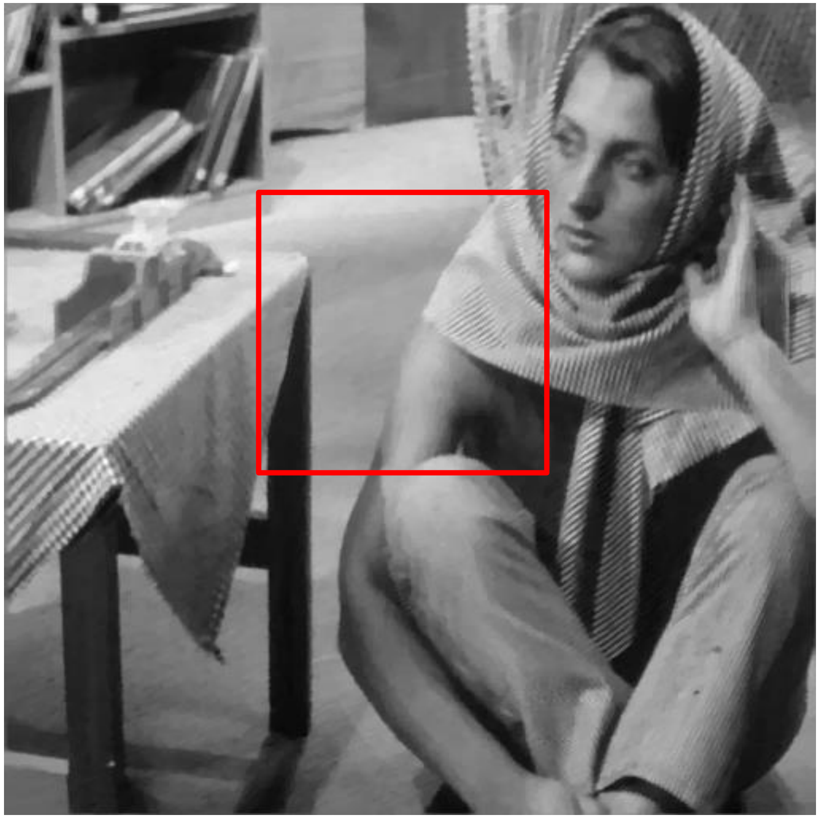
Definition of the Loss terms



- SSE
- SAE
- Huber
  - Change point of the definition is

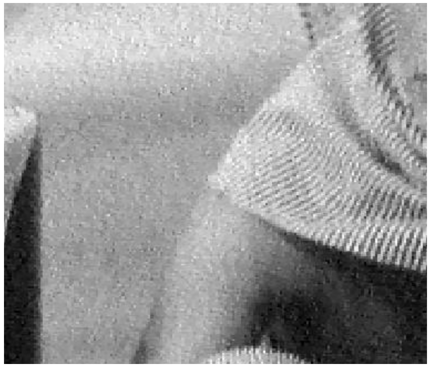
Quality as function of the loss term

- Let see an example – Gaussian noise is applied:



The deconvolved image

Sum of absolute errors

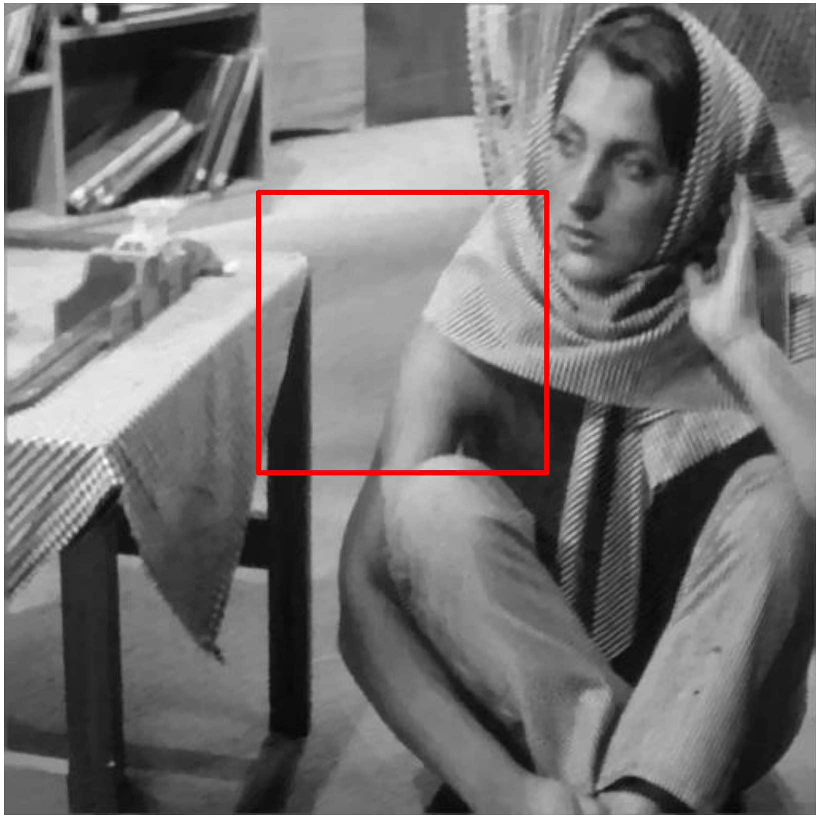


Huber penalty function

A Region of interest

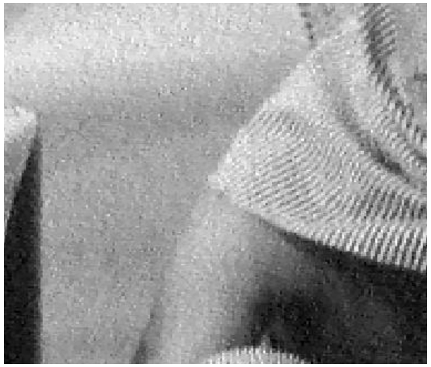
Quality as function of the loss term

- Let see an example – Gaussian noise is applied:



The deconvolved image

Sum of squared errors



Huber penalty function

A Region of interest

Conclusions of the experiments

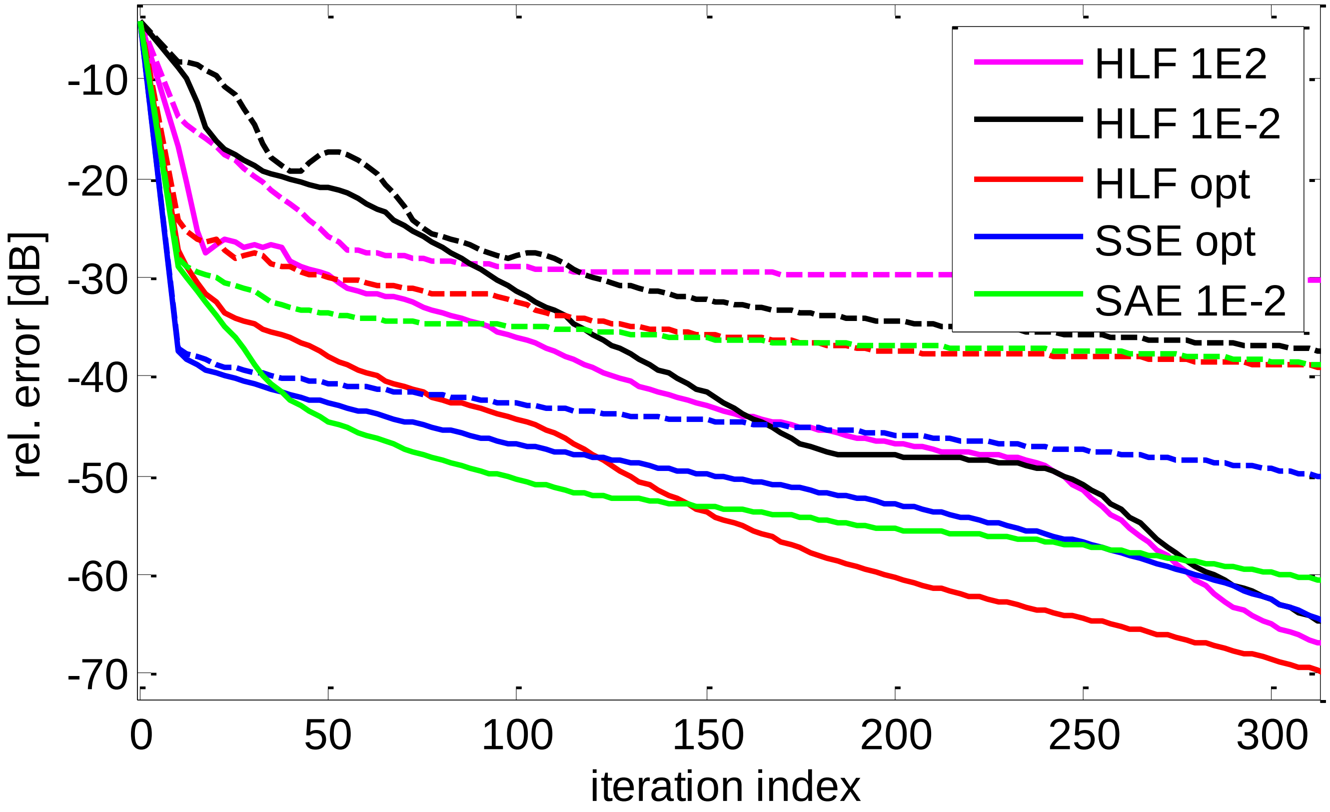
- Quantitative comparison with benchmark images:
  - Babara, Goldhill, Cameraman and Lena images
  - Additive Gaussian noise with different variance
  - Distortion: uniform blur / linear motion blur
  - ISNR was applied for quantifying the results
- General conclusions:
  - Better quality of images if the domain constraint used
  - SSE is only better than SAE in too noisy cases (BSNR<25 dB)
  - In every test cases there were a Huber function based solution which was the best
    - There exist change point value which …

Convergence properties of the optimization

- Convergence properties of the ADMM:
  - If the objective function is proper, closed and convex it converges to the solution in finite number of iterations
  - The speed significantly depends on the value of the hyper-parameters (AL penalty weights)
- How to calculate the optimal values:
  - Technically and theoretically it's very difficult (impossible?)
  - Instead of this, these weights are adjusted automatically at the end of every iteration:
    - In order to equalize the norms of the primal and the dual residuals

Automatic penalty weight modification

- Strictly accelerates the convergence rate in the case of every examined loss term:



Thank you for your kind attention!

Alternating Direction Method of Multipliers (ADMM)

- An improved iterative dual ascent method:
  - Utilize the idea of dual decomposition
  - Improvement by introducing new tags to the cost
    - Penalizing the primal feasibility gap
  - Effective if the criteria function can be decomposed into easily optimizable parts
    - E.g. parts which minimum can be calculated analytically
  - Also the optimization method of the Split Bregman algorithm

Alternating Direction Method of Multipliers (ADMM)

- Let see an example:
  - First step – introduce penalty of primal feasibility gap:

$$\begin{aligned} \min.\quad & f(\mathbf{x}) + g(\mathbf{y}) \\ \text{s.t.}\quad & \mathbf{y} = \mathbf{x} \end{aligned} \quad \Rightarrow \quad \begin{aligned} \min.\quad & f(\mathbf{x}) + g(\mathbf{y}) + (\rho/2)\cdot\|\mathbf{y}-\mathbf{x}\|_2^2 \\ \text{s.t.}\quad & \mathbf{y} = \mathbf{x} \end{aligned}$$

  - Then the dual problem optimized iteratively:

$$\max_{\boldsymbol{\eta}} \min_{\mathbf{x},\mathbf{y}} f(\mathbf{x}) + g(\mathbf{y}) + (\rho/2)\cdot\|\mathbf{y}-\mathbf{x}+\boldsymbol{\eta}\|_2^2$$

1. Minimizing over the primal variables one by one
2. Updating the dual variable by gradient ascend step
3. Optional – adjustment of the AL penalty weight