

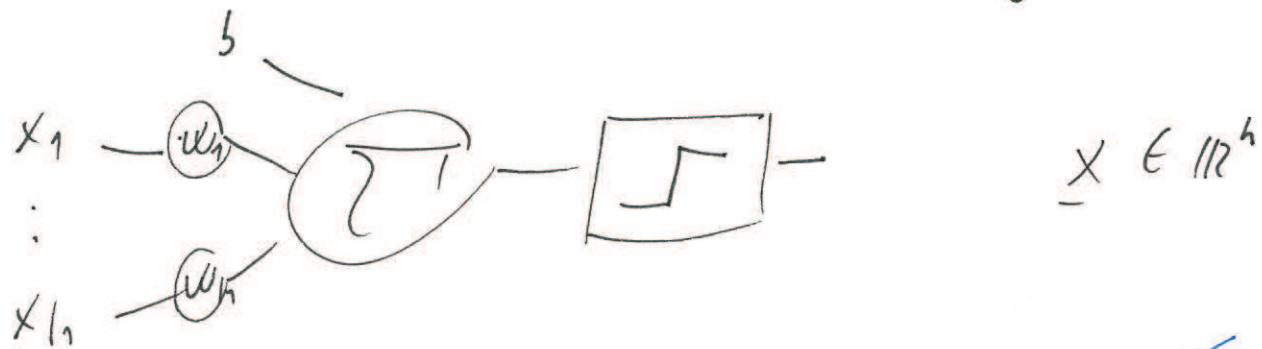
- McCulloch ; Pitts ; Hebb
- '58: Rosenblatt perceptron
- '60: Widrow
- '69: Minsky
- '76 ('69, '74): Univerisitatorijosztes MLP
- '98: Statistical Learning Theory
- '99: SVM
- '12: (NN) - Moky hozzájárulás hálózat

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Perceptron:

$$y(x) = \text{sgn}(\underline{\omega}^T \underline{x} + b) \quad b \in \mathbb{R}$$

$$= \text{sgn}(\underline{\omega}^T \underline{x}') \quad \underline{x}' = \begin{bmatrix} \underline{x} \\ 1 \end{bmatrix} \quad \underline{\omega}' = \begin{bmatrix} \underline{\omega} \\ b \end{bmatrix}$$



$$d(k) = \text{sgn}(\underline{\omega}^T \underline{x}(k))$$

$$\underline{\omega}(k+1) = \underline{\omega}(k) + \eta \cdot \epsilon(k) \cdot \underline{x}(k) \quad \eta \in \mathbb{R}_{++}$$

$$X^+ := \left\{ \underline{x} \in X : d(k) = 1 \right\} ; \quad X^- := \left\{ \underline{x}(k) \in X : d(k) = -1 \right\}$$

1. /  $d(k) = 1 \quad g(\underline{x(k)}) = -1$

$$\underline{w(k+1)} = \underline{w(k)} + 2\lambda \cdot \underline{x(k)} \quad / \underline{x(k)^T}.$$

$$\underline{x(k)^T w(k+1)} = \underline{x(k)^T w(k)} + 2\lambda \cdot \underbrace{\| \underline{x(k)} \|_2^2}_{\geq 0} \underbrace{\geq 0}_{\geq 0}$$

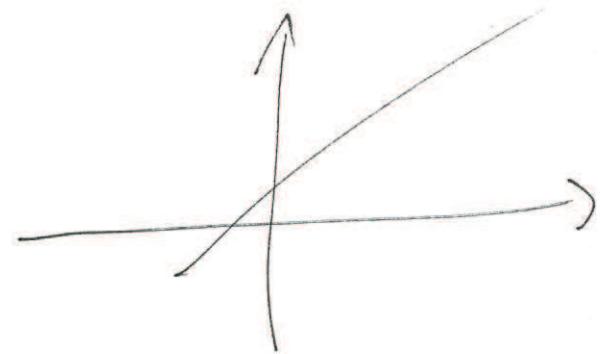
2. /  $d(k) = -1 \quad g(\underline{x(k)}) = 1$

$$\underline{w(k+1)} = \underline{w(k)} - 2\lambda \cdot \underline{x(k)} \quad / \underline{x(k)^T}.$$

$$\underline{x(k)^T w(k+1)} = \underline{x(k)^T w(k)} - \underbrace{2\lambda \cdot \| \underline{x(k)} \|_2^2}_{\geq 0}$$

3. /  $d(k) = g(\underline{x(k)})$

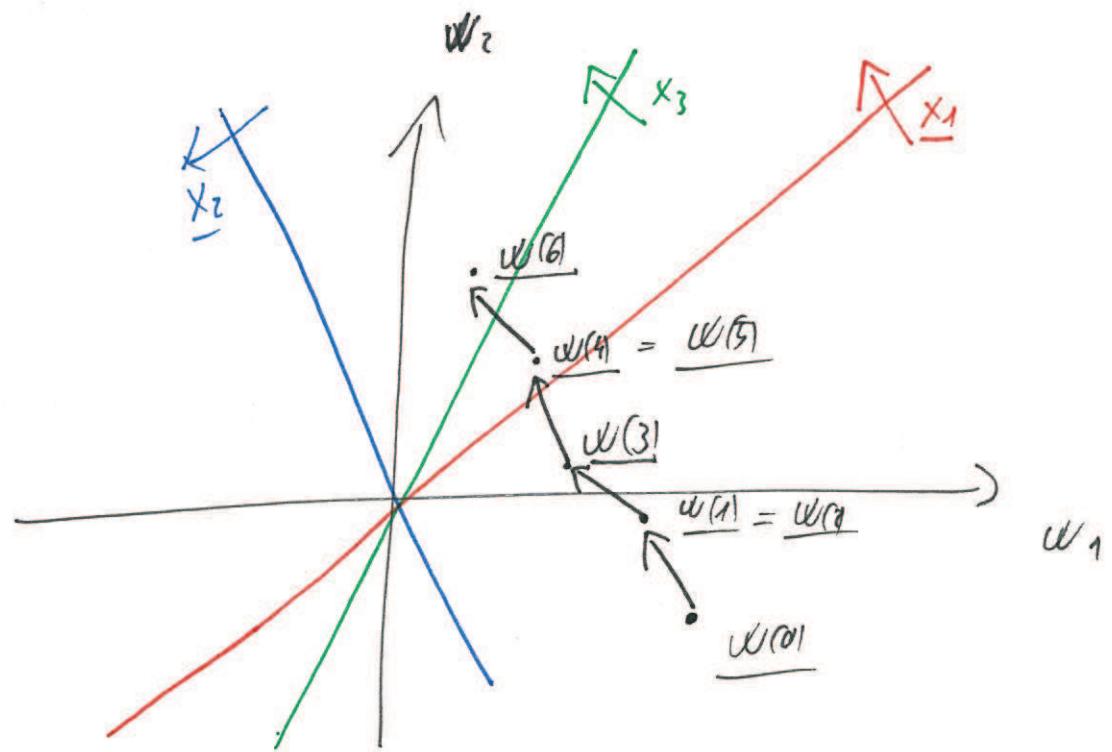
$$\underline{w(k+1)} = \underline{w(k)} + \underbrace{\lambda \cdot \varepsilon(k) \cdot \underline{x(k)}}_0$$



$$\underline{x_1}, d_1 = 1$$

$$\underline{x_2}, d_2 = -1$$

$$\underline{x_3}, d_3 = 1$$



$$\exists \underline{w}^*: \underline{x}^T \cdot \underline{w}^* > 0 \quad \forall x \in X_1 \quad -5-$$

$$\underline{x}^T \cdot \underline{w}^* < 0 \quad \forall x \in X_{-1}$$

$$X^* = \{x \in X_1\} \cup \{-x : x \in X_{-1}\} \quad \forall x \in X^* \quad \underline{w}^{*T} x \geq b \\ b \in \mathbb{R}_{++}$$

$$\|x\|_2 \leq M \quad \forall x \in X^*$$

$$\underline{w}(k+1) = \underline{w}(k) + \lambda \cdot \tilde{\epsilon}(k) \cdot \underline{x}(k) \quad f(\underline{w}^{*T})$$

$$\underline{w}(k+1) = \underline{w}(0) + 2\lambda \cdot \sum_{i=1}^k \underline{x}(h_i) / \underline{w}^{*T} \quad \text{f.f.h. } \underline{w}(0) = 0$$

$$\underline{w}^{*T} \underline{w}(k+1) = \underbrace{\underline{w}^{*T} \cdot \underline{w}(0)}_{=0} + 2\lambda \cdot \sum_i \underbrace{\underline{w}^{*T} \underline{x}(h_i)}_{\geq b}$$

$$\underline{w}^{*T} \underline{w}(k+1) \geq 2\lambda \cdot b$$

$$\|\underline{w}^*\|^2 \cdot \|\underline{w}(k+1)\|^2 \geq (\underline{w}^{*T} \underline{w}(k+1))^2 \geq 4\lambda^2 b^2$$

$$\|\underline{w}(k+1)\|^2 \geq \frac{4\lambda^2 b^2}{\|\underline{w}^*\|^2}$$

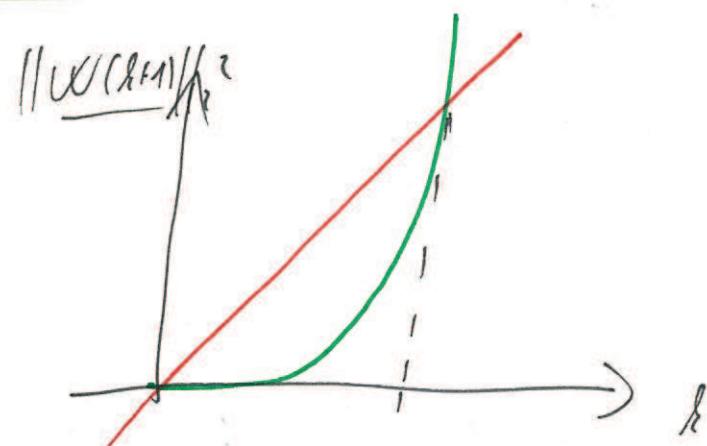
$$\underline{w(\lambda+1)} = \underline{w(\lambda)} + 2\gamma \cdot \underline{x(\lambda)}$$

$$\|\underline{w(\lambda+1)}\|_2^2 = \|\underline{w(\lambda)} + 2\gamma \underline{x(\lambda)}\|_2^2 = (\underline{w(\lambda)} + 2\gamma \underline{x(\lambda)})^\top (\underline{w(\lambda)} + 2\gamma \underline{x(\lambda)})$$

$$\|\underline{w(\lambda+1)}\|_2^2 = \|\underline{w(\lambda)}\|_2^2 + \underbrace{4\gamma \underline{w(\lambda)^\top} \underline{x(\lambda)}}_{\leq 0} + 4\gamma^2 \|\underline{x(\lambda)}\|_2^2$$

$$\|\underline{w(\lambda+1)}\|_2^2 \leq \|\underline{w(\lambda)}\|_2^2 + 4\gamma^2 \|\underline{x(\lambda)}\|_2^2$$

$$\boxed{\|\underline{w(\lambda+1)}\|_2^2 \leq \underbrace{\|\underline{w(0)}\|_2^2}_{0} + 4\gamma^2 \lambda \cdot \left( \|\underline{x(\lambda)}\|_2^2 \right) M}$$



Nelm liberalisah step. Pl.:

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