CALIBRATION ALGORITHM FOR CURRENT-OUTPUT R-2R LADDERS

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Abstract: The paper presents a new calibration algorithm for current output R-2R ladders. The R-2R ladders are popular because they are easy to realise, but the resistance tolerances limit their linearity properties. However, it is possible to achieve higher linearity with self-calibration techniques. A formerly published algorithm by Cutkosky is available for the so-called Cutkosky R-2R ladders. A similar algorithm can be applied to the current-output ladders, which are widely used as MDAC or DAC. In the paper the Cutkosky divider is shortly discussed, and then a new description is presented as basis of the calibration algorithm. The calibration algorithm is discussed in details, and the error budget is analysed. A simulator was also written to verify the theory.

Keywords: binary voltage dividers, Cutkosky-divider, DA converter, Gauss method, R-2R ladder, self-calibration

1 INTRODUCTION

The simple and popular method for converting a digital number into a quantified analog signal is based on R-2R ladder networks. The benefits of these ladders are the relatively good resolution and the very simple structure, which makes them easy to realize. There are at least three different version of the basic ladder: voltage-output, current-output (sometimes also called inverse) (Fig. 1.) and Cutkosky-divider (Fig. 2.).

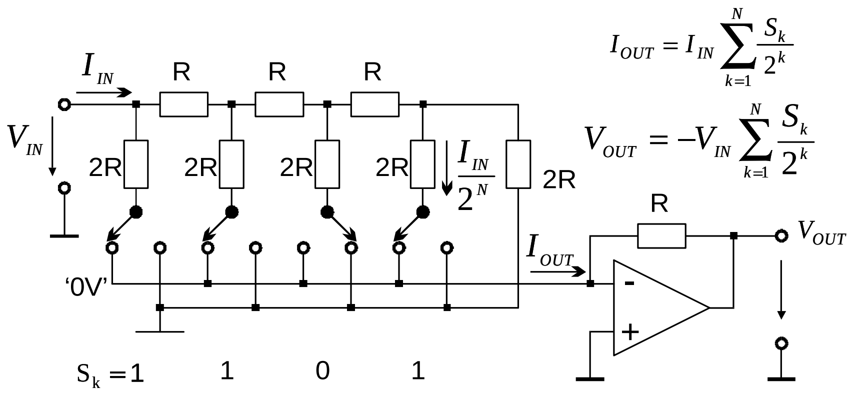

Figure 1. Inverse ladder

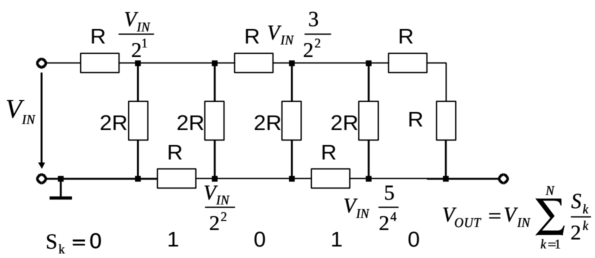

Figure 2. Cutkosky-ladder

The first two variants are well known, but not the last one. The Cutkosky-divider is a modified R-2R ladder in order to make it easy to calibrate [2]. It is often used in national standards laboratories and in other applications, where extremely high precision is needed [4, 5, 6]. Since the first publication of the divider by R. D. Cutkosky in 1978, its parameters, benefits and drawbacks have been analyzed in details by many authors. To illustrate the possibilities of this special ladder, there are realized ladders up to a resolution of 25 bits [5]. The calibration method of the divider invented by Cutkosky [2], is an application of the classical Gauss method.

2 DESCRIPTION OF THE INVERSE R-2R LADDER

The analysis of the ladder is simple in the ideal case. However, the general description of the ladder with small variations of the resistances around their nominal value is a more complex task. Even for a 3-bit ladder it is hard to give the relation of the input and the output analytically. For the calibration of the ladder, a new description is needed, which makes the further analysis easier.

The solution can be found in a special R-2R ladder invented by Cutkosky. Although from the first sight it seems to be completely different from the inverse ladder, the generalization of Cutkosky's results can solve the problems. The Cutkosky ladder uses the general R-2R ladder structure (2R input resistance is seen from each divider stage) with a modified switching conception. Cutkosky described the ladder in a completely unusual way. He defined new variables, which characterizes the goodness of the voltage divider stages.

$$V_1 = V_{IN}\frac{R_1}{R_1+R_2} = V_{IN}\frac{1}{2}\frac{R_1+R_2}{R_1+R_2} + V_{\mathrm{IN}}\frac{1}{2}\frac{R_1-R_2}{R_1+R_2} = \frac{1}{2}(V_{IN}+\varepsilon), \quad \varepsilon = \frac{R_1-R_2}{R_1+R_2} \tag{1}$$

$$V_2 = V_{IN}\frac{R_2}{R_1+R_2} = V_{IN}\frac{1}{2}\frac{R_1+R_2}{R_1+R_2} - V_{\mathrm{IN}}\frac{1}{2}\frac{R_1-R_2}{R_1+R_2} = \frac{1}{2}(V_{IN}-\varepsilon) \tag{2}$$

In the case of a 1-bit Cutkosky-ladder – which is one resistance divider – Eq. 1., 2. show the description. The resistor values R_1, R_2 are transformed to variable e, and therefore the number of parameters is halved. The N-bit ladder consists of cascaded 1-bit dividers, so the algorithm can be easily applied to an N-bit ladder (for the exact analysis of the Cutkosky ladder see [2]).

Now Cutkosky's description should be applied to an inverse ladder. The main difference between the Cutkosky-divider and the current-output ladder is that the Cutkosky-divider is a voltage divider, and the current-output ladder is a current divider network. The signal, which is carrying the information, is the magnitude of current in the inverse ladder. These two ladders are dual networks from this point of view. Therefore, the presented description must be modified to cascaded current dividers. This means only changing some of the indexes and polarities as it can be seen for a 1-bit inverse ladder in Eq. 3., 4. :

$$I_1 = I_{IN}\frac{R_2}{R_1+R_2} = I_{IN}\frac{1}{2}\frac{R_1+R_2}{R_1+R_2} - I_{\mathrm{IN}}\frac{1}{2}\frac{R_1-R_2}{R_1+R_2} = \frac{1}{2}(V_{IN}-\varepsilon), \quad \varepsilon = \frac{R_1-R_2}{R_1+R_2} \tag{3}$$

$$I_2 = I_{IN}\frac{R_1}{R_1+R_2} = I_{IN}\frac{1}{2}\frac{R_1+R_2}{R_1+R_2} + I_{\mathrm{IN}}\frac{1}{2}\frac{R_1-R_2}{R_1+R_2} = \frac{1}{2}(V_{IN}+\varepsilon) \tag{4}$$

After this short introduction which showed the idea of the new description, an N-bit inverse ladder will be discussed in details. The ladder is partitioned to N current divider. For the k-th divider the input current $I_k^{''}$, the divided current that is the input of the next divider is $I_{k+1}^{''}$, and divided current which can be switched to the output is $I_k^{'}$ (Fig. 3.).

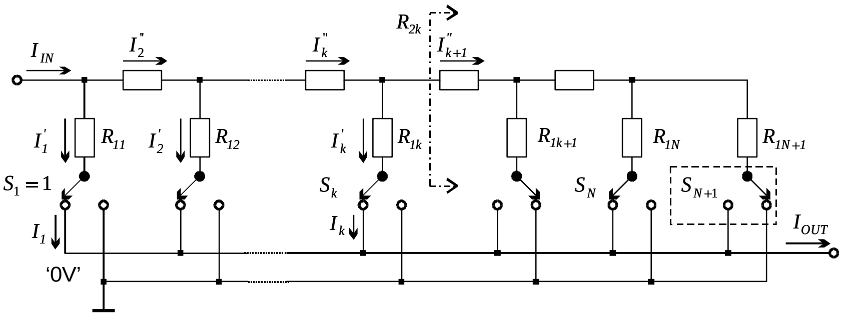

Figure 3. Inverse ladder as cascaded current dividers

The relation between these variables is using the new e_k variable rather than the resistances is determined by the simple current dividing rule, if e_k defined as

$$\varepsilon_k = \frac{R_{1k} - R_{2k}}{R_{1k} + R_{2k}} \tag{5}$$

$$\frac{I_k^{'}}{I_k^{''}} = \frac{R_{2k}}{R_{1k} + R_{2k}} = \frac{1}{2} - \frac{\varepsilon_k}{2} \tag{6}$$

$$\frac{I_{k+1}^{''}}{I_k^{''}} = \frac{R_{1k}}{R_{1k} + R_{2k}} = \frac{1}{2} + \frac{\varepsilon_k}{2} \tag{7}$$

Let's calculate currents $I_{k+1}^{''}$ and $I_k^{'}$ as a function of e_k terms, and the result is:

$$I_{k+1}^{''} = I_{IN} \frac{1}{2^k} \prod_{i=1}^{k} (1+\varepsilon_i), \quad \mathrm{k} \geq 1 \tag{8}$$

$$I_k^{'} = I_{IN} \frac{1-\varepsilon_k}{2^k} \prod_{i=1}^{k-1} (1+\varepsilon_i), \ \mathrm{k} \geq 2 \tag{9}$$

From k=1 to N the output current is the sum of $S_k I_k^{'}$ terms:

$$\begin{aligned} I_{OUT} &= \sum_{k=1}^{N} S_k I_k^{'} = I_{IN} \left[S_1 \frac{1-\varepsilon_1}{2} + \sum_{k=2}^{N} S_k \frac{1-\varepsilon_k}{2^k} \prod_{i=1}^{k-1} (1+\varepsilon_i) \right] \cong \\ &\cong I_{IN} \left[\sum_{k=1}^{N} \frac{S_k}{2^k} \left(1 - \varepsilon_k + \sum_{i=1}^{k-1} \varepsilon_i \right) \right] \end{aligned} \tag{10}$$

The result is a new description, in which the static errors are represented by the e_k terms. The uncertainty of the bits increases going from the MSB to the LSB.

3 CALIBRATION

Equation 10. describes the behaviour of an inverse ladder with any resistor values, even for non R-2R relations. The resistor values are hidden in the e_k terms but it doesn't make any change. These terms represent the errors of the dividers, and in the ideal case their value is zero. To calibrate the ladder, the e_k terms must be measured somehow, and then a correction factor can be computed for each bit and therefore for any code (Eq. 11.).

$$I_{OUTCorr} = I_{IN} \left[\sum_{k=1}^{N} \frac{S_k}{2^k} \left(1 - e_k + \sum_{i=1}^{k-1} e_i \right) \right] + I_{IN} \left[\sum_{k=1}^{N} \frac{S_k}{2^k} \left(e_k - \sum_{i=1}^{k-1} e_i \right) \right] = I_{IN} \sum_{k=1}^{N} \frac{S_k}{2^k} \tag{11}$$

The determination of the e_k terms is critical from the point of calibration. It would be possible to measure the output current ($I_k^{'}$) of each divider stage step by step. However, in this case currents from $2^{-1} I_{IN}$ to $2^{-N} I_{IN}$ should be measured, what would make it practically impossible to realise, because there is no instrument available with such a wide dynamic range. Fortunately, there is another solution. The current dividers produce two approximately equivalent currents, $I_k^{'}$ and $I_{k+1}^{''}$. The difference of $I_k^{'}$ and $I_{k+1}^{''}$ equals to

$$\Delta_k = I_k^{'} - I_{k+1}^{''} = -I_{IN} \frac{\varepsilon_k}{2^{k-1}} \prod_{i=1}^{k-1} (1+\varepsilon_i) \cong I_{IN} \frac{\varepsilon_k}{2^{k-1}}, \tag{12}$$

if the second order terms of e_k are neglected. As it can be seen from the difference of the two measured currents, the unknown e_k could be easily determined. Because of the subtraction, it is possible to use the so-called difference method. It means that if the same current offset is used during the two measurements (M_1, M_2 in Eq. 13.), the difference will not change, but the absolute value of the measurements will be in a smaller range:

$$\Delta_k = M_1 - M_2 = \left(I_k^{'} - I_k^{offset}\right) - \left(I_{k+1}^{''} - I_k^{offset}\right) = I_k^{'} - I_{k+1}^{''} \tag{13}$$

The offset current must be chosen nominally equal to $I_k^{'}$ or $I_{k+1}^{''}$, and then the signal being measured is

$$M_1 = I_{IN}\frac{1-\varepsilon_k}{2^k}\prod_{i=1}^{k-1}(1+\varepsilon_i) - I_{IN}\frac{1}{2^k} \cong I_{IN}\frac{\sum_{i=1}^{k-1}\varepsilon_i - \varepsilon_k}{2^k} \tag{14}$$

$$M_2 = I_{IN}\frac{1+\varepsilon_k}{2^k}\prod_{i=1}^{k-1}(1+\varepsilon_i) - I_{IN}\frac{1}{2^k} \cong I_{IN}\frac{\sum_{i=1}^{k-1}\varepsilon_i + \varepsilon_k}{2^k} \tag{15}$$

The first measurement is made with all switches OFF except the *k*-th one. In this case the output current is $I_k^{'}$. After that the current $I_{k+1}^{''}$ should be measured, what is not so simple. However, if an auxiliary switch is inserted in the ladder (see S_{N+1} in Fig. 3.), the required current can be measured with all switches from *k*+1 to *N*+1 with position ON. The output current will be $I_{k+1}^{''}$ with this binary code.

The hereby-described results correlate with Cutkosky's calibration method. The previous mentioned expression for D_k is the same as for the Cutkosky ladder [2]. Although the inverse ladder and the Cutkosky ladder seem to be very different for the first view, they are really similar to each other. The Cutkosky ladder contains cascaded voltage dividers with voltage input and output signals, and the inverse ladder contains cascaded current dividers with current input and output signals. In most cases, the inverse ladder is used with voltage input, which is converted to current by the input resistance of the ladder, and the output current is also commonly converted to an output voltage by a simple C/V converter. So it can be said, that these networks show duality properties for some point of view.

4 ERROR BUDGET OF THE CALIBRATION ALGORITHM

The further analysis of the ladder needs a carefully considered error budget. Some of the error sources are technology-dependent, and it must be kept in mind when dealing with error sources.

- The main error source is the *tolerance of the resistors*. Before any other examinations, this cardinal error must be studied. However, it is quite simple with the new description. The resistances are hidden in the e_k terms, as it was mentioned earlier (Eq. X.). During the analysis, it was *never* supposed that there is a certain relation between the value of the resistors – 2N *different* variables were used instead – and therefore, the description is valid for *any* combination of the resistances. So the inaccuracy of the resistors can be neglected with respect to the measurement device (see later).
- Another error source is the *non-ideal behaviour of the switches*. The analysis is limited only for DC signals this time. Before analysing the effects of a real switch, it must be mentioned that the analysis is not finished yet, because the proper model of an integrated switch depends on the technology. Further examinations are planned with co-operation of IC manufacturers to map the error sources. Figure 4. shows the model of an analog switch:

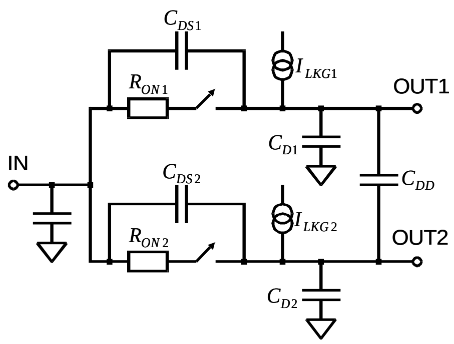

Figure 4. Model of an analog switch

- The effect of R_{ON} is corrected, if the switches have the same R_{ON} for each channel. Practically, it is not true, so separate this resistance to a 'typical' part – which is the same for each channel – and to a 'deviation' one – which represents the variation of R_{ON} for the channels. Then the typical part is corrected, because it can be interpreted as a part of the R_{1k} resistances. The deviation part can not be corrected, and represents an error source, which will remain after the calibration.
- Leakage currents are a critical problem of the calibration. There are ON and OFF leakage currents, which are different. This part of the error budget has not been properly analyzed, because the lack of the correct model of an integrated switch. Some questions, which therefore could not been answered yet:

  → can the leakage currents be simple added or there is a correlation between them
  → are they level dependent
  → the effect of feed-through

➢ The measurement device has *finite accuracy*, adding another error source. The equations considering the accuracy of the meter (worst case calulations):

$$M_1 = \left[I_{IN} \frac{1-\varepsilon_k}{2^k} \prod_{i=1}^{k-1} (1+\varepsilon_i) - I_k^{offset} \right] (1 \pm h_M) \tag{16}$$

$$M_2 = \left[I_{IN} \frac{1+\varepsilon_k}{2^k} \prod_{i=1}^{k-1} (1+\varepsilon_i) - I_k^{offset} \right] (1 \pm h_M) \tag{17}$$

$$\begin{aligned} \Delta_k &= I_{IN} \frac{\varepsilon_k}{2^{k-1}} \prod_{i=1}^{k-1} (1+\varepsilon_i) + h_M \left(\frac{I_{IN}}{2^{k-1}} \prod_{i=1}^{k-1} (1+\varepsilon_i) - 2 I_k^{offset} \right) \cong \\ &\cong I_{IN} \frac{\varepsilon_k}{2^{k-1}} + h_M \frac{I_{IN}}{2^{k-1}} \sum_{i=1}^{k-1} \varepsilon_i \end{aligned} \tag{18}$$

Expressing the measured e_k and substituting it to Eq. 11.:

$$e_{k,measured} = \frac{2^{k-1}}{I_{IN}} \Delta_k - h_M \sum_{i=1}^{k-1} e_i = e_k - e_k h_e \tag{19}$$

$$\begin{aligned} I_{OUTCorr} &= I_{IN} \left[\sum_{k=1}^{N} \frac{S_k}{2^k} \left(1 - e_k + \sum_{i=1}^{k-1} e_i \right) \right] + I_{IN} \left[\sum_{k=1}^{N} S_k \left(e_k - e_k h_{ek} - \sum_{i=1}^{k-1} (e_i - e_i h_{ei}) \right) \right] = \\ &= I_{IN} \sum_{k=1}^{N} \frac{S_k}{2^k} \left(-e_k h_{ek} + \sum_{i=1}^{k-1} e_i h_{ei} \right) \end{aligned} \tag{20}$$

It means that the inaccuracies of the ladder caused by the resistor tolerances are corrected by a factor of the instrument accuracy.

5 CONCLUSIONS

The paper presented a new description for current-output R-2R ladders. The new description based on the former work of Cutkosky, implementing it to the inverse ladder. The main benefit of the new description is that the further analysis of the ladder is simpler. The calibration of the ladder based on the new results, and only slight changes should be made (an auxiliary switch) to realise it. The analysis of the calibration procedure is not finished yet. As it was mentioned, the study of the error budget remained a research topic. The Cutkosky ladder uses three precision switches for each bit but the current-output ladder needs one switch per bit and an auxiliary one for the calibration. This reduction in the number of the switches can be an interesting feature of the inverse ladder.

The calibration may be used both in the case of a DAC or an MDAC which are based on R-2R ladders.

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