MULTIPATH CROSS-CORRELATION FLOWMETERS

V. Skwarek and V. Hans
University of Essen, Institute for Measurement and Control
45117 Essen, Germany

Abstract: The ultrasonic cross-correlation flowmeter is not only a reliable measurement device due to recent improvements in signal processing; it can even be used for gaining further knowledge about the properties of turbulent flow. This paper shows as well considerations about fluctuations, measuring time and ergodicity as arrangements and necessities for multi path measurements. Results are used for averaging and first tomographic approaches for the visualization of turbulent flow.

Keywords: ultrasonic cross-correlation flowmeter, multi path measurement, ergodicity, tomography

1 INTRODUCTION

The ultrasonic cross-correlation flowmeter was already intensively explored in the early 70es by Beck and affiliates with the main results published in [1]. Generally, these results can be summarized in a few words: From the scientific point of view many new theories about turbulent fluids and their interaction with ultrasound are suggested. But the accuracy of the measures and the instrument range are too poor for commercial use compared with other principles. The main reason for these results, causing the nowadays very bad image and acceptance of the device, bases on the lack of signal processing facilities of the former times.

Meanwhile, these stereotypes are invalidated: With the support of modern digital high speed computers the device is an equivalent competitor concerning the quality of the results. According to the requirements on measurement hardware, processor speed and memory effort there are unanswerable advantages to some similar devices.

The main focus of this paper is less put on industrial but on scientific use of the flowmeter. The state of current research enables the simultaneous use of several ultrasonic paths. Herewith questions concerning ergodicity, averaging and tomography are to be answered. Obtained data are used for first tomographic approaches to determine influencing structures, to model the turbulent fluid and to optimize the flowmeter.

2 BASIC MEASUREMENT PRINCIPLE

2.1 Fluid-dynamic background

The main working principle of the ultrasonic cross-correlation flowmeter is already mentioned in its name: two ultrasonic continuous signals in a row take a course in a radial direction through a turbulent fluid (figure 1).

![Figure 1. General design of an ultrasonic cross-correlation flowmeter.](image)

Turbulent structures modulate the signal as well in amplitude (AM) as in phase (PM). Due to the stochastic properties of turbulent structures, each modulation pattern is unique – like a fingerprint. With a short distance between the ultrasonic barriers the modulations at both sensors show significant similarities, only with a time shift $\tau$ according to the travelling time of the turbulences between the barriers. Therefore the CCF, a mathematical measure for a similarity between two signals, features an
easy detectable absolute extremum for this \( \tau \). The type of the extremum – minimum or maximum – depends on the direction of the beams to each other. For avoiding such discussions, the absolute value of a CCF is only taken into consideration. Consequently, the extreme value will always be a maximum (figure 2).

With a known travelling time and distance \( d \) between the barriers, the average speed ideally results from

\[
\nu = \frac{d}{\tau}.
\] (1)

2.2 Signal processing

According to these basic theoretical considerations about the measurement principle, it seems to be easy, to obtain results, which are in any relation to the mean velocity of the fluid. But reality behaves differently as many scientists in this area had to experience. Very comprehensive work was done by Poppen [2]. He developed methods and principles of modern signal processing for measuring a gas flow with comparable little effort:

In a first step the received ultrasonic signal has to be demodulated. This is done with a technique called “undersampling with integer submultiples of the carrier-frequency”, which is well known in radio science. It works like a low-pass filter with the carrier remaining as dc-part. But especially for higher velocities [3], it got discovered that the AM and PM parts of the ultrasonic signals have to be demodulated individually in a complex demodulation [4]. Without getting lost in details, a few words about the principle: The mathematical basis is a Hilbert-transform [5], converting the original function into two perpendicular parts. Interpreting them as real- and imaginary-part of a pointer in the complex domain, the length and angle of the pointer can be easily obtained. In practical measurements the Hilbert-transform is performed by taking double samples with a phase-shift of 90° according to the carrier frequency instead of a single-sample-undersampling.

After these transformations the AM and PM signals for each ultrasonic barrier are available. The according signal types of both barriers are cross-correlated with the discrete CCF

\[
\varphi_{xy}(kT_s) = \lim_{N \to \infty} \sum_{n=-\infty}^{N} s_1(nT_s) s_2((k + n)T_s).
\] (2)

Ideally, its maximum indicates the average travelling time \( \tau \) of the modulating structures between the barriers in multiples of the sampling time \( T_s \) (figure 2). Equation (1) converts \( \tau \) into a velocity.

2.3 Results

Further processing steps like velocity-adaptive filtering or CCF-correction algorithms help to improve the results again. With a statistical analysis over a wide range of velocities, the calibration curve in figure 2 is obtained. It shows the average of the \( r_{dev} \)-factor in the left and its scattering in the right diagram.

The \( r_{dev} \)-factor

\[
r_{dev} = \frac{\nu_{\text{meas}}}{\nu_{\text{ref}}}
\] (3)

specifies the deviation between the measured speed \( \nu_{\text{meas}} \) and the referenced speed \( \nu_{\text{ref}} \). From a superficial point of view it is expected to take ideally a value around \( r = 1 \). But this expectation is wrong because the reference integrates the speed over an area whereas the ultrasonic beam measures along a line only. Therefore equation (3) changes to
Figure 3. r-factor and scattering of a cross-correlation measurement.

\[
\begin{align*}
\text{r}_{\text{dev}} &= \frac{1}{2R} \int_0^R v(r)dr \\
&= \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R v(r)drd\varphi.
\end{align*}
\]

Assuming an approximation for the radial velocity distribution such as a potential law

\[
v(r) = v_{\text{max}} \left( 1 - \frac{r}{R} \right)^n
\]

according to Nikuradse [6] with a speed dependent exponent \( n \), \( r_{\text{dev}} \) amounts to

\[
r_{\text{dev}} = 1 + \frac{1}{2n}.
\]

This means a total value of \( r_{\text{dev}} = 1.08..1.07 \) for Reynoldsnumbers = 4000..300 000. Taking these facts into consideration, the measured curve for \( r_{\text{dev}} \) fits quite good into the expected values. In all, the measured curve shows a negative offset to the theoretical one. The probable reason for this offset is the difference between the theoretical assumption of the ultrasonic beam as a line. In reality it covers a non-neglectible volume of the test rig and measures not only the center-line of velocities but even slower velocities at the left and right of the center-line. Consequently the real measured speed is lower than expected.

The rapid decrease at the beginning of the curve is based on two different possible reasons:

- Either structures with lower velocities change more than structures with higher velocities between the ultrasonic barriers. Therefore the CCF detects preferably structures with higher velocities especially at a lower average speed of the fluid.
- Or the faster structures at the center of the pipe carry a higher energy content taking more influence on the ultrasonic beam. With increasing average speed of the fluid the energy content of the wall-near structures and herewith the influence on the ultrasound increases too.

Independently of which explanation is more probable, the deviation is between the expected and the measured curve is explainable.

The scattering in the right diagram in figure 3 is even more important than the average: Whereas a deviation of mean values is reducible by calibration factors and look-up-tables, the scattering is hard to handle. In the last consequence this is the main aspect deciding, whether a device fulfills standards for commercial use. But meanwhile – unlike early cross-correlative devices – the scattering could not only be reduced into limits for household-gas-measurements. It even decreases with increasing velocities, which is a central aspect for commercial accounting applications.

3 MULTI PATH ARRANGEMENT

The main reason for the scattering and a possibility for a reduction will be explained in this section:

It is simply the problem of performing a single path measurement. This is in contrast to the definition of an average flow which demands a spatial averaging at least over the cross section of the pipe – as already indicated in equation (4). Turbulent fluctuations, the basis for a cross-correlation measurement, cause random local changes in the velocity profile amounting up to \( \pm 5\% \) of the local mean velocity. A measurement at another position of the pipe increases the chance that a further structure is met,
equalizing the profile disturbance. Therefore a simultaneous measurement with several ultrasonic beams, a so-called “Multi Path Arrangement (MPA)” has to be used.

Figure 4. Multipath test rig with sensors in star-arrangement (left) and grid-arrangement (right).

First examinations about MPA and problems were introduced in [7] with some basic consideration about the feasibility of measurements and the design of the pipe (figure 4). With better algorithms for signal-processing, that work is particularly continued. Simplifying assumptions are the existence a of fully developed flow profile and diametric ultrasonic paths with finite dimensions, crossing exactly in the center of the pipe. For preliminary examinations a three path configuration is taken into consideration as shown in figure 4 (left).

Measurements using the three single paths and their common average are shown in figure 5. The range of the set speed had to be limited for the introductory measurements to 20m/s.

Figure 5. Results of a Multi Path Measurement with sensors in star arrangement.

As already mentioned before, it is still not possible to estimate the quality of the r-factor. All three paths as well as their average run in similar intervals which are at least plausible compared with the expected value. The real improvement of averaging over multiple paths is obvious in the right diagram: The scattering of all three paths shows a different and non-monotonous behavior, which is significantly reduced by their common averaging. This is an indication for oppositely effecting fluctuations at the same time but at different positions in the pipe. Consequently the mean value over several paths will equalize and suppress these effects.

Better measures for a fully developed flow profile are to be expected by averaging over multiple paths.

4 AVERAGING AND ERGODICITY

In the next step the scattering is not reducible in only two but all spatial dimensions of the pipe by registering all fluctuations. The intention is the certainty of reliable measures within standardization limits of ±8% of the actual value for a low mass flow and ±4% for a high mass flow. Assuming a Gaussian distribution, not only the simple scattering (= 68.27% of the measures) but at least three times the scattering (= 99.73% of the measures) should fit into the standardization limits.
Since a “real ensemble measurement” at each position of the whole pipe cannot be performed, other possibilities for averaging have to be taken into consideration: Either one prolonged data set can be used for calculating a long-term CCF or an average is calculated over several short term measurements. The decision, which of these methods should preferably be used, is a topic about ergodic theory briefly discussed in the next section:

Usually, a fully developed flow is considered to be stationary [8][9][10], fulfilling the condition for calling a random process “ergodic”. Then “… its ensemble averages equal appropriate time averages” [11]. Therefore, a long-time measurement at a fixed position replaces the ensemble average at a fixed time over the whole process.

This long-time measurement leaves the question unanswered about how to handle the prolonged set of data. To alternatives are given: It can either be used for calculating one long CCF or it is split into several short records, each with a CCF on its own and a final common average.

The proposed “longer correlation time” sounds to be the accordance to the “time average” of the ergodic theorem. But a more detailed analysis shows a tiny but important difference between the definition and the realization: The definition demands a measurement at a fixed position i.e. a fixed point or a line such as an ultrasonic path. But in reality a path consists of many resonating atomic particles, exchanging impulses. Fractions of these impulses are originally caused by an ultrasonic transmitter. From this point of view it becomes clear that an ultrasound measurement cannot be modeled by a fixed linear path but only by these moving impulses, particles or wavefronts, respectively.

Up to this point there is still no real restriction for the ergodic theorem because with a long (infinite) measurement time at least a theoretical possibility is left for an interaction between each ultrasonic impulse and the statistical entity of the fluid. But the measurement time is not equal to the record length of the data but the duration of interaction between ultrasound and the fluid. Therefore, it depends only on the time a particle needs for the expansion between transmitter and receiver which again is directly connected to the distance and the speed of sound. The record length takes no influence on the result.

Consequently, a long-time measurement should better be interpreted as an ensemble which has to be averaged. Therefore, it has to be split into several data records, each with a CCF of its own. The mean value of the final results accords to the definition of an ergodic ensemble average.

5 TOMOGRAPHIC APPROACHES

The application of an instantaneous use of several ultrasonic paths is extendable from a simple averaging for a fully developed profile to a visualization of turbulent flow. In this paper only a brief overview about the cross-correlation ultrasound tomography and the arising problems will be given.

The idea came up with the examination of disturbed profiles: Simple averaging is not sufficient due to missing rules about weighting the results over the single paths. At least a basic knowledge about the general shape of the profile is necessary. Therefore, a tomography has to be performed. The results of the CCF, which are measures for the average velocities over the ultrasonic path, are used for a reconstruction of the axial velocity distribution of the fluid. An integration over this profile returns the total average velocity of the fluid.

But the main problems come up with the definition of boundary conditions for tomographies: One parallel projection is ideally represented by the entity of an infinite number of parallel beams. For an ideal reconstruction again an infinite number of projections is necessary [12]. These are demands which cannot be fulfilled by the momentary designed test rig due to its very limited dimensions. With respect to crosstalk between the sensors and the macroscopic size of the beams only two perpendicular projections with three beams each are performed. Furthermore, the parallel expansion of the ultrasonic beams is not ensured because the modulating effects are not precisely known up to now. No reliable mathematical algorithms are available for such few data with not linear expansion paths so that the expectations to the results have to be reduced to an absolute minimum. Instead of a full reconstruction with a perfect averaging process only an intention about the interaction between ultrasound and the fluid can be yield. The comparison with other visualization methods such as PIV give an idea about the possible origins of the modulations in the received ultrasonic signals.

Therefore a new combination of mathematical algorithms with a suitable sensor setup have to be taken into consideration. Following first reflections of Radon [13], one of the pioneers of tomography, a qualitative picture is obtained by “smearing” the projections back along their projecting paths and adding all the single pictures up to one final result. Nowadays this method is called “summarizing tomography”.

In figure 6 such reconstructed velocity distributions are shown for a fully developed profile (left) and behind of a single elbow (right). The perspective is against the direction of the flow.
Characteristic properties for each particular flow profile are obvious: The fully developed flow is symmetric to the center of the pipe – apart from problems with an exact symmetric sensor arrangement. The single elbow causes as expected a shift of the maximum speed to the wall of the pipe in the bending plane.

6 SUMMARY AND OUTLOOK
Using Multi-Path-Arrangement for ultrasonic cross-correlation flowmeters the obtained results for a fully developed flow are improvable by averaging over all paths in a "star arrangement". Furthermore, stochastic changes of the spatial velocity distribution due to fluctuations are reducible by calculating an ensemble average. Unlike other measurement principles this must not be performed by time averaging over infinite measurement intervals as the considerations about ergodicity show.

For future research a new playground is opened with the beginning of cross-correlation tomographies: The first approaches produce promising results so that improved algorithms, an optimized design of the test rig and a better sensor arrangement help to visualize turbulent flow.

7 ACKNOWLEDGEMENT
The research was supported by the grant of the "Deutsche Forschungsgemeinschaft" (Forschergruppe "Strömungsmechanische Grundlagen der Durchflußmessung", Me 484/29-1).

REFERENCES