ESTIMATION OF EPICYCLOIDAL GEAR CONTOUR

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Abstract: There are described guidelines of creating epicycloids and their application in engineering. Authors propose a scanning method as a way of estimation of real deviation of contour for epicycloidal gears. Measuring results and their interpretation is presented as well.
Keywords: epicycloidal gear, measurement on CMM, deviation from contour

1 INTRODUCTION

Instruments for measurement of deviations of flank pitch line in involute gears [1] cannot be used for checking epicycloidal gears. The reason is the way of generation of involute and epicycloid are different. Epicycloid is described by a point on the rolling circle by the radius r when it rolls upon a fixed circle, tangent externally to the fixed circle by the radius R [1, 2], while involute is described by a point of a straight line when it rolls upon the base circle by the radius r_z.

Epicycloid finding application in gears should meet condition $R/r \geq 2$ where quotient belongs to the set of natural numbers N. Epicykloids and involutes can occur as: common, curtate and prolate ones.

Deviations of outlines of epicycloidal transmissions can be estimated with co-ordinate measuring machines (CMM) providing that they are equipped with appropriate software. Otherwise, point set obtained from CMM is equidistant to the measured outline and have to be calculated and interpreted appropriately in an arduous way.

2 MATHEMATICAL DESCRIPTION OF EPICYCLOID

An example of a common, curtate and prolate cycloid for R/r = 4 is shown in figure 1. The most popular in technique are curtate cycloids. Common cycloids are used only in special cases.

Any epicycloid can be described with equations:

$$x = (R + r)\cos\omega - \lambda r \cos\frac{R + r}{r}\omega \qquad y = (R + r)\sin\omega - \lambda r \sin\frac{R + r}{r}\omega \tag{1}$$

where: R - radius of the fixed circle [mm], r - radius of the rolling circle [mm],
ω - angle of radius-vector [rad], λ - cycloid shortening index.

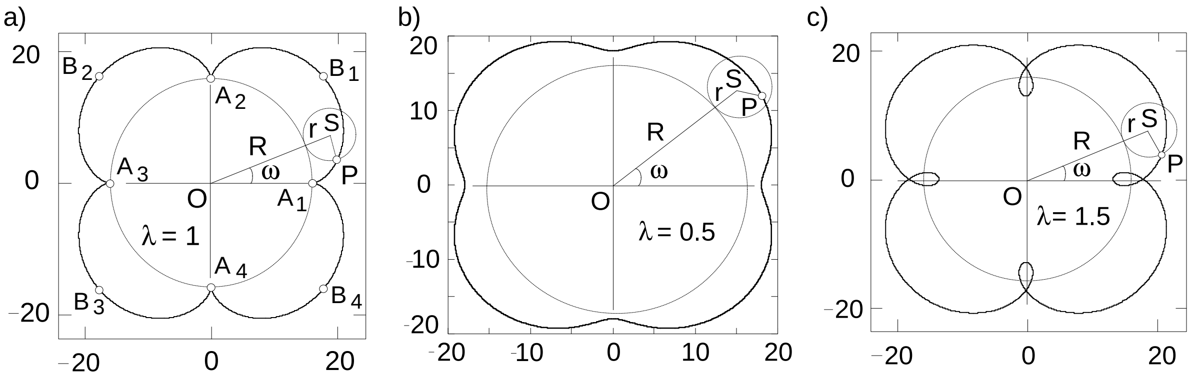

Figure 1. Hypothetical epicycloids for R/r = 4 and different shortening index λ:
a) common epicycloid, b) curtate epicycloid, c) prolate epicycloid.

Prolate epicycloid is obtained for shortening index $\lambda > 1$ and curtate one for $\lambda < 1$. Shape of a curve depends on ratio R/r = m as well. When ratio R/r is an integer then common epicycloid ($\lambda = 1$) is the closed curve with finite numbers of arcs (fig. 1a). Spinodes: $A_1, A_2, ..., A_m$ occurs for radius $\rho = R$ and angle $\omega = 2k\pi/m$, where k = 0, 1, ..., m – 1 and vertexes: $B_1, B_2, ..., B_m$ for radius $\rho = R + 2r$ and angle $\omega = (2\pi/m)(k+1/2)$. The radius of curvature in any point of the epicycloid $r_i = [4r(R + r)/(2r + R)]sin(R\omega/2r)$

and for vertex is $r_B = 4r(R + r)/(2r + R)$. It can be concluded that curvature for one lobe of epicycloid is changeable.

From fig. 1 is noticed that prolate epicycloid ($\lambda = 1.5$) is characterised by undercutting which causes discontinuity of the contour. For that reason this kind of epicycloid is not applied in transmissions.

Cycloid shortening index (tooth height coefficient) λ has some limitations. $\lambda \le 1$ results from application curtate and common curves in construction. It is important that curve should consist from a convex and concave part with the point of inflexion. It is described by $\lambda \ge 1/(1+z)$ for epicycloid and $\lambda \ge 1/(z-1)$ for hipocycloid. Next condition $\lambda \ge (z-1)/(2z+1)$ for epicycloid and $\lambda \ge (z+1)/(2z-1)$ for hipocycloid results from maximal movement of equidistant with reference to the base curve. These conditions are sketched in fig. 2 as application area of cycloid shortening index [5]. Ratio of $0.6 \le \lambda \le 1$ is accepted in practice.

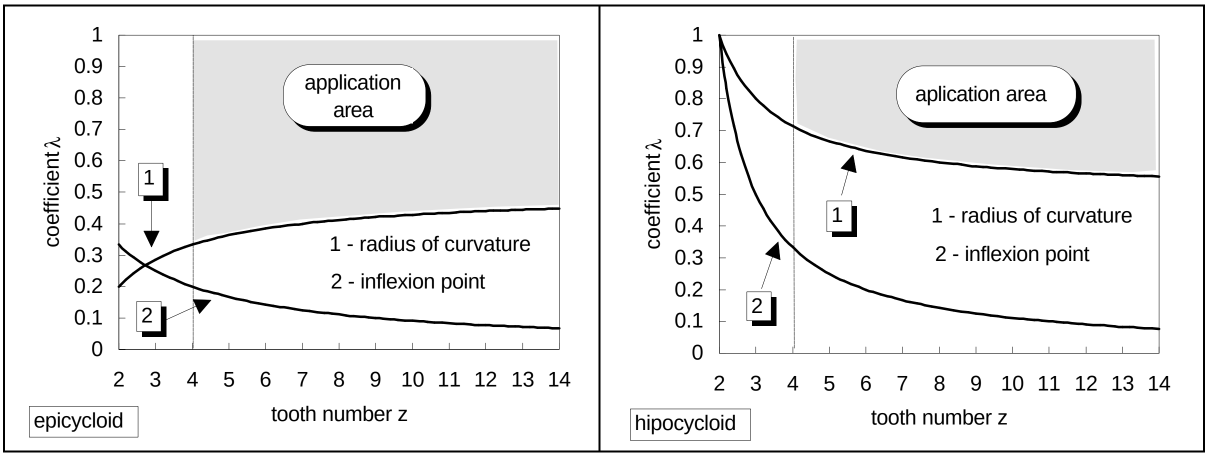

Figure 2. Application area of cycloid shortening index λ: a) for epicycloid, for hipocycloid.

Height of cycloid arc, in other words height of the lobe (tooth) is:

$$h = 2 * \rho * \lambda \tag{2}$$

Value for radius ρ is established according to assumed teeth number. Radius ρ is from several to a dozen or so millimetres for the most applied number of teeth 3 ÷ 15. If tooth number is higher then radius ρ is less, so that not increase dimensions of toothed gears excessively.

3 MEASUREMENT OF EPICYCLOID CONTOUR BY SCANNING METHOD

Figure 3 shows an example of measurement of epicycloidal gear by scanning method. In chosen co-ordinate system $O_mX_mY_m$ there was obtained point set - trajectory of centre S_i of sapphire ball-feeler. This trajectory is moved outside to the real epicycloid by the radius of ball-feeler r_k. Real points

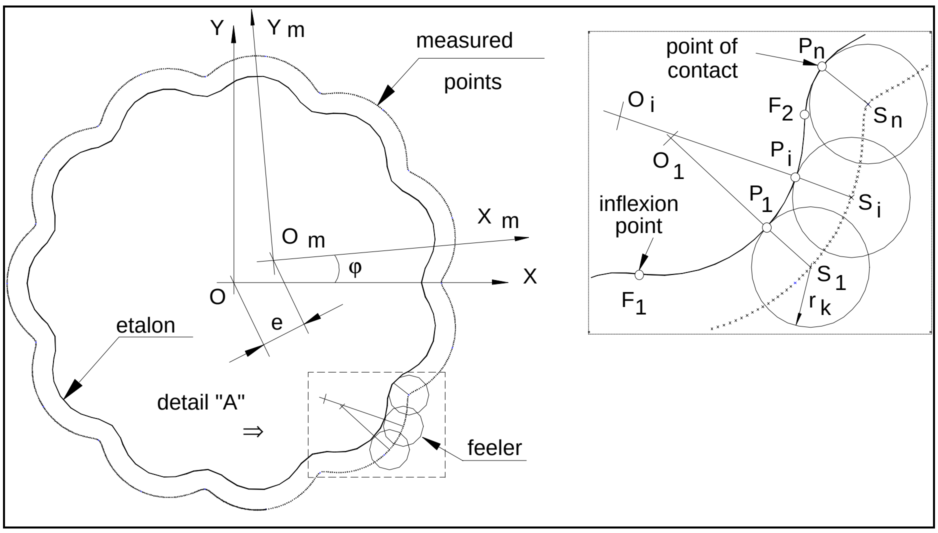

Figure 3. Measured outline of epicycloidal gear: $l = 0.6$, $m = 11$, $r = 6.5$

of epicycloid are lying on straight lines which connect centres of ball-feeler S_i with centres of curvatures of the epicycloid O_i in points of the contact P_i. One lobe of curtate epicycloid has changeable curvature with two inflexion points (fig. 3).

Contour of any curve can be replaced by finite set of circular arcs provided maximum error of approximation Δ_p do not exceed positional errors of co-ordinate measuring machine. Estimation of Δ_p error can be calculated by measurement of distances $\overline{P_i S_i}$ between measured points and etalon (fig. 3). Previously, it is necessary to recalculate measured points into new base co-ordinate system OXY. It means to move initial co-ordinate system $O_m X_m Y_m$ by the value e and rotate it by the angle φ [3, 4].

4 ANALYSIS OF MEASURING ERRORS

The measurement realised by CMM is charged by an error. The value of this error depends on measuring limits and inaccuracy of measurement Δ_m of particular measuring device. For Carl Zeiss CMM (Eclipse 550 with head ST-ATAC) with measuring limit 500 x 550 x 580 inaccuracy can be calculated as: $\Delta_m = 2.9 + L/250$ [µm], where L [mm] is measured distance to base point O_m.

Additional errors are resulted from replacement of epicycloid with etalon consisted of finite number of circular arcs and recalculating obtained data according to new Cartesian co-ordinate system.

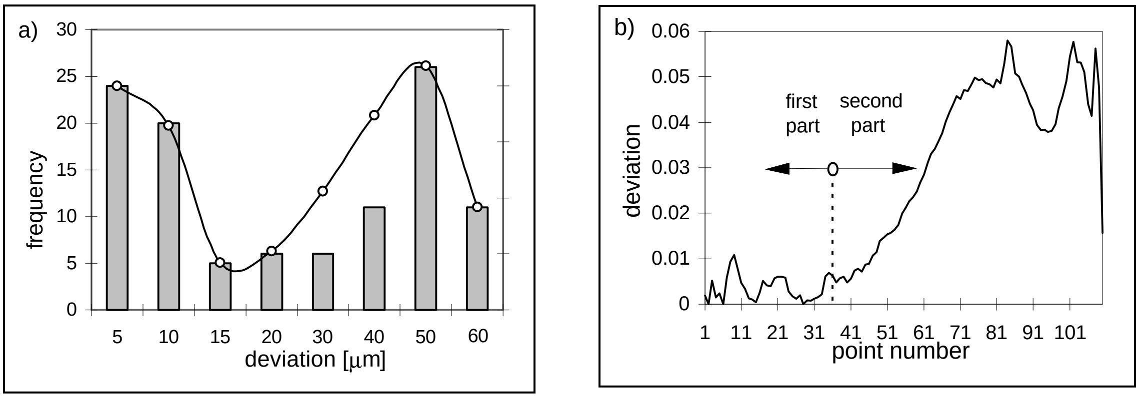

Figure 4. Deviation for one lobe of epicycloid from fig. 3: a) histogram, b) deviation from etalon.

Measuring results of deviations from the contour and their changeability is shown in figure 4. There is an asymmetry between the left and the right part of the contour for the one lobe.

Histogram (fig. 4a) indicates two sets of measuring results of real deviation in the shape of normal distribution. The left-handed set has mean value of 7 µm and standard deviation of σ = 3 µm, and the second set appropriately: mean value - 46 µm and σ = 17 µm. Significant difference in mean values and standard deviations between left and right flank of a lobe proves considerable differences in machining a single lobe contour.

Fig. 4b shows changes of deviation values particular measuring points for one lobe. Point 1 (point P_n in fig. 3 - detail „A”) lies in the lowest part of the outline. Next points were obtained by clockwise movement of the feeler. There is visible that deviation of measured outline from etalon increases considerably up to 58 µm after crossing vertex, and then decreases to value of zero.

Taking under consideration obtained results there is assumed that considerable differences of values may be caused in a machining process for instance by different variables of the process for the left and right part of a lobe or changeable rigidity of the array: machine tool - chuck - workpiece - tool.

5 CONCLUSIONS

1. A limitation for checking cycloidal transmission is lack of special measuring instruments comparing to involute transmission.

2 Mathematical etalon of the contour as theoretical equation or substitute etalon as the set of circles is indispensable to estimate a deviation between the real and theoretical contour.

3. During measurement of the outline deviation by means of CMM it is essential to orientate the real and theoretical co-ordinate systems appropriately. Omitting or incorrect orientation of these co-ordinate system on CMM may lead to considerable errors and inappropriate interpretation of obtained results.

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