THE MEASUREMENT UNCERTAINTY OF A PRESSURE BALANCE 
FOR LOW GAUGE DIFFERENTIAL PRESSURE MEASUREMENT

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Abstract: This paper presents a simplified model for the evaluation of the 
measurement uncertainty of a pressure balance (Type V1600/4D, manufactured by 
Pressurements, Ltd.) used in our pressure calibration laboratory as a standard 
pressure balance for gauge and differential pressure measurement. The gauge 
measuring range of this gas-operated balance runs from a nominal pressure of 20 
Pa to 16 kPa and its minimum differential pressure measurement is 5 Pa. The results 
for measurement uncertainty obtained from the model are based on calibration data 
provided by the accredited calibration laboratory. In the measuring range up to 600 
Pa, its measurement uncertainty is less than ±0.16 Pa, and in the range up to 16 kPa 
it is less than ±0.02 %; this shows good agreement with the manufacturer's data. The 
contributions of certain parameters to the total uncertainty of pressure measurement 
are also presented.

Keywords: measurement uncertainty, pressure balance, gauge and differential 
pressure

1 INTRODUCTION

Low gauge and differential pressure measurement are especially important in gas-flow and liquid-
flow measurement and also in the verification of the air permeability of some devices. The magnitudes 
of these pressures range from a pressure of below 1 Pa to a pressure of 20 (100) kPa. High precision 
and accuracy pressure balances are used for the calibration of such measuring instruments [1, 2, 3].

Standard pressure balances type 1600/4D manufactured by Pressurements Ltd. [4] are used in the 
Laboratory for Measurement in Process Engineering (LMPS) for low pressure and differential pressure 
measurement in the measuring range from 20 Pa to 16 kPa. The pressure balance is a gas-operated 
balance using a conical steel and aluminium piston in a stainless steel cylinder. Pressure is maintained 
by internal regulation of the air flow through the piston and cylinder.

2 OPERATION OF THE PRESSURE BALANCE

The V1600/4D pressure balance operates on the dynamic interaction of the air flow with a non-
cylindrical piston [4].

The component parts are

- piston and cylinder
- regulator and
- selector valves.

A schematic of the balance is shown in figure 1. The main parts labelled in the figure are: 1-piston, 
2-cylinder, 3-insert, 4-double cascade regulator and 5,6-reservoirs. The instrument requires constant 
air flow through the regulator which also isolates the output pressure from supply pressure fluctuations. 
The piston in the cylinder transforms the gravitational force of the pistons and weights $F_g$ into 
pneumatic pressure $p$. When air passes between the piston and cylinder, a back-pressure is 
generated. This back-pressure force $R$ does not exceed 0.0015 $F_g$, which is sufficient to automatically 
centre the piston [4].
Figure 1. Schematic of the air pressure balance.

The pressure balance also enables differential pressure measurements, because it has a second measurement system built-in in order to generate a single static pressure. This protects the instrument from fluctuations in atmospheric pressure.

The most important part of the V1600/4D is the automatic pressure regulator, which consists of pressure chambers and pressure membranes. Pressures $p_1$, $p_2$, $p_3$ and $p_4$ are generated in the chambers. These pressures are dependent on each other. They are transferred via membranes which are of different sizes. Below the piston pressure $p$ is generated, which also affects one of the membranes.

2.1 Mathematical model of the pressure balance

The basic equation which describes the operation of the balance [4] is

$$p = \frac{mg(1 - \frac{\rho_a}{\rho}) + R}{A_0},$$

where $p$ is the outlet pressure of the tester, $m$ is the mass of the pistons and weights, $g$ is local acceleration due to gravity, $\rho_a$ is air density, $\rho$ is the density of the weights, $k$ is a coefficient due to the air buoyancy effect and $A_0$ is the geometric area of the cylinder.

$$A_0 = \frac{\pi d^2}{4},$$

where $d$ is the diameter of the cylinder. The back pressure force $R$ depends on the air mass flowrate $q_m$, the outlet air velocity of the piston-cylinder $v_2$ and the outlet air velocity of the insert $v_1$, see fig. 1,

$$R = q_m v_2 \cos \alpha - q_m v_1,$$

where $2\alpha$ denotes the angle at the peak of the conical piston.

Usually the effective area of the piston-cylinder unit of the pressure balance $A$ is determined. This area is defined as the ratio between the gravity-normal force of the weights and the floating element and the actual pressure. This actual pressure is also the measured pressure $p_m$ which is generated at the outlet of the tester. If this definition of the effective area is used, Eq. (1) could be written

$$p = p_m = \frac{mg k_0}{A}.$$
By comparing Eq. (1) and (4), the effective area of the piston cylinder unit is

\[ A = \frac{\pi d^2}{4} \left( \frac{mgk_p}{mgk_p + R} \right) . \] (5)

The magnitude of the local acceleration due to gravity influences the accuracy of the pressure balance. The balance is often calibrated at the nominal acceleration due to gravity \( g_n \), which is also stated in the pressure balance certificate. If these two values differ from each other \( (g \neq g_n) \), the correction value of the pressure is

\[ p = p_n \frac{g}{g_n} = p_n k_g , \] (6)

where \( p_n \) is the value of the pressure at the reference acceleration due to gravity to which the deadweight tester has been calibrated and \( k_g \) is a correction factor for acceleration due to gravity.

The ambient temperature also has an influence on the magnitude of the pressure. The effective area of the piston-cylinder unit is usually given at the reference ambient temperature of 20 °C. If the actual temperature differs from the reference temperature, the correction factor \( k_t \) has to be applied. This factor depends on the thermal coefficient of expansion for the piston and cylinder

\[ p = \frac{mgk_t}{Ak_t} . \] (7)

3 THE MEASUREMENT UNCERTAINTY MODEL AND ITS EVALUATION

The double standard deviation method is used to express measurement uncertainty [5].

\[ U(y) = 2 \sigma . \] (8)

If a quantity \( y \) depends on a number of independent quantities \( x_i \) \( (i=1,N) \), the absolute uncertainty of quantity \( y \) is

\[ U(y) = \pm \sqrt{\sum_{i=1}^{N} \left( \frac{\partial y}{\partial x_i} \right)^2 U(x_i)^2} , \] (9)

where \( x \) and \( y \) are the directly and indirectly measured quantities and \( U(x_i) \) \( (i=1,N) \) and \( U(y) \) their uncertainties. The uncertainty of the quantity \( y \) could be expressed as the relative uncertainty \( U_r(y) \)

\[ U_r(y) = \frac{U(y)}{y} = \pm \sqrt{\sum_{i=1}^{N} \left( \frac{1}{y} \frac{\partial y}{\partial x_i} \right)^2 U(x_i)^2} . \] (10)

The contribution of the independent quantity \( x_i \) in percent (%) to the total uncertainty of the dependent quantity \( y \) is

\[ c(x_i) = \frac{\left( \frac{\partial y}{\partial x_i} U(x_i) \right)^2}{U^2(y)} \times 100\% . \] (11)

3.1 Simplified measurement uncertainty model of the pressure balance

On the basis of Eq. (5) and Eq. (10) the relative uncertainty of the effective area of the piston-cylinder unit is
If we want to determine the measurement uncertainty of the effective area of the piston-cylinder unit on the basis of Eq. (12), the uncertainties of the individual variables have to be known and evaluated. The values and evaluations are determined on the basis of the calibration certificates [6] and [7], measurements and the data in [1]. The geometrical diameter of the cylinder is 25 mm [7] and its uncertainty \( U(d) \) is \( \leq \pm 0.002 \) mm. On the basis of the piston mass and weights mass data in the certificate [6], their uncertainties \( U(m) \) range from \( \pm 0.1 \) mg to \( \pm 6.3 \) mg. The relative uncertainty of the buoyancy correction \( k_r \) is less than \( \pm 0.003 \% \). This evaluation uses the data for densities \( \rho_a = 1.2 \) kg/m\(^3\) and \( \rho = 8000 \) kg/m\(^3\) which are stated in the certificate. The acceleration due to gravity to which the pressure balance has been calibrated is \( g_n = 9.80665 \) m/s\(^2\), while the local acceleration due to gravity is \( g = (9.80630 \pm 10^{-5}) \) m/s\(^2\). The magnitudes for the back pressure forces are calculated and evaluated from the data on mass and pressures stated in the certificates [6] and [7]. These uncertainties are \( U(R) = \pm 30 \% \) and are evaluated on the basis of the air flow rate measurement at the outlet of the measuring cylinder.

The pressure relative uncertainty of the pressure balance is determined on the basis of Eq. (4) and (10)

\[
U_p = \pm \sqrt{U^2_1(m) + U^2_2(A) + U^2_3(k_r) + U^2_4(g)}. \tag{13}
\]

On the basis of the described algorithm and given magnitudes of the quantities and their uncertainties, the pressure uncertainties are calculated for the whole measuring range. The results are shown in figures 2 and 3. Figures 2 and 3 show the uncertainty limits \( U(p) \) associated with the determination of true pressure, which are obtained from the certificate [7]. Figure 2 shows the measurement uncertainty limits \( U(p) \) for the measuring range from 20 Pa to 16 kPa. Figure 3 shows the same limits of measurement uncertainty in the measuring range from 20 Pa to 1 kPa. These figures also show the uncertainty limits, which are obtained from our model and from the manufacturer. The pressure uncertainties stated by the manufacturer range within the interval of \( \pm 0.1 \) Pa for the measuring range up to 500 Pa, and \( \pm 0.02 \% \) for the measuring range from 500 Pa to 16 kPa. These results are based on an ambient temperature of the piston-cylinder unit of 20 °C.

The pressure uncertainties stated by the manufacturer range within the interval of \( \pm 0.1 \) Pa for the measuring range up to 500 Pa, and \( \pm 0.02 \% \) for the measuring range from 500 Pa to 16 kPa. These results are based on an ambient temperature of the piston-cylinder unit of 20 °C.

4 CONCLUSIONS

On the basis of the calculated values and the values from the calibration certificates, we conclude that the measurement uncertainty of our pressure balance in the measuring range from 20 Pa to 16 kPa are almost within the uncertainty limits which are given by the manufacturer. In the measuring range up to 600 Pa the pressure uncertainty does not exceed \( \pm 0.1 \) Pa, except at pressures \( p = 400 \) Pa and \( p = 600 \) Pa, for which the uncertainties are \( \pm 0.13 \) Pa and \( \pm 0.16 \) Pa (\( \pm 0.027 \% \)), respectively. These results are also supported by the results which were obtained from our simplified uncertainty model. The main influence on the total uncertainty of the pressure is the uncertainty of the effective area of the piston-cylinder unit, while the pressure-back force, in addition to the mass of the pistons and weights, is also a very important quantity. This force has the especially significant influence in pressures which are below 600 Pa.
Figure 2. The uncertainty of the pressure balance in the measuring range from 20 Pa to 16 kPa.

Figure 3. The uncertainty of the pressure balance in the measuring range from 20 Pa to 1 kPa.
REFERENCES


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