QUALITY EVALUATION OF MEASUREMENT INSTRUMENTS

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Abstract: At present, no standardized procedures exist for regulating the process of calculating the certainty of measurement results. Taking geometric measurement activities in the production engineering environment as an example, the presentation indicates methods of establishing mathematical parameters for statistical assessment purposes as a means of ensuring that results can be reliably compared.

When it comes to evaluating the ability of measuring instruments to permit quality standards to be reliably assessed, measuring uncertainty is an important factor. Moreover, even describing the capabilities of measuring instruments is gaining in importance. In order to consider these predominantly statistical descriptive variables with a sufficient degree of reliability, however a standardized or comparable definition is required.

The first part deals with measuring uncertainties. The second part of the contribution indicates problem areas associated with measuring instrument capabilities, together with possible solutions.

Keywords: management of measurement, measuring uncertainty, qualitative process improvement

1 INTRODUCTION

The company-specific implementation of Zero-Defect-strategies in production engineering requires a quality conforming use of manufacturing facilities and the continuous observation of their accuracy. The measuring equipment must be used accurately and safely furthermore for verification and securing of the performance of technological processes. An ever-higher place value belongs therefore to the quality conforming selection of the measuring devices.

The description of the measuring capability achieves an increasing importance beside the traditional assessment of the qualification of measuring instruments for a safe quality evaluation by the measuring uncertainty. The sufficient safe consideration of these mostly statistical description parameters requires however a standardized or comparable definition. Information to the observance of error limits, which are assured by the measurement equipment producers, are often only of general interest for the industrial user. A sufficient safe calibration guaranteed on the basis of standardized calibration regulations and recommendations the „accuracy“ of measure for a lot of standard measuring equipment. But different recommendations are available for description of „safety“ of measuring results (Figure 1).

The differentiated possibilities for the investigation of measuring uncertainties and capabilities are demonstrated and their results are compared later on. This is of deciding technical and economical significant for a safe process evaluation and a reduction of the probability of false quality evaluations.

2 ASSESSMENT OF “MEASURING INSTRUMENT QUALIFICATION”

Summarized norms, recommendations and guidelines with reference to the practical measurement evaluation and monitoring (Figure 1), describe on one hand the measurement capability and on the other hand the determination of the proportion \(u/T\), whereby \(u=\)measuring uncertainty and \(T=\)tolerance.

With the evaluation by the \(u/T\)-proportion, the more comprehensive measuring uncertainty is referred to whereas the measurement capability sets only the random errors of measurement in the proportion to the tolerance. Both coefficients are able to reflect the qualification of a measurement equipment or measuring method for the sufficient safe allocation of a measuring result to the tolerance.
On this is specified a quantitative range which is mainly determined by random errors of measurement and in which the „true value“ of a measure is expected. The random components of the errors of measurements which are on the turn in a random manner in the case of retestings of the same measure under „imaginary equal“ conditions can be describe by statistical parameters.

The systematic errors of measurement keep constantly in the case of retestings in rate and direction or they vary legitimately and can be utilized for a correction of the measuring value and the measuring result.

Unknown systematic errors of measurement which are either not ascertainable or are not recorded out of economical considerations are estimated. Qualification for the evaluation of the proportion of the measuring uncertainty to the tolerance is the practice-relevant determination of measuring uncertainty. About this either estimations or statistical calculations of measuring uncertainty can be used [1,2,3,5].

2.1 Estimations of measuring uncertainty
A simple possibility for a user orientated determination of the uncertainty of measuring devices respectively of components of measuring uncertainty consist of the estimation of equivalent amounts based on experiences. For improvement of estimation of measuring uncertainty an iterative method is described in [5] which found increasing encouragement lately.

2.2 Statistical description of measuring uncertainty
The recording and description of the measuring uncertainty is recommended by practical analyses and statistical evaluations to reduce the error probability of wrong decisions in the measuring process and for quality evaluation of manufactures.
Possibilities for a statistical description of measuring uncertainty, of their single components and their summarization are demonstrated by [3] in the shape of the method of calculation A and general for the measurement of physical sizes by [1]. By [2] are considered the specialties of the length checking practice.

The determination of the relevant errors of measurement is the starting point of the investigations. At first the recorded systematic errors of measurement from the adjustment, setting or calibration are to eliminate by correction of the measurement values respectively the mean of a series of measurement. The corrected measuring result of a measure takes the general form:

$$ y = x_1 - x_2 , $$

in which: $x_1 = \bar{x}_i$ and $x_2 = -K$.

The complete description of a measuring result demands, beside the correction, the indication of the measuring uncertainty as comprehensive characteristic value for the random and estimated unknown systematic variation components. Figure 2 shows the possible systematization of differentiated recommendations for the complete indication of the measuring result with the aid of the measuring uncertainty.

The statistical determination of random errors of measurement and of their single components takes place on the basis of a recorded series of retestings.
The verification that the series of measurement is free of outliers and that the measurement values are independent, is the requirement for the correctness of the ascertaining characteristics for means and deviations. The Dixon test \([1,2]\) is recommended before other outlier tests. The test from Neumann \([6]\) has approved for the verification of randomness respectively freedom of trend.

The relative low number of measurement values which are common for analysis of measuring uncertainty and are stated for example in \([2]\) with \(n=8...25\), requires often the verification that the measurement values run to a normal or student distribution (t-distribution). For this, the classified-free Kolmogoroff-Smirnoff-test was found to give a particular safe decision \([6]\).

For proved or assumed normal or t-distribution takes place the determination of the arithmetic mean \(\bar{x}\) as an estimation for the expected value \(\mu\) and the empirical standard deviation \(s\) as an estimation for \(\sigma\). In instances the range \(R\) (for \(n \leq 25\)) can be also referred to the approximated determination of the standard deviation. These statistical parameters form the basic for a complete description of measuring results (Figure 2).

The standard deviation \(u(y)\) is accordingly used nowadays as an input for different approaches for indication of a complete measuring result. The standard deviation can be calculated by the square addition of the description parameters of the random errors of measurement \(u(x_i)\) and the component \(u(x_j)\). \(u(x_j)\) results primary from the unknown systematic errors of measurement and secondary from the uncertainty which occurs with the adjustment or calibration of the measuring devices and the additional unknown systematic errors of measurement of the calibration or adjustment equipment.

The correct determination of the component \(u(x_i)\) demands often additional detailed knowledge and experiences about the respective measuring, adjusting or calibration method. Since the impact of systematic errors of measurement are unknown, they are counted as random for a simplified practical handling. Thereby it is possible to sum up the single components quadratic. The uncertainty component \(u(x_i)\) of random errors of measurement of „outlier cleaned up“ series of measurement can be specified as the deviation of the mean for a \(n\)-times direct retesting under seemingly equal measuring conditions:

\[
u(x_i) = s_{x_i} = \frac{s_{x_i}}{\sqrt{n}}. \tag{2}\]

If the uncertainty component must be determined for a single value \((n_i = 1)\) or a low number of retestings (for instance \(n_i = 3\)) the empirical standard deviation is to use from a former series of measurement \(s_o\), with a larger number of retestings. In this case it is valid:

\[
u(x_i) = \frac{s_o}{\sqrt{n_i}}. \tag{3}\]

In the manufacturing measurement technology in mechanical engineering this case is typically because the evaluation of a geometrical measurement is almost always done by a single measurement. A triple retesting and an indication of the measuring result as the mean of this three measurements is also practice-relevant in the case of easy handling measurement equipment.

In such an easy and most economic way it is possible to reduce the random component of the measuring uncertainty by the factor \(\frac{1}{\sqrt{3}}\). On this \(s_o\) must be determined by seemingly equal measuring conditions, for instance by 10 to 25 times retestings.

**Measuring uncertainty by DIN 2257**

The complete measuring results according to DIN 2257 \([2]\) from the measuring value which is corrected by the known systematic errors of measurement and from the indication of the measuring uncertainty \(U(y)\) in the form:

\[Y = y \pm U(y). \tag{4}\]

Hereby addicts \(y\) from a single value \(x_i\) or the mean \(\bar{x}\). With the view that a measuring instrument has been calibrated before the measurement with the aid of a measurement standard the measuring uncertainty can be determined with use of the law of quadratic error propagation correspondingly to Figure 2 in the form:

\[U(y) = \pm \sqrt{V\delta_1^2 + V\delta_2^2 + \delta_1^2 + \delta_2^2}. \tag{5}\]
**Expanded measuring uncertainty by GUM**

The description of the measuring result by GUM (Guide to Expressions of Uncertainty of Measurement) [3] takes place with reference to equation (4), whereby $U(y)$ is definite here as expanded measuring uncertainty in the following form:

$$U(y) = k_2 \cdot u(y).$$

General the use of an expansion factor $k_2$ in the range $k_2 = 2...3$ is recommended by [3]. Often this recommendation is reduced again to the use of $k_2 = 2$ what corresponds to the simplified assumption of a statistical safety of 95% with a sample size of $n \Rightarrow \infty$.

**Measuring uncertainty by DIN 1319-3**

This norm allows several indications of a complete measuring result (Figure 2). One possibility consists in the statement of the confidential range of the mean in the form:

$$Y = y \pm VB_x.$$  \hspace{1cm} (7)

In this case as well as the sample size as well the selected statistical safety are considered. The direct inclusion of the standard uncertainty in connection with the measuring value presents a further possibility:

$$Y = y \pm u(y).$$ \hspace{1cm} (8)

This option concerns indeed to sample sizes of $n \rightarrow \infty$.

**Possibilities to the determination of the measurement capability**

The measurement capability is mostly ascertained in analogy to the evaluation of the quality capability of processes. Definitions to the capability investigations almost ever are arranged company-specific because there is no universal definition for the term of measurement capability [7]. At present three methods are used for the determination of the measurement capability.

**Method 1**

The aim of method 1 is the determination of the measurement specific capability characteristics $c_{gm}$ and $c_{gmk}$ [7,8,9].

These characteristics can be determined as follows:

$$c_{gm} = \frac{0.2 \cdot T}{6 \cdot s_w} \quad \text{or} \quad c_{gm} = \frac{0.2 \cdot (6 \cdot \sigma_{process})}{6 \cdot s_w}$$

$$c_{gmk} = \frac{0.1 \cdot T - |\bar{x}_w - x|}{3 \cdot s_w} \quad \text{or} \quad c_{gmk} = \frac{0.1 \cdot (6 \cdot \sigma_{process}) - |\bar{x}_w - x|}{3 \cdot s_w}.$$ \hspace{1cm} (9) (10)

In this case the measurement-specific standard deviation $s_w$ addicts from at least 25 retestings at one measurement standard or at a largely „known“ measuring object which corresponds to the tolerance mean of the test feature.

$c_{gm}$ and $c_{gmk}$ can be determined either with reference to the tolerance or can be calculated with reference to the process on the condition that the process capability is $c_p \geq 1,33$ and therefore $T = 6 \cdot \sigma_{process}$. The values for $c_{gm}$ permits interference over the capability of the measuring method to provide reproducible measuring results. The value $c_{gmk}$ considers additionally systematic errors of measurements which can be deduced from the difference between the mean of the measurement series of retestings $\bar{x}_w$ and the nominal size of the measurement standard $x_i$. These have the consequence of a calibration or correction. At present usual demands are to be meet for the measuring capability often in the range $c_{gm}, c_{gmk} \geq 1,0$ or 1,33.

This calculation method presupposes that the series of measurement of retestings is free of outliers. If the total measurement range of a measuring device is used it is recommended to the investigation of the linearity over three equally distributed over the measurement range measuring points with the aid of measurement standards.
By [7,8] the fixation of suitable actions for monitoring and for documentation of the time related variations of the random and systematic errors of measurement is proposed because the evaluation by the method 1 is a current evaluation of the state.

Eventual modifications of the errors of measurement are observed at this by measurements under repeatable conditions in definite time intervals over a determined period of time.

The verification of the observance of the border cases of the measuring capability by method 1 is the condition for the realization of method 2 and 3 for the investigation of the total range of deviation of the measuring method.

**Method 2 and 3**

With the equipment and work piece related method 2 for evaluation of the capability of the measurement equipment by [7,8,9] additional influences are included for the evaluation of the measuring process:

- different measurement personal,
- deficiencies of the measuring object (work-piece),
- deficiencies of the set up of the work-piece in the measurement devices,
- differentiated regulations for measurement and interpretation.

At this the determination of the total standard deviation $s_M$ takes place under the conditions of the later use of the measuring method – by way of example by the inclusion of three different operators repeated of 5 to 10 series-produced pieces. The for an evaluation to be considered total standard deviation $s_M$ is to determine from the repeatability $WP$ and the comparison precision $VP$.

The evaluation of the measuring method takes place in connection with the tolerance or $6 \cdot \sigma$ by the following border cases:

$$ s_M / T \text{ or } s_M / 6 \cdot \sigma_{\text{process}} \begin{cases} \leq 0.2 & \text{-acceptable} \\ > 0.2 \ldots \leq 0.3 & \text{-executory acceptable} \\ > 0.3 & \text{-too large} \end{cases} $$

In case of an automated measuring device the influence of different operators is omitted and the evaluation takes place by method 3. On this two series of measurement for 25 serial pieces are taken under the same measuring conditions.

The influence of the measurement object on the total deviation was ascertained recently by the determination of the part variation (PV) too [9]. The summing up of the part variation PV with the total standard deviation $s_M$ results in the total variation $TV$ expanded by the object-related influence which shall enable an expanded evaluation.

This deviation analysis and evaluation which includes manifold components of influence in general does not permit a separate measurement-related interpretation. This is to perform before by method 1 and can be referred to a comparison with the proportion $u/T$.

The investigations by method 2 can be used against it for the analysis of the influence related to the production process and can point out approaches for the optimization of the process.

**Are $u/T$ and measurement capability comparable?**

Nowadays for a evaluation of the qualification of a measuring device or a measuring method are the following requirements practice-relevant:

$$ u/T = 0,1 \ldots 0,2 \text{ and } c_{gm} = 1.0 \ldots 1.33 $$

Due to the mathematical conditions $c_{gm}$ can be compared directly only with the parameter $u/T$ or the reciprocal value $T/u$. For the tendency display of the hereby arising connections according to figure 3 it is necessary to set $u = 3 \cdot s_w$ to make the simplifying adoption that $n \rightarrow \infty$ and $P = 99.97\%$.

Related to the respective deviation from that it takes place for instance for a assumed border case of $u/T = 0,1$ and $c_{gm} = 1.0$ the following relation corresponding to the equations (9) and (10):

$$ s_u = s_w. $$

The retention of the $u/T$-proportion is to get much more difficult in a normal case, but it is more related to the real practical conditions, because $s_w$ describes only the repeatable standard deviation and $u$ includes all random and unascertainable errors of measurement. That means that for instance the number of retestings which influences directly the confidential range of the measuring result is unconsidered for the determination of the capability of measurement $(n = 25 \ldots 50)$. 
Simplified there is a statistical safety of 99,97 % for $n \to \infty$ adopted. For the adoption of a normal distribution the confidential range results to $\pm \lambda \cdot s$ with $\lambda = 3$ as 6s.

The determination of the measuring uncertainty takes place in contrast to that with reference to the $t$-distribution which enables a safe adoption in the case of a low number of retestings ($n \leq 25$). Indeed the confidential range open out against the confidential ranges of the capability of measurement on it for instance for $n = 25$ by 15 % and for $n = 50$ by 9 %.

Also in practice to find repeated measurements of the same workpiece $n_i$ and the direct calculation of the mean for the calculation of the measurement value do not find a consideration by the model of the capability of the measurement. This possibility also known as “$n$-law” for the reduction of the action of random errors of measurement and for the reduction of the confidence interval too figures particularly for the quantization of the influence of the measurement equipment.

The determination of the capability of measurement by method 1 is furthermore limited to the registration of the random errors of measurement and does not include the unknown, estimated systematic deviations of the measurement and the influences of uncertainty of the calibration.

3 EFFECTS OF THE MEASURING UNCERTAINTY

The measuring uncertainty influences the in Figure 4 shown areas [6]. The use of knowledge over the technical especially the error-geometrical performance of measurement devices and methods is limited in the practice often to the allocation between a measuring instrument and the geometrical quality characteristics. The draft of DIN EN ISO 14253-1 [4] evaluated in detail the limitations of a safety quality classification of work pieces whose actual sizes are located in closeness of the tolerance limit (Figure 5).

In the case of a quantitative evaluation of processes for the improvement of the safeness of decision it is recommended in [4] a reduction of the measuring uncertainty by „suitable actions“ or „according to contract arrangement“. The first problem solving possibility results in new approaches for optimization between technical and economical aspects. A second possibility is given by such a way, that the production of work-pieces is done within restricted, so-called manufacturing tolerances.

The difference value between the pretended work-piece tolerances by the designing engineer and the manufacturing tolerances will be determined basically by the expected measuring uncertainty or the uncertainty of measuring method. An unconditional qualification for an efficient result consists of, that the fixing of tolerances are effected by functional as well as economical aspects and that there are no fixed so-called „angst tolerances“.

In addition to linear subtraction of the measuring uncertainty from the work-piece tolerance, different possibilities of a square subtraction have been developed by Berndt [10] in the fifties.

Those were defines later by Leinweber [10] under consideration of normal distributed process models in form of the „Golden rule of the measurement technology“ with $u/T = 0,1...0,2$ so that the unsteady range of decision of quality classification is ignorably small under determined requirements.

Modern production structures require at the same time error rates in ppm-ranges (parts per million) which require against new basic approaches for the technically measured and statistical evidences of their observance.
Figure 4. Effects from the measuring uncertainty on selected fields in production engineering

Figure 5. Influence of the measuring uncertainty on the safety of decision for quality evaluation of products by [4]
In this case it is only possible to get sufficient certainties of decisions for the quality classification of products or assemblies by utilization of statistical superposition. Thereby the investigation of the untruly accepted and untruly rejected products is also possible on the basis of a yet to be defined statistical process model for fixed $u/T$-proportions and demanded permissible error rates.

The effect of the measuring uncertainty is also of significance for the use of quality control charts for monitoring, and control of the behavior of position and deviation of variable inspection test features. At this point it influences the effectiveness of quality control charts and has the consequence of false or omitted actions in the production process. Statements at the grade of influences for shifting of the process position or the increase of process deviation are possibly by the determination of the expected action probability.

The objects as well as for the definition and the calculation of the measuring uncertainty, the evaluation of their influence to the qualitative process evaluation and control or possible limitations of tolerances, may be not handled as single results. They are to handle always as a technical and economical method of optimization. The technically measured expenditures for measures of the reduction of the measuring uncertainty are set against to the costs for rework and scrap respectively the costs in consequence of errors in the case of delivery of faulty products.

The technical performance limits of the manufacturing facilities and the measuring methods must be considered hereby.

4 EXPERIENCES ABOUT THE DETERMINATION OF THE MEASUREMENT UNCERTAINTY

Within closer co-operation with different industrial concerns have been investigated different groups of complex measuring equipment for the geometry inspection in mechanical production areas. Specialties of the practical determination of the measurement uncertainty will be presented by selected examples of the car manufacture and the possibilities of simplified practice-relevant determination of the measurement uncertainty will be deduced.

Abbreviations and symbols

- $a, b$: Amplitudes of a unsymmetrical rectangle distribution; ($b = a$ in case of a symmetrical rectangle distribution)
- $C_{GM}, \bar{C}_{GM}$: Indices of capability of measurement
- $f_1, f_2$: non-recorded systematic errors of measurement
- $k$: Factor for indication of confidence range of a single value
- $k_2$: Extension factor by [4]
- $K$: Correction value
- $MA$: Errors of measurement
- $n$: Number of measurements/ Retestings
- $n_i$: Number of retestings for calculation of the measurement value $x_i$
- $P$: Statistical safety
- $R$: Range
- $s$: Standard deviation, general
- $s_m$: Total standard deviation
- $s_0$: Standard deviation from a former series of measurements
- $s_u$: Standard uncertainty, general
- $s_w$: Standard deviation of retestings
- $s_{si}$: Standard deviation from i single measurements $x$
- $s_{\bar{x}_i}$: Standard deviation of the mean $\bar{x}_i$
- $t$: Quantile of the $t$-distribution
- $T$: Tolerance
- $u$: Measuring uncertainty, general
- $u(x_1)$: Standard uncertainty of random errors of measurement
- $u(x_2)$: equivalent standard deviation to the unknown systematic errors of measurement
- $u(y_i)$: Standard uncertainty
- $U(y)$: Measuring uncertainty, extended measuring uncertainty
- $VB_1$: Confidence range of the measure
- $VB_2$: Confidence range of the calibration
- $VB_{\bar{x}_i}$: Confidence range of the mean
- $VB_{x_i}$: Confidence range of the single value
\(x_i\)  
\(- i\) times single value

\(x_1\)  
Equivalent value to the mean \(\bar{x}_i\) of a series of measurement

\(x_2\)  
Equivalent value to the correction value \(-K\)

\(x_r\)  
Nominal value of the adjusting standard

\(\bar{x}\)  
arithmetic mean of the sample

\(\bar{x}_i\)  
arithmetic mean of a sample from \(i\) single values

\(\bar{x}_w\)  
arithmetic mean of retestings

\(y\)  
Measuring result; Estimate for \(Y,Y^*\)

\(Y,Y^*\)  
complete measurement results

\(\mu\)  
Estimate for the mean of population

\(\lambda\)  
Quantile of the normal distribution

\(\sigma\)  
Estimate for the distribution of the population

\(\sigma_{\text{process}}\)  
Estimate for the deviation of the manufacturing process

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