CALIBRATION OF A SYSTEM FOR THE MEASUREMENT OF COMPLEX VOLTAGE RATIOS

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Abstract: A calibration method for a system designed to measure complex voltage ratios is presented. The method is based on the application of an AC transformer bridge. The system comprises a multichannel data acquisition board with simultaneous sampling, input adaptation circuits and dedicated software. An analysis of the accuracy of the proposed calibration method is made. Experimental tests of the calibration system were performed for signals differing in phase by $\pi/2$ and $\pi$ at frequencies from 50Hz to 1kHz.

Keywords: calibration, complex voltage ratios, bridge circuit.

1 INTRODUCTION
Evaluation of metrological parameters of many AC measurement circuits requires knowledge of complex voltage ratios. A ratio of two AC voltages is measured to determine e.g. a ratio of precise inductive voltage dividers and measurement transformers, as well as processing characteristics of instrumentation amplifiers and current-voltage transducers. This measurement approach is applied in comparison (AC bridge) and compensation methods.

The availability of high performance data acquisition cards (DAQ) and appropriate software makes it possible to implement modern digital signal processing (DSP) methods in measurements of such complex voltage ratios. Theoretically, increasing sampling frequencies and resolutions of common commercial measuring cards allow for an increased measurement accuracy of amplitude ratios and phase differences between two input voltages due to application of DSP methods in an extended frequency range [1,2,3].

Exact estimation of the error of complex voltage ratio measurement realised using relatively complicated DSP algorithms is difficult and the final evaluation of system applicability requires a practical verification. This is because quite often-measuring cards need to be extended with additional circuits that increase their transformation range and input impedance. Consequently, it is necessary to calibrate such systems.

2 CALIBRATION PROCEDURE
In order to determine errors of complex voltage ratios when measuring voltages differing in phase by $\pi$, a suitable transformer bridge was used in [4,5]. A simplified block diagram of the corresponding measurement circuit is shown in Fig.1.

![Figure 1](image-url)  

**Figure 1.** System for the measurement of complex voltage ratios (the signals differ in phase by $\pi$).
In the circuit supplied by voltage $U$ two impedances, $Z_1$ and $Z_2$, of the same type (e.g. resistors or capacitors) are compared. The voltage drops on the compared impedances constitute at the same time input signals to the calibrated system. The bridge is initially equalised by appropriately selecting the transformation ratio, $k$, of an inductive voltage divider (IVD). The remaining unbalanced voltage $U_b$ of the bridge is measured using a phase-sensitive detector. The values of the IVD transformation ratio, $k$, and unbalanced voltage components make it possible to determine the complex ratio of the compared voltages with a high precision. The relation between the quantities mentioned above is given by

$$\frac{U_2}{U_1} = k \left( \frac{1 + \frac{1}{k} \frac{U_b}{U}}{1 - \frac{U_b}{U}} \right)$$

(1)

It is convenient to express the remaining unbalanced voltage with reference to the supplying voltage of the bridge. The relative unbalanced voltage $U_b$ is defined as follows

$$U_b = \frac{U_b}{U} = \frac{U_{bp} + jU_{bq}}{U} = U_{bp} + jU_{bq}$$

(2)

where $U_{bp}$ and $U_{bq}$ represent properly in-phase and quadrature components of the unbalanced voltage, respectively. When the bridge is nearly balanced, (1) becomes

$$\frac{U_2}{U_1} \approx k \left[ 1 + U_b \left( 1 + \frac{1}{k} \right) \right]$$

(3)

The ratio $K$ of two ac voltages can be express by a nominal value of their ratio $K_n$, as well as amplitude $\alpha$ and phase $\beta$ errors

$$K = \frac{U_2}{U_1} = K_n \left( 1 + \alpha + j\beta \right)$$

(4)

Combining (3) with (4) and assuming that for a virtually balanced bridge the condition $k = K_n$ is met, we obtain

$$\alpha = U_{bp} \left( 1 + \frac{1}{k} \right), \quad \beta = U_{bq} \left( 1 + \frac{1}{k} \right),$$

(5)

which can then be used to calculate the amplitude and phase errors of the two-voltage ratio.

In the above considerations errors in the IVD ratio have been assumed negligible. If this assumption is not valid, when comparing impedances of relatively small values, then we should replace (5) with

$$\alpha = \alpha_{IVD} + U_{bp} \left( 1 + \frac{1}{k_n} \right), \quad \beta = \beta_{IVD} + U_{bq} \left( 1 + \frac{1}{k_n} \right)$$

(6)

where $k_n$ means the nominal ratio of the IVD and $\alpha_{IVD}$ and $\beta_{IVD}$ are respectively amplitude and phase errors of the IVD ratio.

To determine the errors when measuring voltages which differ in phase by $\pi/2$, a digital quadrature bridge was used and a permutation technique described in [6] was applied. The bridge of Fig. 2 is composed of two voltage sources with a high resolution of amplitude and phase adjustment, two binary inductive voltage dividers BIVD$_1$, BIVD$_2$, phase-sensitive detector $D$, and compared components: a resistor and a capacitor. Two measurements are required to determine the complex
voltage ratio \( U_1 / U_2 \). During the first measurement executed in the setting of Fig. 2, a preliminary balance is accomplished via BIVD1 and BIVD2 dividers. Hence we have two values \( C \) and \( D \). Figure 2. System for the measurement of complex voltage ratios (the signals differ in phase by \( \pi / 2 \)).

of the IVD ratio \( a_1, b_1 \) and an unbalanced voltage \( U_{b1} \). The second measurement is performed for interchanged supplying voltages, which results in values \( a_2, b_2 \) and \( U_{b2} \), respectively. From (2), neglecting a short-time instability of the voltage sources and compared components, we obtain

\[
\alpha = \frac{1}{2} a_1 b_1 \left( a_2 U_{bq2} - a_2 U_{bq1} + b_2 U_{bs1} - b_2 U_{bs2} \right) \\
\beta = \frac{1}{2} a_1 b_1 \left( a_2 U_{bs1} - a_2 U_{bs2} + b_2 U_{bq1} - b_2 U_{bq2} \right)
\]

3 TECHNICAL REALISATION OF THE CALIBRATION SYSTEM AND EXPERIMENTAL RESULTS

The main components of the calibration system shown in Figs.1 and 2 are two 20-bit IVDs, a two-phase voltage generator and a phase-sensitive detector, which are connected to a PC. This permits to easily process measurement results and to efficiently use software calibration procedures.

The calibration system was used to compare precise commercial resistors (VISHAY) and gas dielectric capacitors. Special attention was paid to constructing a buffer stage. In the circuit of Fig.1 the buffer is provide the signals from floating sources to the tested DAQ card. A very good symmetry of both the buffer stage channels is necessary for correct work of the measuring system. The proposed system is designed for measurements in a frequency range of 50Hz to 1kHz. The calibration process is performed for signals differing in phase by \( \pi \) and \( \pi / 2 \) with the amplitude ratio ranging from 0.1 to 1.

As a tested card, the National Instruments DAQ card, AT 2150 was used. This card and the applied software (LabWindows/CVI) makes it possible, as a result of applying the FFT method, to calculate amplitude ratio and phase difference of compared signals. Examples of results are presented in Table1.

Correct determination of the phase difference between the voltages obtained from the compared impedances in the bridge circuit requires knowledge of IVD phase errors. Taking account of the IVD phase errors is of paramount importance during measurements made at higher frequencies as well as while comparing resistors or capacitors of considerably differing nominal values. Choosing nominal values of the compared components, we have on one hand to limit them from above, due to a potential influence of the shunting impedance of the DAQ input stage. On the other hand too small values of the impedances decrease the bridge accuracy. For example, while comparing impedances of a ratio of about 10:1 additional comparatively large errors in the IVD ratio occur owing to connection of measured impedance to the IVD output (IVD ratio error). Moreover, an asymmetry of indirect stage channels must be taken into account. For these reasons, the calibration process must be preceded by several measurements that finally permit to minimize the influences of all those disadvantages.
Table 1. Phase difference error (expressed in µrad) for signals differing in phase by a nominal value $\pi$ and measured through the AC bridge and DAQ card.

<table>
<thead>
<tr>
<th>$f$ [Hz]</th>
<th>ratio 1:1</th>
<th>ratio 10:1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DAQ Bridge</td>
<td>DAQ Bridge</td>
</tr>
<tr>
<td></td>
<td>phase error uncertainty</td>
<td>phase error uncertainty</td>
</tr>
<tr>
<td>200</td>
<td>94 95 $\pm$ 18</td>
<td>384 392 $\pm$ 18</td>
</tr>
<tr>
<td>400</td>
<td>135 111 $\pm$ 20</td>
<td>716 904 $\pm$ 63</td>
</tr>
<tr>
<td>1000</td>
<td>354 328 $\pm$ 25</td>
<td>1570 2040 $\pm$ 80</td>
</tr>
</tbody>
</table>

The uncertainty about the phase error in the voltage ratio of Table 1 results form the uncertainties about the phase errors in the IVD and buffer stage, as well as that about the out-of-balance voltage of the bridge.

The results shown in Table 1 were obtained for the DAQ card and for signals of efficient values 2V/2V and 2V/0.2V. For signals considerably different amplitudes (2V/0.2V) larger phase errors were observed.

An error in the amplitude difference of the compared signals was also determined. The deviation from the results obtained by means of a DAQ card and a bridge did not exceed 50ppm (a ratio of 10:1) while the estimated uncertainty of the measurement with bridge was on level $\pm$10ppm.

Results of preliminary measurements of the voltage ratios for signals which differ in phase by $\pi/2$ confirm the possibility of calibrating systems for the measurement of complex voltage ratios also in circuits with two generators (Fig. 2).

4 CONCLUSION

In the paper the calibration procedure based on an AC bridge for the measurement of complex voltage ratios based on a DAQ card and digital signal processing has been presented. The conducted experiments show that good-quality commercial DAQ cards equipped with suitable software allow for measurements of phase errors with uncertainty of the order of tens µrad (0.001deg), which is satisfactory result in many applications. An increase in the precision of phase error measurements with DSP methods will demand use of essentially better hardware e.g. systems based on high-resolution sampling voltimeters [1].

REFERENCES


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