TOLERANCE-FIT OF GEOMETRIC ELEMENTS AND SCULPTURED SURFACES AND PROFILES

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Abstract: For quality control in the case of workpieces measured by CMM both a sufficient number and location of measuring points and the correct evaluation for determining the minimum deviations of size, position and form for the entire workpiece guaranties economic manufacturing. The idea to fulfil this general objective function opens a new set of tasks besides of the well known data fitting for individual features. The paper deals with new solutions for data fitting of compound features with partly different tolerances oriented on the minimum zone objective function.
Keywords: co-ordinate metrology, CMM workpiece geometry, data fitting

1 INTRODUCTION

The co-ordinate measuring technique has grown up to an important tool for geometric quality control in automotive and machine building industry, in plastics forming etc. The growing functionality of the mechanical parts and higher requirements to the accuracy by smaller tolerances are leading to measuring and evaluation strategies for determining geometrical deviations and out of tolerances of geometric workpiece features without any errors. The certain determination of size, form and positional parameters out of tolerance requires co-ordinate measuring machines (CMM) with uncertainty less than 20% of the tolerance to be checked, measuring strategies for a sufficient number and optimised position of measuring points as well as powerful software and evaluation routines according to the given rules.

The quality decision for checked workpieces will be wrong if the evaluation routines are insufficient or wrong as following:

- If form deviations of any geometric element are not calculated according to the minimum zone principle as defined in standard ISO 1101.
- If the dependence between form deviation, size and positional deviation as well as the influence of workpiece misalignment are neglected and consequently the positional deviations are calculated in too large;
- If the co-ordinate system optimisation for profile testing does not consider the different tolerances of the individual profile parts the evaluation will not show the minimum of form deviation.

Thus it follows that for quality control also in the case of workpieces measured by CMM with sufficient number and location of measuring points only the correct evaluation for determining the minimum deviations of size, position and form for the entire workpiece guaranties economic manufacturing. The idea to fulfil this general objective function opens a new set of tasks besides of the well known data fitting for individual features. The paper deals with new solutions for data fitting of compound features with partly different tolerances oriented on the minimum zone objective function.

2 DATA FITTING ACCORDING TO THE MINIMUM ZONE PRINCIPLE

The common way for parameterisation of geometric elements in co-ordinate measuring technique are based on data fitting algorithms according to the least squares principle (Gaussian). Geometric elements hereby are usually single features such as straight line, plane, cylinder, cone, sphere etc., compound features such as polygons consisting of a set of straight lines, multi-cylinders, profiles of connected arcs, free form profiles or sculptured surfaces as well as patterns of features such as bores of flanges etc.. The parameterisation of geometric elements by data fitting according to least squares method as well as minimum zone principle has to satisfy the important condition of **o**rthogonal distance **r**egression (ODR). Thus the residua to be minimised have to be calculated perpendicular to the best fitting feature [1]. Especially for least squares parameterisation a lot of solutions and algorithms are well known [13, 17].

Besides of the least squares solutions the data evaluation by best-fit algorithms according to the minimum zone principle (Chebyshev) as so called MiniMax problems are becoming more and more important. One of the fundamental tasks is the calculation of form deviation of geometric elements according to ISO 1101 [2]. The parameters of the geometric element are to be determined in such a way that the orthogonal form deviation becomes a minimum. There exist only a few publications and solutions for ODR MiniMax solutions until now [1, 14, 15, 18] and the solutions and algorithms differ reasonably between the individual geometric elements. This is also the reason why the minimum zone approximation in CMM is restricted only on simple geometric elements and not very common in CMM.
But there are also very serious accuracy problems of minimum zone fitting because of its sensitivity for data point errors. If the data set to be fit contains random errors (usually in case of scanning points) or any outliers these wrong points will determine the final solution. Thus the data point validation is a very important step in advance of the the minimum zone fitting procedure. The data validation should comprise two steps as following:

- Elimination of outliers by means of common methods. If a large number of points have been measured a few points may be deleted. But there have to be used algorithms which do not delete valid points describing local form deviation.
- Low pass filtering in order to decrease local random errors. The filtering procedure must also work as an ODR filter with cut off length according to the properties of the random errors as they are common in the case of CMM with scanning control.

The most serious disadvantage of common minimum zone algorithms is the fact that they are working with constant zone width for the whole data set or geometric element. As described above the evaluation of measuring data of real workpieces require algorithms for different zone width defined by the individual tolerances for compound features.

The figure 1 shows this problem in a simple example for profile testing of an edge consisting of two straight lines. The comparison of the measured points with the given tolerance zones often leads to a situation described in fig. 1a: While the tolerance zone of the horizontal section of the profile is not fully used in the other section there are points out of tolerance. The consequence is a wrong decision about the product quality.

The tolerance fit as the minimax solution with defined zone width avoids this problem (fig. 1b). The tolerances are considered by the bestfit algorithm and the position of the measured points with respect to the nominal profile are optimized under consideration of the distances f_i and their tolerances.

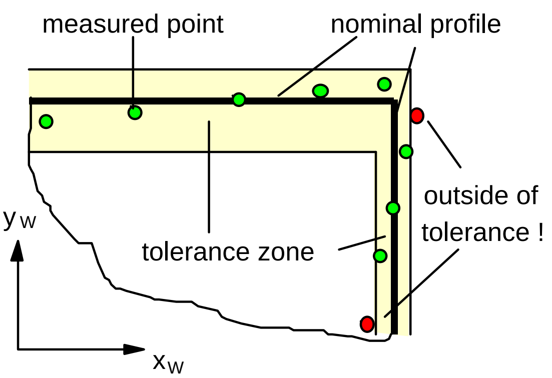

Figure 1a. Measured profile and nominal profile

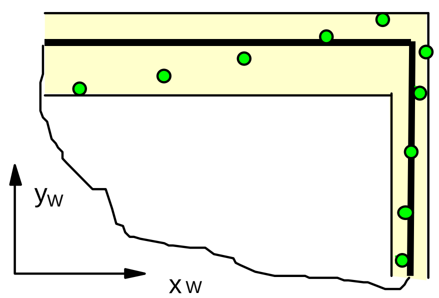

Figure 1b. Profile with tolerance fit

3 ALGORITHMS AND MATHEMATICAL SOLUTION FOR TOLERANCE FIT

3.1 Principle of tolerance fit solution for geometric elements

For parameterisation of geometric elements from measuring data according to the tolerance fit principle problem a number of methods and algorithms are available. The following methods are especially oriented on this task.

1. The weighed least squares method [9] with the objective function

$$\sum\sum\frac{f_{i,j}^2}{T_j^2} \Rightarrow Min$$

with T_j Tolerance of the feature j
f_{ij} deviation of point i of feature j

2. The fixed tolerance fit solution [12] according to the objective function
$$Max\left(f_{ij} - T_j\right) \Rightarrow Min$$
In this case the measuring points are fitted inside of the defined tolerance zones.

3. The proportional tolerance fit [10] according to the objective function
$$\underset{i=1...n}{Max}\left(\frac{T_0}{T_j}\left|f_{i,j}\right|\right) \Rightarrow Min$$

The first method is well known, very easy to handle and also very robust [9]. But it is only a good approximation of the tolerance fit problem. The latter ones have been developed by the authors and they are part of powerful CMM software packages.

3.2 The proportional tolerance fit method

The basic step for data fitting and parameterisation of geometric elements is to find the analytic representation by an equation suited for ODR. A simple representation is the implicit equation (1)
$$F(x,y,z;a_1,a_2,\ldots,a_m) = 0 \tag{1}$$
with x, y, z spatial co-ordinates
$a_1, .. \, a_m$ parameters of the geometric element
$\left|grad(F)\right| = 1$ scaling condition.

Real measuring points (x_i, y_i, z_i) do not satisfy this equation and a residuum f is left
$$f_i = F(x_i, y_i, z_i;\ a_1, a_2, \ldots, a_m) \neq 0 \tag{2}$$
The objective function for data fitting according to the common minimum zone principle is defined as following
$$Q = \underset{i\,=\,1...n}{Max}\left(\left|f_i\right|\right) \quad \Rightarrow \quad Min \tag{3}$$
In order to consider different tolerance zone width of the simple or compound geometric element the residuum must be weighted by means of a weighing factor as tolerance T_i related to a mean or standard tolerance T_0 and it follows
$$Q = \underset{i=1...n}{Max}\left(\frac{T_0}{T_i}\left|f_i\right|\right) \Rightarrow Min \tag{4}$$
The non-linear equation (4) may be solved on the common way by linearisation and optimisation of the unknown parameters a_1 .. a_m.
$$\begin{aligned} F(x_i,y_i,z_i;\ a_1,a_2,\ldots,a_m) &= F(x_i,y_i,z_i;\ \tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_m) + \frac{\partial F}{\partial a_1}\Delta a_1 + \cdots + \frac{\partial F}{\partial a_m}\Delta a_m \\ &= f_i + \mathbf{J}_i\,\Delta\mathbf{a} \end{aligned} \tag{5}$$
with $\mathbf{J}$ as the Jacobian matrix of the feature equation (1) for estimated parameters $\tilde{a}_1$ .. $\tilde{a}_m$.

If we assume the variable Φ as half of the actual zone width it follows constraints from the corrected residuum for each measuring point P_i (i=1 .. n)
$$\left|f_i + \mathbf{J}_i\,\Delta\mathbf{a}\right| \le \Phi \tag{6}$$
or in explicit form for the upper and lower tolerance limit the following constraints
$$\begin{aligned} f_i + \mathbf{J}_i\,\Delta\mathbf{a} &\le \frac{T_i}{T_0}\Phi \\ -f_i - \mathbf{J}_i\,\Delta\mathbf{a} &\le \frac{T_i}{T_0}\Phi \end{aligned} \tag{7}$$
For the whole set of measuring points it follows finally a linear system of constraints

It must be solved for all measuring points P_i (i=1 .. n) simultaneously by linear programming with the objective function $\Phi \Rightarrow Min$ for the following linear system perhaps as Simplex table for solving by the well known Simplex method for variables with unconstrained sign
$$\begin{pmatrix} \mathbf{A} & \mathbf{t} \\ -\mathbf{A} & \mathbf{t} \end{pmatrix}\begin{pmatrix} \Delta\mathbf{a} \\ \Phi \end{pmatrix} \ge \begin{pmatrix} \mathbf{f} \\ -\mathbf{f} \end{pmatrix} \tag{8}$$
with

A	Jacobien Matrix for all points (*n* equations),
t	normalised tolerance vector
f	vector of the residua

$$\mathbf{A} = \begin{pmatrix} \mathbf{J}_1 \\ \vdots \\ \mathbf{J}_n \end{pmatrix} \qquad \mathbf{f} = \begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix} \qquad \mathbf{t} = \begin{pmatrix} \frac{T_1}{T_0} \\ \vdots \\ \frac{T_n}{T_0} \end{pmatrix}$$

The solution of the linear system by gives the parameter correction vector $\Delta\mathbf{a}$

$$\Delta\mathbf{a} = \left(\Delta a_1 \quad \Delta a_2 \quad \cdots \quad \Delta a_m\right)^{\mathrm{T}} .$$

In the case of a non-linear fitting problem the parameters have to be calculated iteratively

$$\mathbf{a}_{k+1} = \mathbf{a}_k + \Delta\mathbf{a}_k \tag{9}$$

until the weighed corrections a small compared with a given limit ε (e. g. 0.1 µm).

$$\sum_{j=1}^{m} g_j \left|Äa_{k,j}\right| \le e \tag{10}$$

The weighing facors g_i must be chosen in such a way that all weighed part are of the same dimension. The solution for the tolerance fitting factor Φ shows whether the points are inside of the defined tolerance zones.

$$\frac{2\Phi}{T_o} = \begin{cases} \le 1 & \text{all points in tolerance} \\ > 1 & \text{some points out of tolerance} \end{cases} \tag{11}$$

As result of the tolerance fit solution as described above the measuring points of the individual parts of the geometric element lie inside of the tolerance range T_i. Thus the solution does work like a geometric element with rubber tolerances witch do trim down proportional until the reduced limits touches the least possible number of points (usually as the number of parameters).

3.3 The fixed tolerance fit method

The principle of this solution shall be described for a free form profile with the following initial situation:

- The nominal surface / profile is presented by (CAD-) points. Between the points it is to be interpolated by planes (surface) or lines (profile). The nominal surface is divided into patches and for every patch *j* is given a symmetrical tolerance T_j.
- The actual surface / profile is presented by measured points.
- The nominal surface and the measured points are given in one workpiece coordinate system defined by datum elements. If it is not so an initial transformation must be done, see [12].
- It is evaluated the foot point of every measured point with respect to the nominal surface.

Now the tolerance fit leads to the evalution of a coordinate transformation of the measured points with respect to the nominal surface in such a way that after this transformation all points lie inside the tolerances or (if this cannot be realised) the distance of outliers is minimised. This method uses the orthogonal distance between every point and its foot point shown in figure 2.

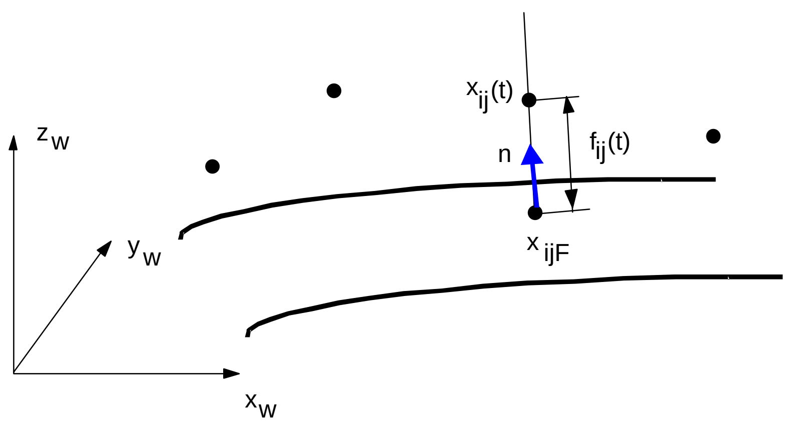

Figure 2. The orthogonal distance a between transformed measured point and its foot point.

In fig. 2 mean:

x_W, y_W, z_W: axis of the workpiece coordinate system

$\mathbf{x}_{ij}(\mathbf{t})$ coordinates of the measured point i at the patch j of the surface after the transformation

$$\mathbf{t} = (t_x, t_y, t_z, a_x, a_y, a_z)^{\mathsf{T}} \tag{12}$$

with t_x, t_y, t_z as translation parameters and a_x, a_y, a_z as rotation parameters.

$\mathbf{x}_{ijF}$ coordinates of the foot point of the measured point i on patch j before / after the transformation,

$\mathbf{n}$ normal vector.

Then the distances $f_{ij}(\mathbf{t})$ which will be minimized can be described by

$$f_{ij}(\mathbf{t}) = \left\| \mathbf{x}_{ij}(\mathbf{t}) - \mathbf{x}_{ijF} \right\| \tag{13}$$

The consideration of the tolerances T_j leads to the minimax problem:

$$\max_{\substack{i=1,\dots,n \\ j=1,\dots,m}} (f_{ij}(\mathbf{t}) - T_j) \rightarrow Min! \tag{14}$$

which is equivalent to:

$$\begin{aligned} & d \rightarrow Min! \\ & f_{ij}(\mathbf{t}) - T_j - d \leq 0 \end{aligned} \tag{15}$$

This problem (15) is a problem of nonlinear programming. It can be solved iteratively [5,7]. The parameter d of the solution includes the information about the whole surface/profile:

$$d = \begin{cases} \leq 0 & \text{all points in tolerance} \\ > 0 & \text{some points out of tolerance} \end{cases}$$

4. APPLICATION OF TOLERANCE FIT FOR TESTING OF BORE PATTERNS AND PROFILES

4.1 Fit of bore pattern as an example of a 2D ODR-Problem

Testing of positional tolerances of bore pattern is one of quality testing tasks to be solved by the tolerance fit method linked to co-ordinate transformation in order to fit the positional deviations into the defined tolerance areas for the feature positions.

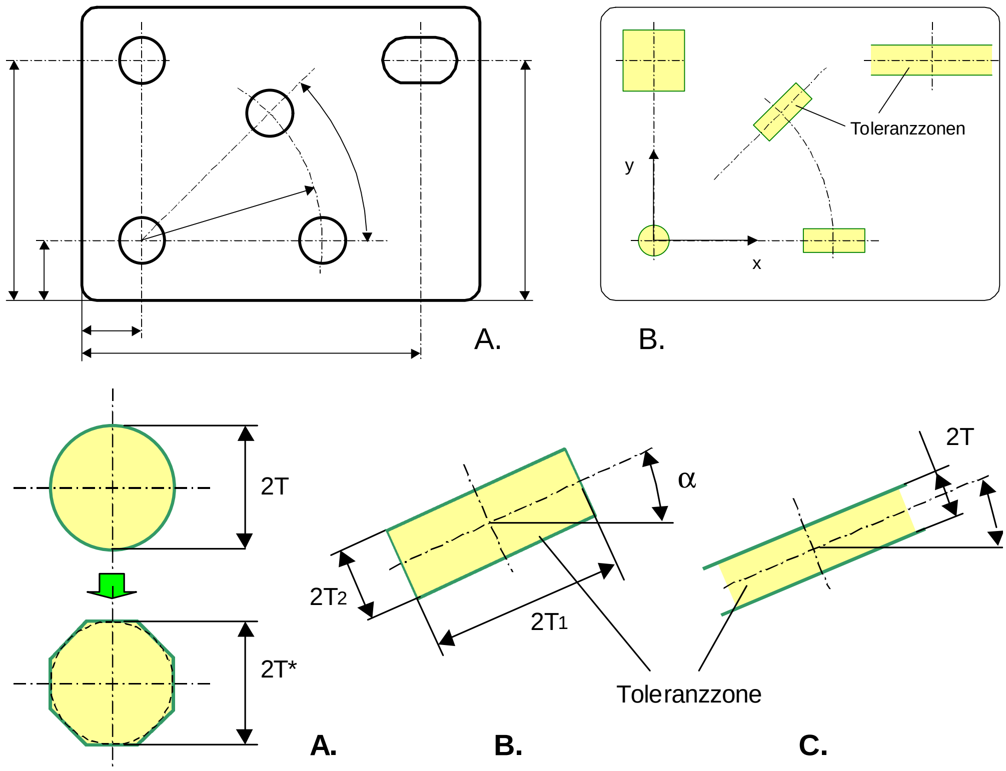

Figure 3. Fit of bore pattern with different geometric elements

Fig. 3 shows an example for bore pattern with different geometric elements and different positional tolerances. The individual features are measured by means of CMM and calculated by common best fit routines (least squares or minimum zone). The problem is to check the feature position in comparison with the defined tolerance areas by the tolerance fit method. Therefor the translation and rotation of the workpiece co-ordinate system is to be optimised.

The task is to fit points (feature midpoints) into the defined tolerance areas as there are the following types.

- Circular tolerance areas for positional tolerances according to ISO 1101;
- Rectangular tolerance areas with defined angular orientation
- Slot tolerances as one-dimensional tolerance with defined orientation.

In order to get linear constraints for the tolerance areas even for the circular tolerance field it must be substituted by a regular polygon perhaps with 8 or 12 sides as shown.

For the tolerance fit as described above each straight boundary line gives a constraint for the linear system for optimisation of the transformation parameters. Depending of the number and types of tolerance fields the total number of constraints is (in case of an 8-sided polygon for circular tolerance fields)

$$N = 2\,n_s + 4\,n_r + 8\,n_k \tag{16}$$

with n_s, n_r, n_k as the number of slot, rectangular and circular tolerance fields. The linear system can be solved by the common Simplex method for sign-unconstrained variables and gives the solution for the optimised transformation parameters as well as for the tolerance fitting factor Φ. The following fig. 4 shows the result of the tolerance fit problem for a bore pattern of a flange. The tolerance areas in nominal position, the optimised real position of the features as well as the reduced deviation areas used proportional to the given tolerance fields are shown.

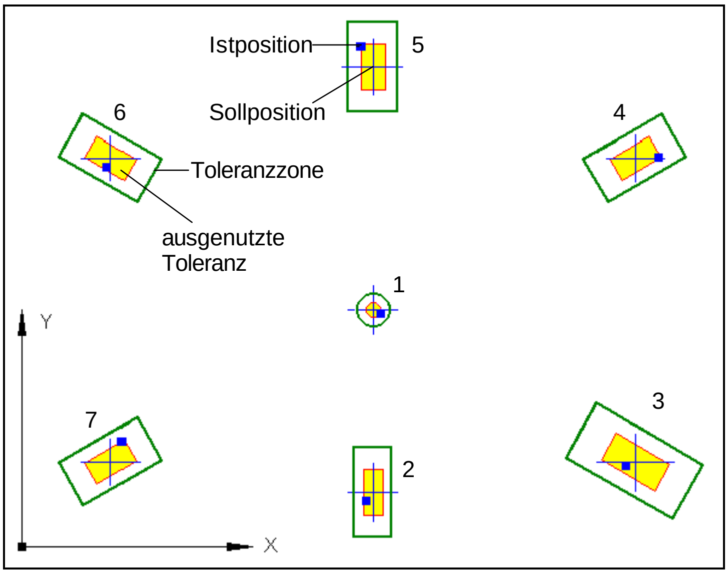

Figure 4. Bestfit of bore pattern of a flange

4.2 Tolerance fit of a profile

The fixed tolerance fit method is an integrated part of the software WinWerth of Werth Messtechnik Gießen [20]. It is demonstrated in fig 5 to check the profile of a washer part.

Fig. 5a shows the nominal profile with the tolerance requirements. The function of the parts leads to different tolerances of profile sections.

The actual part was scanned in any position with an optical CMM and fig. 5b contains the coordinates of the scanned points (more than 4.000 points).

The comparision of the actual points an the nominal profile is demonstrated in fig 5c. It is easily to see that a lot of profile sections are outside of the required tolerances, especially at the profile section with the higher tolerance tolerance requirements.

The situation after using the tolerance fit method gives fig 5d. The outliers are minimized to some small sections. Left down one can see the resulting added co-ordinate transformation with respect to the nominal actual comparision only (rotation a_z=3,0898°, transformations t_x=0.046 mm and t_y=-0.034 mm).

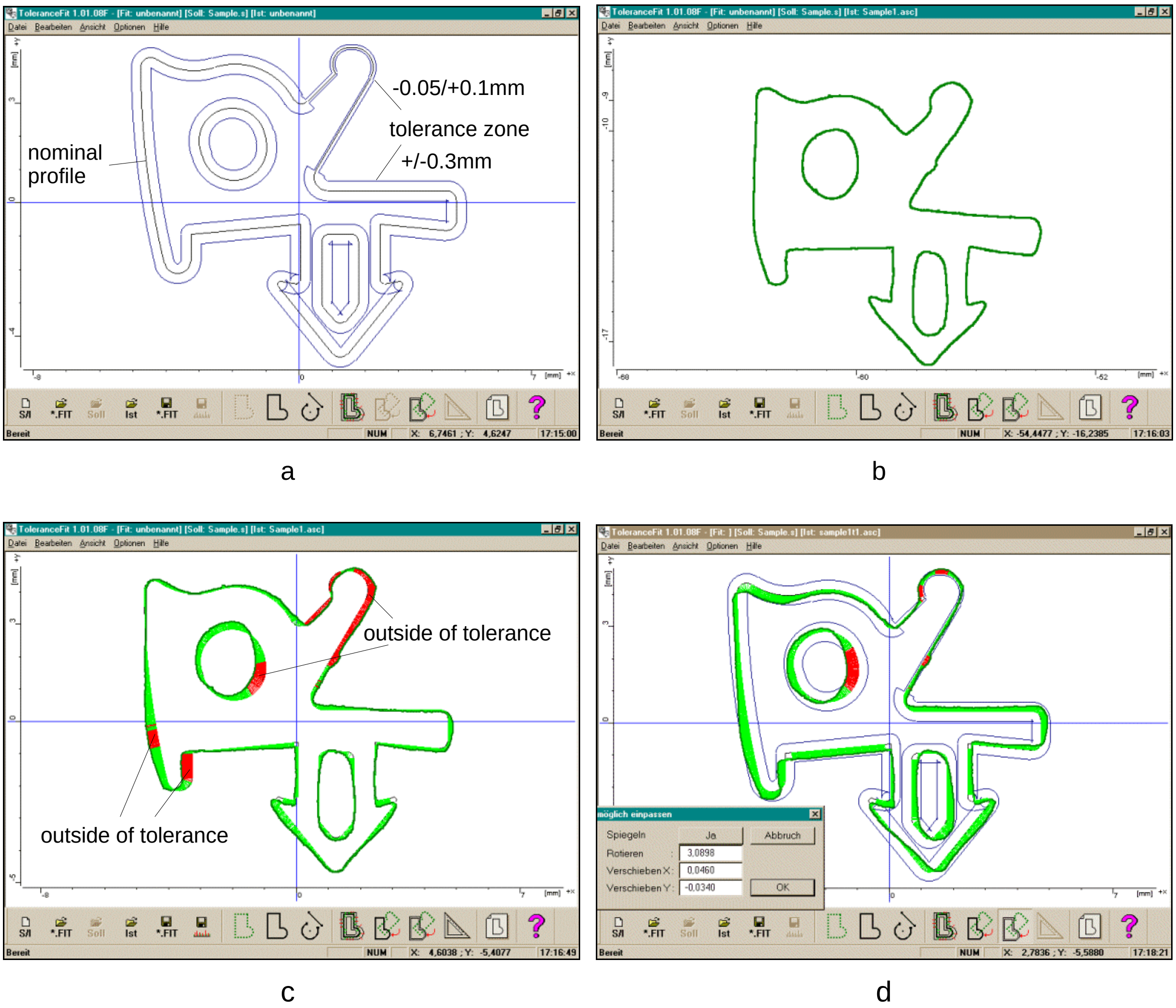

Figure 5. Tolerance fit of a profile [20]

5 SUMMARY

The paper describes the tolerance fit method as a modification of the minimum zone best fit. Two new solutions for tolerance fit of geometric elements according to defined tolerance zones are discussed.

Both methods discussed here uses optimisation algorithms in connection with smoothing algorithms and elimination of outliers, but these are well known procedures of evaluation of geometrical measurements.

The tolerance fit is a powerful software tool of coordinate measuring machines to assure the quality of geometric elements and sculptured surfaces and profiles. It checks the workpiece geometry under consideration of the required tolerances and avoids wrong quality decisions and unnecessary rework.

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