

# SUPER-HIGH CAPACITOR ANALYZER WITH COMPENSATION OF COMMON-MODE ERROR

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**Abstract** The paper is devoted to measurement of the super-high capacitor (from 0.1F to 200F) parameters with alternative current. The authors chose the mathematical model super-high capacitors and their equivalent circuit. They investigated of chosen mathematical model with equivalent circuit and proposed the super-high capacitor analyzer block diagram with compensation of common-mode error.

**Keywords** - super-high capacitor, equivalent circuit, low-frequency characteristics, electrical parameters, compensation of common-mode error.

## 1. SUPER-HIGH CAPACITOR EQUIVALENT ELECTRICAL CIRCUIT

To design analyzers of super-high capacitors electrical parameters, it is necessary to choose the mathematical model of such capacitor, which would correspond to features of super-high capacitors more fully [1].

Experimental researches, which were carried out by authors show that capacity of super-high capacitors with an alternating-current at standard frequencies of 50 Hz and 100 Hz in tens and hundreds time differ from capacity in case of measuring with the charge – discharge method.

Besides, the researches indicate that values of capacity increase when frequency of measuring signal decreases [2].

It is explained with inadequacy of super-high capacitors model to usual capacitors model, which use a simple two-element equivalent circuit.

Therefore, by analyzing results of experimental researches, the common mathematical model of super-high capacitors can be figured in a graphic aspect, as a series connection of an active resistance  $R(\omega)$  and  $C(\omega)$  fig.1.

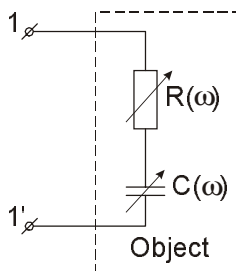


Fig.1. The super-high capacitors common mathematical model.

The super-high capacitors common mathematical model (fig.1) does not allow finding out and normalizing parameters of such capacitors. To solve this problem we shall use the super-high capacitors equivalent circuit fig.2, which simulates the relaxation polarization and losses in such capacitors.

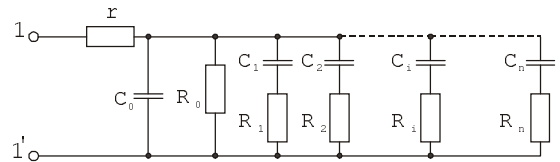


Fig.2. The super-high capacitors equivalent circuit.

In this equivalent circuit:  $r$  is an equivalent active resistance of losses;  $C_0$  is a geometrical (non-inert) capacity;  $R_0$  is an active leakage resistance;  $R_iC_i$  are  $n$  - relaxation circuits.

By analyzing this equivalent circuit, it is possible to make a conclusion that mathematical model of the super-high capacitors consists of the set of  $R_iC_i$  circuits, which amount  $n$  is prior unknown.

Now, it is logical to suppose that the representation of the model as  $n$  - cells  $R_iC_i$  will depend on an amount of measurement frequencies. It means, that for the model considered an amount of absorption  $R_iC_i$   $n$  - cells, on the one hand, is restricted by a frequency grid of a measuring sine signal and distributing ability of the measuring device on capacity  $\Delta C$  and an active resistance  $\Delta R$ .

On the other hand, the number of  $n$  - cells in the model should be restricted by the requirements to normalized characteristics and parameters, which are necessary to know to design and to use the super-high capacitors.

By analyzing the equivalent circuit on fig.2 and omitting the complicated mathematical transformations the expression for equivalent complex resistance  $Z(j\omega)$  looks like:

$$Z(j\omega) = r + \frac{\frac{1}{\omega R_0} + \sum_{i=1}^n \frac{\omega R_i C_i^2}{1 + \omega^2 R_i^2 C_i^2}}{\omega \left( \left( \frac{1}{\omega R_0} + \sum_{i=1}^n \frac{\omega R_i C_i^2}{1 + \omega^2 R_i^2 C_i^2} \right)^2 + \left( C_0 + \sum_{i=1}^n \frac{C_i}{1 + \omega^2 R_i^2 C_i^2} \right)^2 \right)} + \frac{1}{j\omega} \left( \frac{C_0 + \sum_{i=1}^n \frac{C_i}{1 + \omega^2 R_i^2 C_i^2}}{\left( \frac{1}{\omega R_0} + \sum_{i=1}^n \frac{\omega R_i C_i^2}{1 + \omega^2 R_i^2 C_i^2} \right)^2 + \left( C_0 + \sum_{i=1}^n \frac{C_i}{1 + \omega^2 R_i^2 C_i^2} \right)^2} \right) \quad (1)$$

Taking into account, that  $Z(j\omega)=R(\omega)+1/j\omega C(\omega)$ , let's write the expressions of dependencies  $C(\omega)$  and  $R(\omega)$  for an equivalent circuit of fig.2.

$$C(\omega) = \frac{\left( \frac{1}{\omega R_0} + \sum_{i=1}^n \frac{\omega R_i C_i^2}{1 + \omega^2 R_i^2 C_i^2} \right)^2 + \left( C_0 + \sum_{i=1}^n \frac{C_i}{1 + \omega^2 R_i^2 C_i^2} \right)^2}{C_0 + \sum_{i=1}^n \frac{C_i}{1 + \omega^2 R_i^2 C_i^2}} \quad (2)$$

$$R(\omega) = r + \frac{\frac{1}{\omega R_0} + \sum_{i=1}^n \frac{\omega R_i C_i^2}{1 + \omega^2 R_i^2 C_i^2}}{\omega \left[ \left( \frac{1}{\omega R_0} + \sum_{i=1}^n \frac{\omega R_i C_i^2}{1 + \omega^2 R_i^2 C_i^2} \right)^2 + \left( C_0 + \sum_{i=1}^n \frac{C_i}{1 + \omega^2 R_i^2 C_i^2} \right)^2 \right]} \quad (3)$$

The dependency diagrams of an equivalent capacity and equivalent resistance on frequency of a measuring sine signal are presented in fig.3 and fig.4.

The experimental parameters of an equivalent circuit (at  $n=4$ ) are:

$r=0.0017\text{Om}$ ;  $C_0=6.4\text{F}$ ;  $R_0=1000\text{Om}$ ;  $C_1=50\text{F}$ ;  $R_1=0.0023\text{Om}$ ;  $C_2=167\text{F}$ ;  $R_2=0.0046\text{Om}$ ;  $C_3=126\text{F}$ ;  $R_3=0.022\text{Om}$ ;  $C_4=41\text{F}$ ;  $R_4=1.3\text{Om}$ .

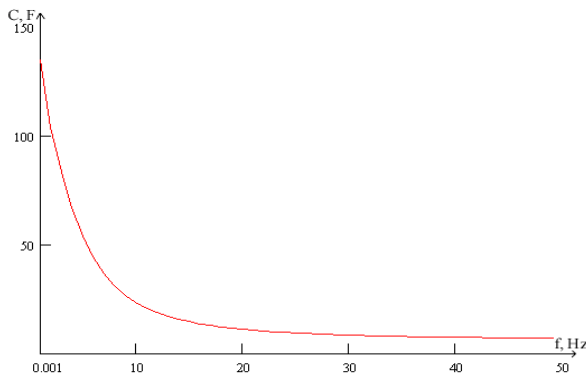


Fig. 3. The dependency diagram of an equivalent capacity on the frequency of a measuring signal.

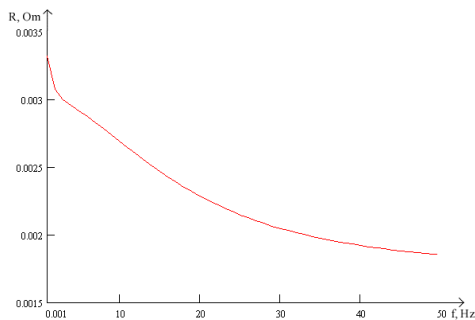


Fig. 4. The dependency diagram of an equivalent active resistance on a frequency of a measuring signal.

The determination of the equivalent circuit parameters is a basic measuring problem during development, manufacture and maintenance of super-high capacitors. The theoretical and experimental researches of equivalent capacity  $C(\omega)$  and active resistance  $R(\omega)$  frequency dependencies show, that the determination of these element is a complicated measuring problem.

The complexity this problem is that the amount of absorption  $R_i C_i$   $n$  - cells for real super-high capacitors is prior unknown.

To solve this problem authors propose the iterative - recursive method for measuring electrical parameters of an super-high capacitor equivalent circuit.

The essence of the offered iterative - recursive method is:

1) at the first stage of the iterative approximation, it is necessary to determine an amount of absorption  $R_i C_i$   $n$  - cells, which need to be known at development, manufacture and maintenance of super-high capacitors;

2) at the second stage it is necessary to define the equivalent circuit numerical values of super-high capacitors electrical parameters  $r$ ,  $C_0$ ,  $C_i$ ,  $R_i$  by recursive calculations.

At the first stage the amount of absorption  $R_i C_i$   $n$  - cells is determined by the following criteria:

1) the frequency range  $f_{min} \dots f_{max}$  in which the super-high capacitors are used;

2) by a step of digitization on frequency  $\Delta f$ , which is determined with rate of a frequency characteristic variation;

3) by normalizing performances and parameters, which are necessary for knowing at development, manufacture and maintenance of super-high capacitors.

The capacitors with over high capacities are used in infra low frequency a range, therefore the first criterion restricts a range of measuring frequencies:

$$f_{min} = 0.001\text{Hz} \quad T_{max} = \frac{1}{f_{min}} = 1000\text{s}$$

$$f_{max} = 100\text{Hz} \quad T_{min} = \frac{1}{f_{max}} = 0.01\text{s}$$

The second criterion restricts a step of digitization on frequency  $\Delta f$ , which is determined by an iterative approximation. On the first step of parameter measurement iterations of frequency dependencies in bandwidth of the chosen frequency band from 0.001Hz up to 100Hz is executed per decade. In this case grid of measuring frequencies makes:

$$f = \{0.001; 0.01; 0.1; 1; 10; 100\} \text{ Hz.}$$

On consequent step of iterations the magnitude of variation of measuring parameters  $C_i(f_i)$  and  $R_i(f_i)$  (or  $\text{tg}\delta_i(f_i)$ ) between adjacent frequencies are analyzed. At considerable variations of the pointed measuring parameters a step of digitization on frequency  $\Delta f$  diminish, and the measurements yield between adjacent frequencies, on which the measurements on the previous step of iterations were manufactured.

The third criterion defines admissible magnitude of variation of an instrument parameters  $C_i(f_i)$ ,  $R_i(f_i)$  or  $\text{tg}\delta_i(f_i)$  between adjacent frequencies. This criterion is determined in

normalizing performances and parameters, which are necessary for knowing at projection, manufacture and maintenance of super-high capacitors. On this first stage of an iterative - recursive method is completed.

At the second stage the electrical parameters numerical values  $r$ ,  $C_0$ ,  $C_i$ ,  $R_i$  of equivalent circuit elements are determined. These recursive calculations are founded on experimental researches of parameters of super-high capacitors.

For determination of two unknowns of magnitudes  $C_i$  and  $R_i$ , it is necessary to meter values of an equivalent capacitor capacitance on two frequencies accordingly  $f_i$  and additional  $f_{ai}=(f_i-1)/2$ .

In this case, the formulas will look like:

$$C_i = \left( C_{mi} - C_0 - \sum_{k=1}^{i-1} \frac{C_k}{1 + 4\pi^2 f_i^2 R_k^2 C_k^2} \right) \times \left( 1 + \frac{f_i^2 \left( C_{ai} - C_{mi} + \sum_{k=1}^{i-1} \frac{C_k}{1 + 4\pi^2 f_i^2 R_k^2 C_k^2} - \sum_{k=1}^{i-1} \frac{C_k}{1 + 4\pi^2 f_{ai}^2 R_k^2 C_k^2} \right)}{f_i^2 \left( C_{mi} - C_0 - \sum_{k=1}^{i-1} \frac{C_k}{1 + 4\pi^2 f_i^2 R_k^2 C_k^2} \right) - f_{ai}^2 \left( C_{ai} - C_0 - \sum_{k=1}^{i-1} \frac{C_k}{1 + 4\pi^2 f_{ai}^2 R_k^2 C_k^2} \right)} \right) \quad (4)$$

$$R_i = \frac{1}{\left( C_{mi} - C_0 - \sum_{k=1}^{i-1} \frac{C_k}{1 + 4\pi^2 f_i^2 R_k^2 C_k^2} \right) \left( 1 + \frac{f_i^2 \left( C_{ai} - C_{mi} + \sum_{k=1}^{i-1} \frac{C_k}{1 + 4\pi^2 f_i^2 R_k^2 C_k^2} - \sum_{k=1}^{i-1} \frac{C_k}{1 + 4\pi^2 f_{ai}^2 R_k^2 C_k^2} \right)}{f_i^2 \left( C_{mi} - C_0 - \sum_{k=1}^{i-1} \frac{C_k}{1 + 4\pi^2 f_i^2 R_k^2 C_k^2} \right) - f_{ai}^2 \left( C_{ai} - C_0 - \sum_{k=1}^{i-1} \frac{C_k}{1 + 4\pi^2 f_{ai}^2 R_k^2 C_k^2} \right)} \right)} \times \sqrt{4\pi^2 \left[ f_i^2 \left( C_{mi} - C_0 - \sum_{k=1}^{i-1} \frac{C_k}{1 + 4\pi^2 f_i^2 R_k^2 C_k^2} \right) - f_{ai}^2 \left( C_{ai} - C_0 - \sum_{k=1}^{i-1} \frac{C_k}{1 + 4\pi^2 f_{ai}^2 R_k^2 C_k^2} \right) \right]} \quad (5)$$

where  $C_{mi}$  and  $C_{ai}$  - are measured values of an equivalent capacities accordingly on frequencies  $f_i$  and  $f_{ai}$ .

The formulas (4) and (5) are recursive, because in them the numerical values of magnitudes  $r$ ,  $C_0$ ,  $C_1$ ,  $R_1$ ,  $C_2$ ,  $R_2$ , ...,  $C_{i-1}$ ,  $R_{i-1}$  were defined on the previous measuring frequencies of a selected frequencies grid accordingly  $f_{max}$ ,  $f_1$ ,  $f_2$ , ...,  $f_{i-1}$  and  $f_{a1}$ ,  $f_{a2}$ , ...,  $f_{ai-1}$ .

## 2. SUPER-HIGH CAPACITOR ANALYZER BLOK DIAGRAM WITH COMPENSATION OF COMMON-MODE ERROR

To successfully apply any super-high capacitor, full understanding of its specifications are required. The modern capacitor analyzers are continually been improved, providing the customer with ever-increasing accuracy.

In general, most super-high capacitor analyzers use a direct transformation measurement methods and its varieties [2]. For example there are a voltmeter-ampmeter method, a three voltmeter method and others.

It is important to note, however, that a considerable proportion of present-day super-high capacitor measuring circuits use the series connection of unknown object with standard resistor  $R_0$ .

The most typical block diagram for super-high capacitor analyzers is shown in Fig.5.

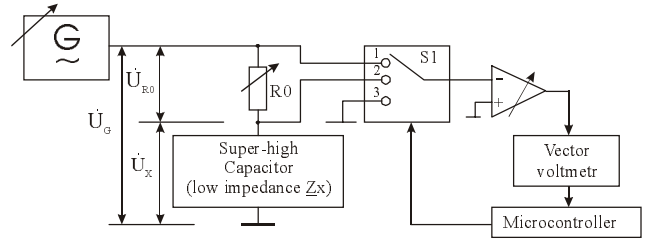


Fig.5. A simplified typical block diagram of super-high capacitor analyzers.

According to circuit theory, the super-high capacitor unknown impedance  $Z_x$  is defined by expression:

$$\underline{Z}_x = \frac{\dot{U}_x}{\dot{I}_x} = \frac{\dot{U}_x}{\dot{U}_G - \dot{U}_x} R_0 = \frac{\dot{U}_x}{\dot{U}_G - \dot{U}_x} R_0 \quad (6)$$

In this case it is necessary to subtract complex voltage  $\dot{U}_G$  from complex voltage  $\dot{U}_x$  or to measure the complex voltage drop  $\dot{U}_{R_0}$  on standard resistor  $R_0$ .

It is important to note that in first case the unknown object is 4-connected circuit because current connections and potential connections are separate.

In contrast to the unknown object, the standard resistor  $R_0$  is 3-connected circuit in measuring complex voltage  $\dot{U}_G$ . Moreover, the vector voltmeter analog-digital converter (ADC) operates in different voltage ranges because complex voltage  $\dot{U}_x$  is a portion of complex voltage  $\dot{U}_G$ . These factors increase additional errors.

In this case both unknown object and standard resistor  $R_0$  are 4-connected circuits in measuring complex voltages  $\dot{U}_x$  and  $\dot{U}_{R_0}$ . Besides, voltage values  $\dot{U}_x$  and  $\dot{U}_{R_0}$  we can do resembling with regulating a value of standard resistor  $R_0$ . So that it is improving an accuracy in measuring of super-high capacitor low impedance  $Z_x$ .

But measuring the complex voltage drop  $\dot{U}_{R_0}$ , it is necessary to take into consideration a common-mode error of instrumentation amplifier A1 because the complex voltage  $\dot{U}_x$  is a common-mode voltage for it.

Typical values of common-mode rejection (CMR) for modern instrumentation amplifiers are 60dB to over 100dB [4]. Virtually, these values are more less for a closed-loop gain instrumentation amplifiers. Therefore a common-mode error becomes significant and increasing to over 1% [4].

In order to solve this measuring problem, authors propose the super-high capacitor analyzer block diagram with compensation of common-mode error, as shown in Fig.6.

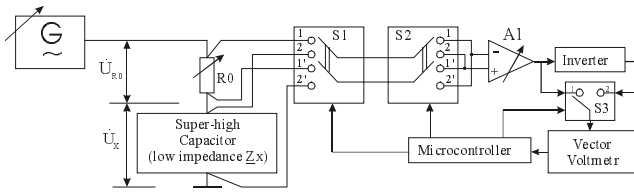


Fig.6. A simplified block diagram of super-high capacitor analyzers with compensation of common-mode error.

According to Fig.6., the complex voltage drop  $\dot{U}_{Ro}$  is measured twice. At first, switches S1 and S2 are in position 1-1' and switch S3 is in position 1. In this case the complex voltage drop  $\dot{U}_{Ro}$  passes to a vector voltmeter directly.

After that switches S2 and S3 are connected to others positions 2-2' and 2 correspondingly. The complex voltage drop  $\dot{U}_{Ro}$  passes to a vector voltmeter through inverter. Furthermore, it is a invert with respect to instrumentation amplifier inputs.

It is seen that complex voltage drop  $\dot{U}_{Ro}$  is inverted twice: the first time due to a switch S2 and the second time due to an inverter.

Whilst common-mode voltage is inverted only one time by means of inverter. Therefore, by adding both measuring results by microcontroller, we compensate the common-mode error in total sum because common-mode voltage has a different signs.

This is chief virtue of offered block diagram. Due to the common-mode error compensation, a measuring accuracy is improved.

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