A FINITE-MEMORY DISCRETE-TIME CONVOLUTION APPROACH FOR THE NON-LINEAR DYNAMIC MODELLING OF S/H-ADC DEVICES

D. Mirri [(1)], G. Pasini [(1)], P. A. Traverso [(2)], F. Filicori [(2)], G. Iuculano [(3)]

[(1)] Department of Electrical Engineering, University of Bologna, Viale Risorgimento 2, 40136 Bologna, Italy
Phone +39-051-2093472 Fax +39-051-2093470 e-mail: domenico.mirri@mail.ing.unibo.it
[(2)] Department of Electronics, Computer Science and Systems, University of Bologna
[(3)] Department of Electronics, University of Firenze, Via S. Marta 2, 50125 Firenze, Italy

Abstract - *A non-linear dynamic black-box model for Sample-Hold and Analog to Digital Conversion devices (S/H-ADC) is proposed in this paper. It derives from the expansion of a Volterra-like series previously introduced by authors and it is based on a discrete convolution in the time domain which describes the non-linear dynamics of the system. The model allows a functional description of the S/H-ADC non-linear behaviour as the sum of two blocks which take separately into account the static and the purely-dynamic contributions. The model characterization can be carried out by means of a simple and reliable measurement procedure and the model analytical representation allows an easy implementation in the framework of commercial available CAD tools for circuit analysis and design. Preliminary experimental results which validate the proposed approach are presented in the paper, together with some examples which show model capabilities in predicting device behaviour under non-linear operating conditions.*

Keywords - S/H-ADC device, non-linear dynamic model, Volterra series.

1. INTRODUCTION

Characterization and modelling of S/H-ADC devices are of basic importance in the performance analysis of measurement instruments based on digital signal processing. In the aim of identify suitable procedures for the correction of the device non-idealities, an high accuracy in the prediction of device behaviour under large-signal operating conditions is needed: this can be obtained only if the model takes into account not only the non-linearities associated with the S/H-ADC static characteristic, but also those which deal with device dynamics.

In order to characterize the non-linear dynamic effects it has been previously shown that a S/H-ADC device with input signal $s_I(t)$ can be described through the black-box functional model of Fig.1 [1,2]: the actual device is modelled as an ideal instrument sampling a signal $u(t)$ which is the result of the input/output relationship of a non-linear dynamic system (non-linear system with memory). The non-linear block is then described by the cascade of a purely-linear dynamic system with a virtually non-finite memory and of a non-linear one associated with finite, relatively short memory effects (Fig.2).

The first block of such a cascade is simply characterized by means of its pulse response in the time domain, while the latter can be conveniently modelled by the truncation to the first order integral of a modified Volterra series [3], previously proposed for the characterization of non-linear dynamic systems under the hypothesis of "short" duration of non-linear dynamic effects. More precisely, the output signal of the non-linear system in Fig.2 can be further described as the sum of the outputs of a memoryless non-linear block and a purely-dynamic non-linear one, if the duration of its memory can be considered "short" with respect to the typical minimum period of the input signal $s(t)$. This hypothesis represents the only important constraint required on the behaviour of a non-linear dynamic system in order to successfully apply to it the approach proposed in [3], and it can be considered satisfied in the case of a S/H-ADC device. In fact, the ideal behaviour for this family of systems is characterized by purely-linear dynamic effects (which are taken into account by the first block of the cascade in Fig.2), while the non-linear dynamics represent an error source due to the presence, in the system, of active devices, which are usually described by fast dynamics and can be associated with short memory.

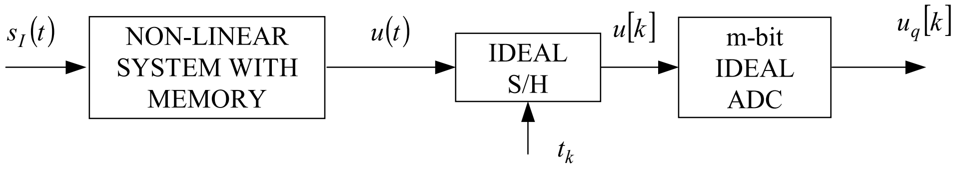

Fig. 1 - Functional model for the characterization of non-linear dynamic effects in the cascade of a S/H circuit and an m-bit ADC (S/H-ADC device).

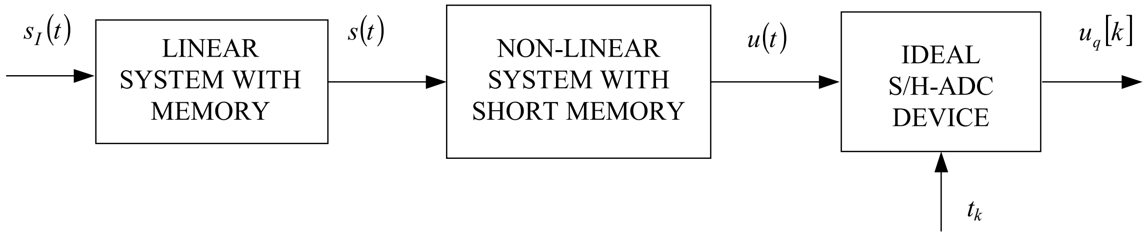

Fig.2 - Functional model of the S/H-ADC device pointing out the separation of the non-linear system with memory of Fig.1 into the cascade of two blocks, taking into account respectively the linear and non-linear dynamics.

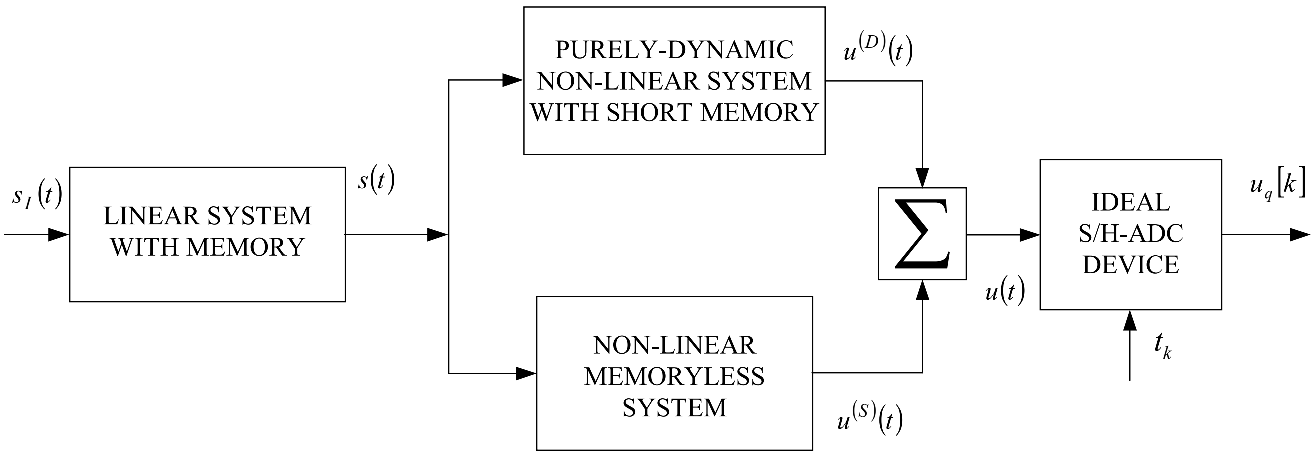

Fig.3 - Final functional model for the S/H-ADC device, showing the purely-linear block with non-finite memory, the memoryless and purely-dynamic with short memory non-linear blocks.

The final S/H-ADC representation is therefore shown in Fig.3. A non-linear dynamic model, suitable for the S/H-ADC characterization in the frequency domain, has been previously proposed by authors, starting from this functional description [2]. In this paper, an accurate S/H-ADC time domain-oriented model, easily identifiable by means of conventional measurement procedures and reliable numerical algorithms, will be presented.

2. THE S/H-ADC DISCRETE-TIME CONVOLUTION MODEL

The purely-linear dynamic block at the input of the functional structure in Fig.2 can be simply characterized by means of the conventional convolution integral of its pulse response $h(\cdot)$ with respect to the signal s_I, extended over a virtually non-finite memory interval:

$$s(t) = \int_{-\infty}^{+\infty} h(\tau) s_I(t-\tau) \mathrm{d}\tau \tag{1}$$

According to the modified Volterra series approach proposed in [3] for the modelling of non-linear dynamic systems with "short" memory, the output of the cascaded non-linear block in Fig. 2 can be described as the sum of two contributions, represented respectively by the output $u^{(S)}(t)$ of a non-linear memoryless block and the output $u^{(D)}(t)$ of a purely-dynamic non-linear block (Fig. 3):

$$u(t) = u^{(S)}(t) + u^{(D)}(t) \tag{2}$$

In (2), the first contribution (i.e. the output of the non-linear static block) can be written as a power series in the signal $s(t)$:

$$u^{(S)}(t) = z_0[s(t)] = y_0 + \sum_{r=1}^{\infty} \frac{a_r}{r!} s^r(t) \tag{3}$$

As far as the contribution $u^{(D)}(t)$ in (2) at the output of the non-linear block is concerned, it can be described by the following expression:

$$u^{(D)}(t) = \int_{-T_A}^{+T_B} w[s(t),\tau] \cdot e(t,\tau) \mathrm{d}\tau \tag{4}$$

where the *dynamic deviation* function

$$e(t,\tau) = s(t-\tau) - s(t) \tag{5}$$

represents the difference between "past" values $s(t-\tau)$ of the input signal of the non-linear system with short memory shown in Fig.2 with respect to the instantaneous value $s(t)$. Some important considerations can be made about the analytical representation of (3) and (4). Recalling the classical Volterra description for a generic non-linear dynamic system:

$$u(t)=\sum_{r=1}^{+\infty}\frac{1}{r!}\int\cdots\int_{-\infty}^{+\infty}h_r(\tau_1,\ldots,\tau_r)\left[\prod_{i=1}^{r}s(t-\tau_i)\mathrm{d}\tau_i\right] \quad (6)$$

it is possible to demonstrate that the coefficients a_r ($r=1,2,...$) of the power series (3) can be expressed by means of the Volterra kernels:

$$a_r=\int\cdots\int_{-\infty}^{+\infty}h_r(\tau_1,\ldots,\tau_r)\prod_{i=1}^{r}\mathrm{d}\tau_i \quad (7)$$

Moreover, also the *weight* function w in (4), which represents the first order kernel of the modified series [3], can be derived from kernels h_r :

$$w[s(t),\tau]=\sum_{m=1}^{\infty}\frac{1}{m!}b_m(\tau)s^m(t) \quad (8)$$

$$b_m(\tau)=\int\cdots\int_{-\infty}^{+\infty}h_{m+1}(\tau,\tau_2,\ldots,\tau_{m+1})\prod_{s=2}^{m+1}\mathrm{d}\tau_s \quad (9)$$

Equation (4) represents a first order convolution integral with respect to the dynamic deviation function $e(t,\tau)$, non-linearly controlled by the signal $s(t)$, and can adequately characterize the S/H-ADC non-linear dynamic effects if the memory interval $[-T_A,T_B]$ duration T_M of the non-linear block in Fig.2 is short with respect to the typical minimum period of $s(t)$ (i.e. the period of the S/H-ADC input signal $s_I(t)$). In such conditions, in fact, the dynamic deviation assumes values that are small even in the presence of large fluctuations of the signal $s(t)$. Thus using expression (2), which is the linearization (i.e. the truncation to the first order integral) of the modified Volterra series with respect to the dynamic deviation $e(t,\tau)$, does not introduce relevant errors in the description of the non-linear block behaviour, since all other integral contributions in the series are higher order infinitesimal of $e(t,\tau)$ [3]. By discretising now the integration domain of (4) (i.e the non-linear memory time) into the summation of (P_A+P_B) intervals of equal width $\Delta\tau$, the same equation can be rewritten as:

$$u^{(D)}(t)\cong\sum_{\substack{p=-P_A\\p\neq 0}}^{+P_B}w[s(t),p\Delta\tau]\cdot e(t,p\Delta\tau)=\sum_{\substack{p=-P_A\\p\neq 0}}^{+P_B}w_p[s(t)]\cdot e(t,p\Delta\tau) \quad (10)$$

where $P_A\Delta\tau=T_A$ and $P_B\Delta\tau=T_B$. Equation (10) represents the discrete-time, non-linearly controlled convolution which takes into account the non-linear dynamics of the S/H-ADC device.

3. MODEL IDENTIFICATION

The feasibility of the extraction of behavioural models, such as those based on the classical Volterra series, for non-linear dynamic systems is strongly limited by the difficulties that are related to the identification of all the kernels which are needed in order to achieve a good prediction accuracy. In many cases only a part of these kernels can be practically characterized, and the model formulation suffers from a strong approximation that reduces its performances. The proposed approach overcomes such a limitation, being its full identification based on a simple and reliable experimental procedure.

It is important to notice that, being possible to impose, without loss of generality, $H(\mathrm{j}0)=1$ to the transfer function of the linear dynamic system, the function $z_0[\cdot]$ coincides with the static characteristic of the entire device. The static block of Fig.3 is thus easily identifiable by means of conventional DC measurements over a suitable interval of input signal values. Moreover, it has been shown [2] that in zero bias, small-signal operating conditions, the non-linear block in Fig.2 can be considered memoryless. Therefore, the linear dynamic block at the input of the system can be characterized through small-signal, zero bias measurements at different frequencies over the input signal $s_I(t)$ typical bandwidth. Finally, by applying to the device a set of R large-amplitude sinusoidal input signals $s_{Ir}(t)$ for different values of frequency and bias point, and by measuring, for each test input signal, a suitable set of M samples $u_r(t_m)$ at the output, a linear system of RM equations in $(P_A+P_B)(N+1)$ unknowns β_{pn} can be obtained from (10):

$$\begin{aligned}&u_r^{(D)}(t_m)=u_r(t_m)-z_0[s_r(t_m)]=\\&\sum_{\substack{p=-P_A\\p\neq 0}}^{+P_B}e_r(t_m,p\Delta\tau)\sum_{n=0}^{N}\beta_{pn}s_r^{\,n}(t_m)\quad(r=1,\ldots,R)(m=1,\ldots,M)\end{aligned} \quad (11)$$

where each function $z_0[\cdot]$ has been represented as a polynomial series truncated to the N-th order with coefficients β_{pn} ($n=0,...,N$). If a large number of test functions is chosen at input of the S/H-ADC device ($RM>>(P_A+P_B)(N+1)$), the model identification can be completed, by solving the over-determined linear system (11) through well-known least square methods. This numerical procedure leads to a reliable solution, without the need for non-linear optimization algorithms which usually suffer from convergence problems, such as the dependence on the starting point due to the presence of multiple local minima.

4. EXPERIMENTAL RESULTS

The proposed time-domain discrete convolution model has been extracted for a S/H-ADC device previously implemented in the framework of SPICE circuit analysis CAD tool. The device has been realized through a sampling switch based on a Schottky diode (MBD701) bridge circuit [4], separated from the input and output by two Op Amps (OPA640). The model has been then fully implemented in the Agilent-ADS package environment, which allows to perform a large set of different kind of circuit analyses, both in time and frequency domain. The simple, time-domain oriented mathematical formulation allowed an easy implementation of the model by means of conventional tools available at the user-interface level of the simulator, with no need for senior facilities, code programming or additional approximations.

In order to validate the proposed approach, several test have been performed, under different operating conditions at the input of the device. In particular, several large-amplitude sinusoidal signals have been applied, for different values of frequency and bias point. Measured and predicted waveforms at the device output have been then compared by computing the mean square error, for each input excitation, over a vector of time domain samples. Tab.1 shows the good agreement between measures and predictions, at moderately low frequencies as well as at high frequencies, where the non-linear dynamic effects in the S/H-ADC behaviour become important.

Tab. 1 – Mean square error, computed over a vector of time samples, between measured and predicted waveforms at the S/H-ADC output. Axis indicate the bias point and frequency values for each sinusoidal signal input.

ε_r^2	1.5 MHz	3 MHz	6 MHz	12.5 MHz	25 MHz	50 MHz
–2.0 V	$2.6\cdot10^{-6}$	$0.7\cdot10^{-6}$	$1.5\cdot10^{-6}$	$1.4\cdot10^{-6}$	$2.1\cdot10^{-6}$	$2.1\cdot10^{-6}$
–1.6 V	$1.7\cdot10^{-6}$	$0.3\cdot10^{-6}$	$0.9\cdot10^{-6}$	$0.9\cdot10^{-6}$	$1.3\cdot10^{-6}$	$1.8\cdot10^{-6}$
–1.2 V	$1.4\cdot10^{-6}$	$2.6\cdot10^{-6}$	$0.8\cdot10^{-6}$	$0.8\cdot10^{-6}$	$1.1\cdot10^{-6}$	$1.8\cdot10^{-6}$
–0.8 V	$1.7\cdot10^{-6}$	$0.3\cdot10^{-6}$	$0.8\cdot10^{-6}$	$0.8\cdot10^{-6}$	$1.1\cdot10^{-6}$	$1.8\cdot10^{-6}$
–0.4 V	$1.2\cdot10^{-6}$	$0.3\cdot10^{-6}$	$2.7\cdot10^{-6}$	$0.8\cdot10^{-6}$	$1.1\cdot10^{-6}$	$1.8\cdot10^{-6}$
0.0 V	$1.1\cdot10^{-6}$	$0.3\cdot10^{-6}$	$0.6\cdot10^{-6}$	$0.7\cdot10^{-6}$	$1.1\cdot10^{-6}$	$1.8\cdot10^{-6}$
0.4 V	$1.0\cdot10^{-6}$	$0.4\cdot10^{-6}$	$0.6\cdot10^{-6}$	$0.7\cdot10^{-6}$	$1.1\cdot10^{-6}$	$1.8\cdot10^{-6}$
0.8 V	$0.9\cdot10^{-6}$	$0.6\cdot10^{-6}$	$0.6\cdot10^{-6}$	$0.8\cdot10^{-6}$	$1.2\cdot10^{-6}$	$2.1\cdot10^{-6}$
1.2 V	$0.9\cdot10^{-6}$	$1.0\cdot10^{-6}$	$41.9\cdot10^{-6}$	$1.1\cdot10^{-6}$	$1.3\cdot10^{-6}$	$2.4\cdot10^{-6}$
1.6 V	$1.1\cdot10^{-6}$	$1.4\cdot10^{-6}$	$1.1\cdot10^{-6}$	$1.4\cdot10^{-6}$	$1.5\cdot10^{-6}$	$2.8\cdot10^{-6}$
2.0 V	$1.1\cdot10^{-6}$	$1.6\cdot10^{-6}$	$1.3\cdot10^{-6}$	$1.6\cdot10^{-6}$	$1.6\cdot10^{-6}$	$2.8\cdot10^{-6}$

Analyses performed by means of the device model allows also to quantify the contribution of non-linear dynamic effects on the S/H-ADC behaviour. In Fig.4 the transfer function predicted by the model for the linear dynamic block of Fig.3 is represented, from 1kHz up to 500 MHz. Since the cut-off frequency of the device can be placed at the value of 10 MHz, it is interesting to investigate the non-linear behaviour of the device for input excitations around this point of the spectrum, where the converter is near to the limit of its region of use.

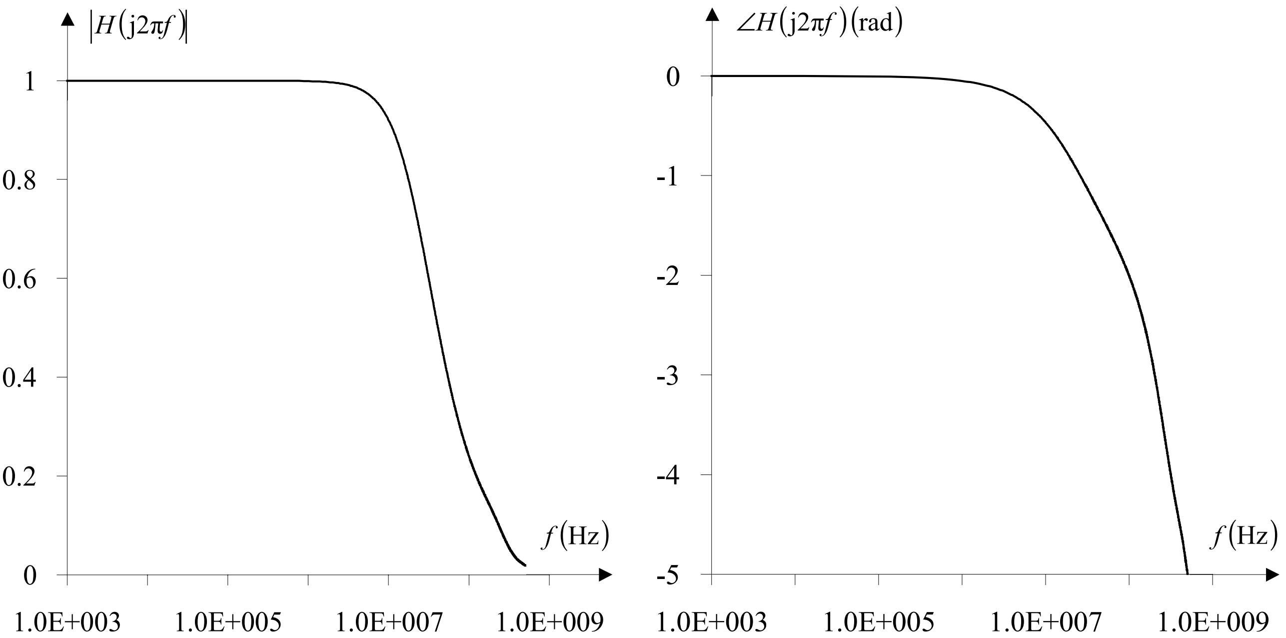

Fig. 4 - Magnitude and phase of the linear dynamic block transfer function, predicted by the model from 1 kHz up to 500 MHz.

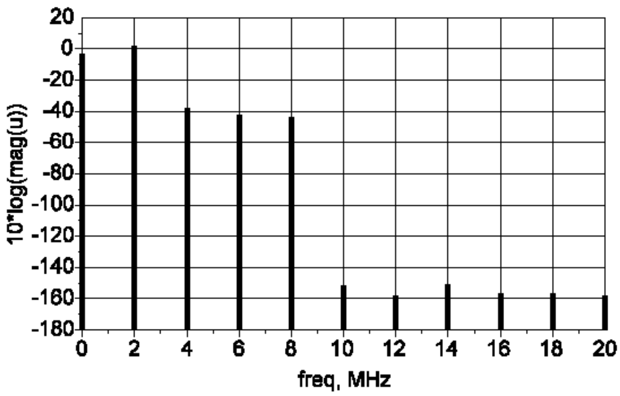

Fig.5 – Magnitude of predicted spectral components (dB units) at the output of the S/H-ADC device, as result of a sinusoidal input signal ($f_0 = 2$ MHz, amplitude: 1.5V, bias: 0.5V)

An harmonic balance analysis has been then performed with a 0.5V bias, 1.5V amplitude sinusoidal input signal at frequency $f_0 = 2$ MHz. Fig.5 shows the spectrum of the waveform at the output of the device. Fig.6 shows the comparison between the measured and simulated device DC characteristic.

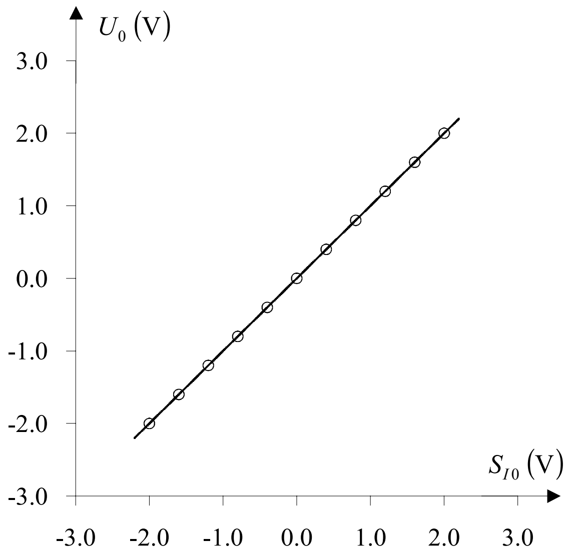

Fig. 6 – Comparison between measured (• – • – •) and predicted (-----) S/H-ADC device static characteristic over the range [-2V,2V].

Besides showing the high accuracy of the model in predicting also the static response, it points out how the memoryless non-linear phenomena are mild in the device behaviour. Thus, all the non-linear effects that cause the presence of harmonics in the output spectrum of Fig.5 can be considered related only to the device dynamics. The proposed model takes into account these non-idealities, not quantifiable by means of conventional models, which usually attribute the non-linear effects of the device only to its non-ideal DC characteristic. Fig.7 represents the predicted waveform

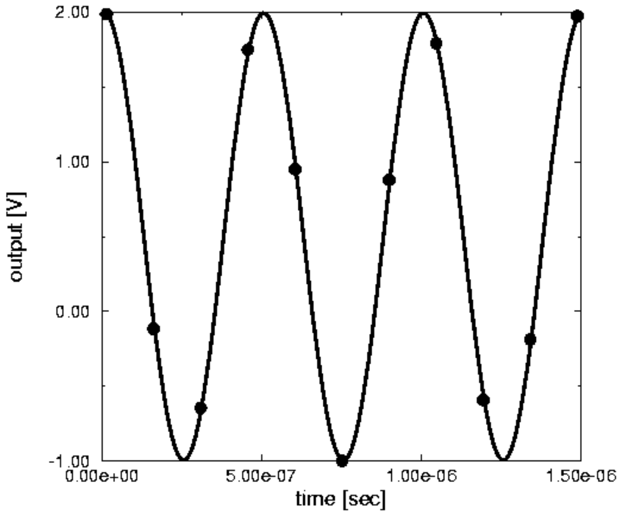

Fig.7 – Measured (• – • – •) and predicted (---) samples at the output of the S/H-ADC device, as result of a sinusoidal input signal ($f_0 = 2$ MHz, amplitude: 1.5V, bias: 0.5V) (See Fig.5)

at the S/H-ADC output whose spectrum is shown in Fig.5, compared to the measured samples.

4. CONCLUSIONS

A new finite-memory discrete-time convolution approach has been proposed for the non-linear dynamic modelling of S/H-ADC devices. The model takes separately into account the static and purely-dynamic non-linearities and can be identified starting from experimental data, obtained through conventional measurement procedures, by applying reliable numerical algorithms. Validation results have been provided, which confirm the model capability to predict accurately the system behaviour, under different operating conditions, from DC up to the frequencies in the region that represents the limit of the device response.

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