DESIGN SIMULATION OF DECIMATION FILTER FOR SIGMA DELTA CONVERTERS

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Abstract - *The purpose of this paper is to present several filter topologies used for decimation of sigma delta modulated digital signals in order to choose the optimised filter architecture with regards to an efficient implementation. The filter is suited for data conversion and measurement applications. A second order 1-bit sigma delta modulator will be considered as the front-end A/D converter. The subsequent digital filter reduces the sampling rate by a factor of 64 and must guarantee a stop band attenuation of 80 dB.*

Keywords – Sigma Delta Converter, Decimation.

1. INTRODUCTION

The A/D converters allow the transition between analogue and digital domains, while sampling the continuous analogue signal and quantifying it on a fixed number of bits. The quantification is an essential stage of the conversion, it introduce to the effective signal a noise supposed white. To increase the SNR (Signal to Noise Ratio), we use the over sampling technique. This solution permits us to increase performances of the converter, because in addition it assures the respect of the Shannon law, and permits to simplify the realization of the low-pass filtering on the chain of conversion[1]. However, to improve the resolution of the converter we need a big factor for over sampling. It is why a supplementary technique is used to increase the SNR: the noise shaping, it is achieved by a sigma delta modulator[2]. The ΣΔ modulator delivers coded information on few bits to an elevated rate. Then it is necessary to do decimation so that information is coded on a raised number of bits (the total resolution of the converter) and to permit a return to the Nyquist frequency. So the data stream at transmissions, storages and numeric treatment is decreased.

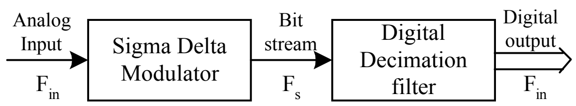

Fig.1 – Block diagram of the sigma delta converter

In recent literature, the authors have proposed a class of high performance linear-phase FIR decimator structures that can be easily integrated in small area[3]. In this paper, the proposed decimators have been designed to work at the output of a second order sigma delta modulator. In order to optimize both the decimator performance and the implementation, the proposed decimators have been designed to consist of several stages with each stage requiring a small number of arithmetic operations. If the resolution of the overall converter is to be at least 12 bits, then it is advisable to have at least a 80 dB stop band attenuation, whereas the maximum pass band droop is desired to be less than 0.1 dB. In such cases, conventional FIR filters require many bits for coefficients representations. In the following sections we focuses the investigations on cascaded structures. The first stage is realized as a comb filter and decimates by a factor of 8 or 16. The unavoidable pass band droop must be compensated in the following low pass FIR filters. In order to compare several filter realizations, three examples of architectures are considered for the design simulation of the decimation filter.

2. SPECIFICATIONS

A considerable effort must be made to meet the specifications with single stage decimation filter. In order to optimise the implementation's requirement, it is useful to implement cascaded filter structures. This approach becomes the more interesting the higher the filter specifications are. The sampling rate of the input signal must be reduced from 1.024 MHz to 16 kHz. The stop band attenuation is 80 dB. The Pass band ripple is little than 0.1 dB. The decimation filter must guarantee a narrow transition band between 8 kHz and 9.6 kHz. The front-end is always a comb filter and the back-end performs the final decimation and compensates the pass band droop of the comb filter. Due to the need of linear phase only FIR filters are used for the purpose of compensation. In the following sections three topologies will be investigated to satisfy the specifications of the over all decimation filters.

3. THE COMB FILTER

The first stage of the cascaded decimator must be the comb filters; they operate at high frequency, which is suitable for this purpose. The comb filters (sometimes called sinc^K filters, where K is the order of the comb filter) have also a simple structure, no multipliers and coefficients are required

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and their architectures are independent to the decimation ratio.

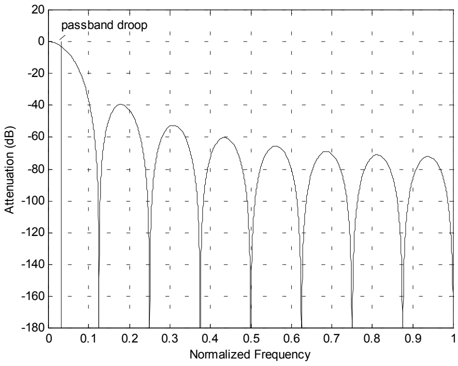

Fig.2 – Magnitude response of comb filter (D=16 and K=3).

The transfer function is given by (1) :

$$H(z) = \frac{1}{D}\sum_{n=0}^{D-1} z^{-n} = \frac{1}{D}\,\frac{1-z^{-D}}{1-z^{-1}} \qquad (1)$$

Where D denote the decimation ratio. The sinc^K is a low pass filter where zeros are placed to the multiple frequencies of Fs/D.

In table I, we notice that more the order of the comb is raised, more the attenuation out band is important. Therefore, it is interesting to use this filter in elevated order to eliminate a maximum of noise, so we reduce the "natural aliasing" phenomena in the pass band of the signal at the down-sampling. The simplicity of implantation of the comb filter can be put in evidence while analyzing the function of transfer. In (1) we recognize an integration function followed by a derivation function. However these two stages are followed of an down-sampling of D, therefore it is interesting to place the decimation operation before the derivation witch will be achieved to reduced frequency and, in addition, will simplify its implantation since the derivation term $(1-z^{-D})$ becomes $(1-z^{-1})$.

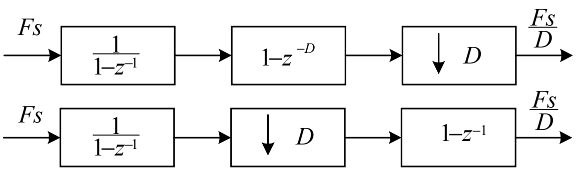

Fig.3 – Structure improvement of the comb filter.

In the design process of the decimation filter, we must carry an attention to the problem of the spectral alias. Limits of aliased band f_{pb}, giving by the equation (2), must be sufficiently attenuated. Concerning our application the minimal attenuation asked off band is As=80dB.

$$f_{pb} = \frac{kf_s}{D} \pm f_p \qquad (2)$$

To eliminate the alias noise of the ΣΔ modulator, CANDY [1] signaled that the order of sinc filter should be at less equal to the order of the modulator more 1. In the case of a second order modulator, a third order sinc filter is required to accomplish a sufficient alias rejection with a decimation rate equal to 16 for a pass band of 8kHz.

Table I - Noise attenuation (A_s) and pass band droop (A_{pd}) of the comb filter.

K	As [DB] D = 4	As [DB] D = 8	As [DB] D = 16	A_{pd} [DB] D = 4	A_{pd} [DB] D = 8	A_{pd} [DB] D = 16
1	23	17	10	0.05	0.22	0.91
2	46	34	21	0.1	0.44	1.82
3	68	51	31	0.16	0.66	2.73
4	**91**	68	42	**0.21**	0.88	3.63
5	114	**85**	52	0.26	**1.10**	4.54
6	137	102	63	0.31	1.33	5.45
7	159	119	73	0.37	1.55	6.36
8	182	136	**83**	0.42	1.77	**7.27**
f_{pb}	0.2343	0.1093	0.0469	0.0156	0.0156	0.0156

4. THE PROPOSED ARCHITECTURES

4.1 The two-stage decimation filter

The first considered topology is the stake in cascade of two comb filters one of order 5 and the other of order 3 having for each one a decimation factor equal to 8. It is the simplest realization of the recursive structure based on moving average filters. The transfer function is:

$$H(z) = \left[\frac{1}{8}\,\frac{1-z^{-8}}{1-z^{-1}}\right]^5 \times \left[\frac{1}{8}\,\frac{1-z^{-8}}{1-z^{-1}}\right]^3 \qquad (3)$$

Every stage of the structure is followed by a decimation that down-samples the signal by 8. Fig.4 illustrate the process of the decimation.

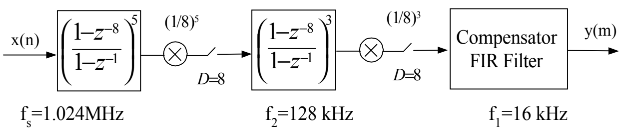

Fig.4 – The first process of decimation.

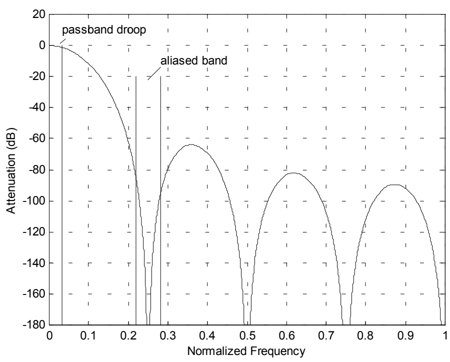

Fig.5 – Magnitude response of the first stage normalized to f_s=1.024 MHz.

The compensation filter is achieved, at last, in FIR structure in single stage while using conventional units of multipliers and accumulators. This filter must satisfy three specifications at the same time. It must compensate the loss in attenuation in the band of interest, guarantee a narrow band of transition and give up a sufficient attenuation of the attenuated band. It leads us to an elevated order of the filter (roughly N=645) gotten by simulation with MATLAB software.

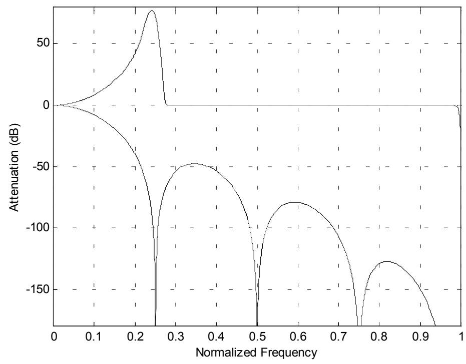

Fig.6 – The compensator filter normalized to f_l=128 k Hz.

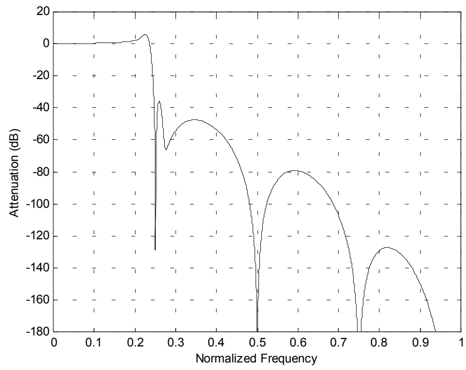

Fig.7 – The overall response of the decimation filter.

4.2 The Comb FIR compensator cascade

In this second example of conception, a "sharpened" comb filter is used like a decimator in the first stage instead of the conventional comb filter as we presented it in the preceding example [4]. The "sharpened" filter is used a third order of which the transfer function is given by (4).

$$H(z) = 3\times\left[\frac{1}{D}\,\frac{1-z^{-16}}{1-z^{-1}}\right]^{6} - 2\times\left[\frac{1}{D}\,\frac{1-z^{-16}}{1-z^{-1}}\right]^{9} \qquad (4)$$

An adequate structure of the filter can be applied while choosing a frontal decimation by 16 for an overall decimation executed in three steps, D=16×2×2.

The interest to use a sharpened that it doesn't present a great pass band droop as shown in table II, so, we don't need a compensator filter[5]. The Fig.8 exposes the block diagram for this example.

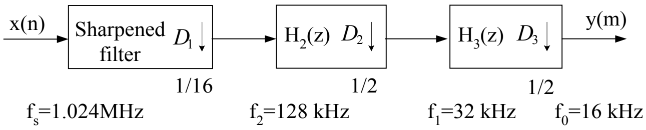

Fig.8 – The second process of decimation.

Table II - Noise attenuation and pass band droop of the Sharpened filter.

K	A_s [dB] D = 4	A_s [dB] D = 8	A_s [dB] D = 16	A_{pd} [dB] D = 4	A_{pd} [dB] D = 8	A_{pd} [dB] D = 16
1	36.4	25.2	13.3	0.0009	0.0161	0.243
2	**81.6**	58.4	32.7	**0.0037**	0.0622	0.852
3	127.1	**92.2**	53.1	0.0082	**0.1347**	1.709
4	172.7	126.1	73.9	0.0145	0.2311	2.743
5	218.2	160	**94.7**	0.0225	0.3489	**3.911**
f_{pb}	0.2343	0.1093	0.0468	0.0156	0.0156	0.0156

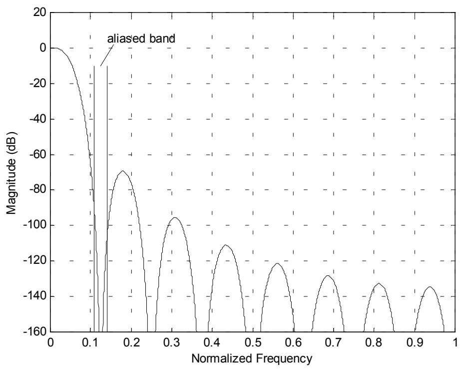

Fig. 9 – Magnitude response of the sharpened normalized to f_s=1.024 M Hz.

The second stage must achieve the remaining decimation in D=D1×D2=4. However, execute the remaining decimation in only one block means that an FIR filter with 200 coefficients is required to assume 80 dB of attenuation in the stop band. That's why we split it in two FIR filters H_2 and H_3. Filters are designed while using the MATLAB software, coefficients of $H_2(z)$ and $H_3(z)$ are calculated with the REMEZ exchange algorithm and they are, respectively, 80 and 100.

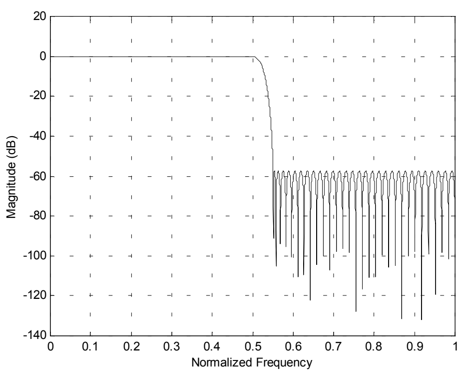

Fig.10 – Magnitude response of $H_2(z)$ normalized to f_2=64 k Hz.

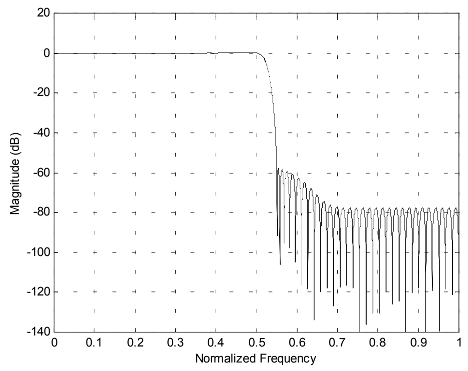

Fig.11 – Magnitude response of $H_3(z)$ normalized to f_1=32 k Hz.

4.3 The Comb Half-Band cascade

The object of this structure is the cascade of the comb filter with two half-band filters. The Fig.12 gives the block diagram of this architecture. The objective is to design a multi-stage decimation filter where the intermediate stage possesses with the more flexible specifications then lower requirements at the implementation level.

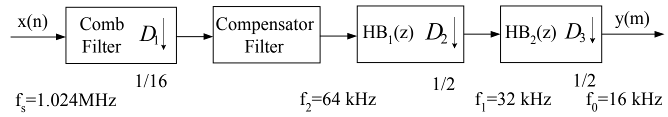

Fig.12 – The third process of decimation.

The first half-band filter HB_1 in this architecture has the most expanded transition band, it is submitted to an input sampled at frequency f_2=64 kHz, its transition band spreads from 0.125 to 0.375 normalized to f_2.

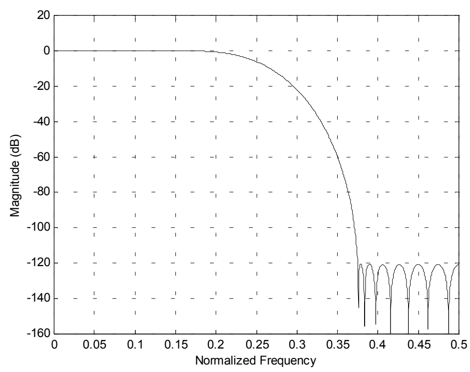

Fig.13 – The frequency response of HB_1 normalized to f_2=64 kHz.

To achieve the desired transition band, a FIR of 22 coefficients is required. With this configuration we get a pass band droop of 8 μ dB. The Fig.14 shows the curling in frequency response of the first filter HB_1, on the other hand, the Fig.13 present its overall response.

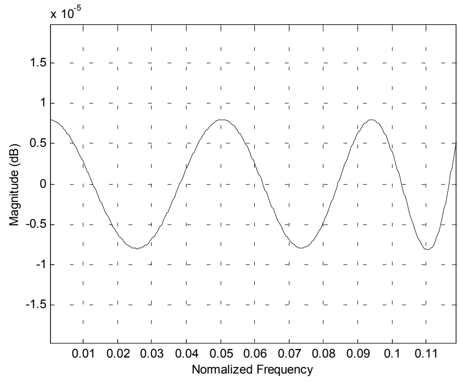

Fig.14 – The pass band ripple of HB_1.

The last half-band filter HB_2 has the strictest specifications with a pass band at 0.25 and a stop band at 0.27; frequencies are normalized to 32 kHz. The narrow transition band determines the order of the last stage while applying (5) with As=80 dB and Δf=0.02, we get a pass band ripple δ_p = 0.012 dB by (6) [6].

$$N=\frac{-13-20Log_{10}\sqrt{\delta_p\delta_s}}{14,6.\Delta f}+1 \qquad (5)$$

$$As=20.Log_{10}\left(\frac{1+\delta p}{1-\delta p}\right) \qquad (6)$$

The Fig.15 shows the overall frequency response of the first half band, while the Fig.16 represents a relative section of the pass band of this filter.

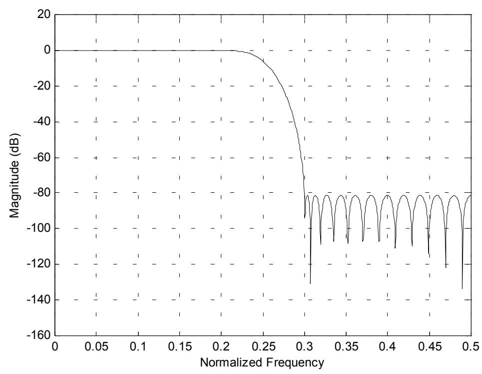

Fig.15 – Magnitude response of the second half-band filter normalized to f_1=32 kHz.

Coefficients for the first and the second half-band filter have been calculated while using MATLAB Software and applying the REMEZ exchange algorithm, their orders are respectively 28 and 22[7].

In addition the coefficients of the two half band filters can be generated symmetrically with REMEZ exchange algorithm and be implemented in polyphase structure, which reduce their order (11 and 14) and their work's frequencies. The proposed cascade of the half band filters is not conceived to decrease the loss in the pass band of the comb filter. Therefore, a compensation filter is always necessary.

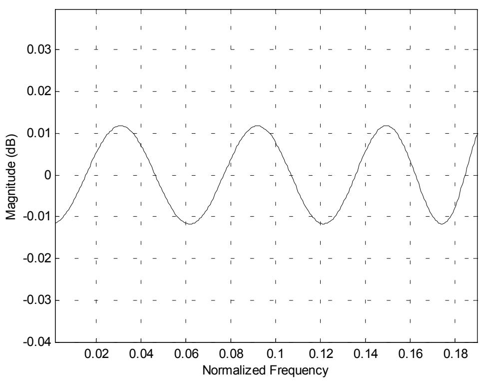

Fig.16 - Pass band ripple of HB_2.

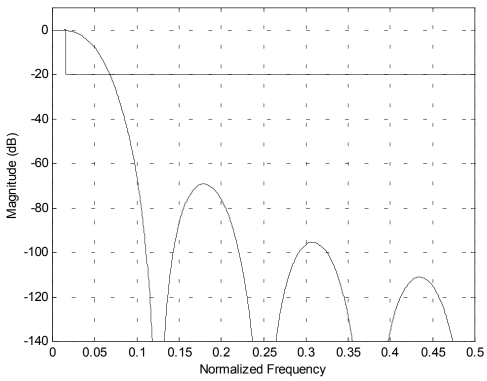

Fig.17 - The desired compensator filter normalized to 1.024 MHz.

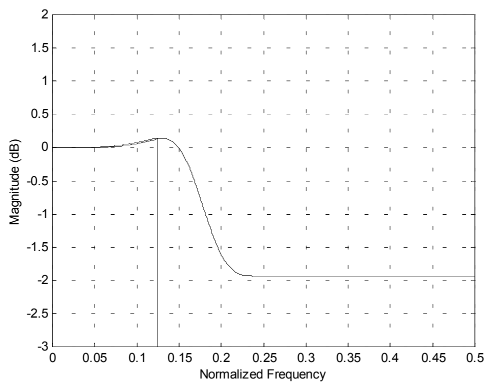

Fig.18 - The designed compensator filter.

To limit the order of the compensator filter we have opt for a symmetrical structure, the filter order is 40 but only 20 are different.

Fig.17 and Fig.18 represent, the loss in amplitude and the frequency response of the compensator filter. The overall frequency response of the decimation filter is illustrated in Fig.19.

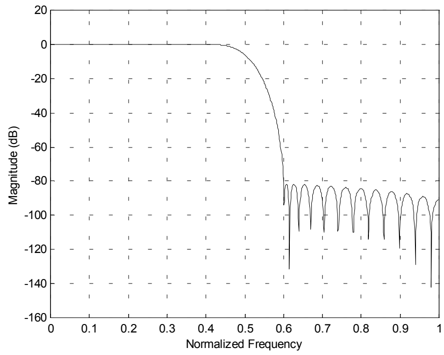

Fig.19 - The overall response of the decimation filter.

5. COMPARAISON

In this paper three topologies are presented for the decimation filter. The first topology, shown in Fig.4, consist in the cascade of two comb filters witch reduce the hardware requirement, but this structure need a compensator FIR filter with an elevated order (645). In the second topology, shown in Fig.8, we used the sharpened comb filter to avoid the pass band droop and, so, the compensator filter. However, we notice that the orders of the following decimators FIR filters are also great (80 and 100). In the third topology, shown in Fig.12, we use a classic comb filter with two half-band filters. It seems that this structure is the most suitable for decimation. In fact, due to the low order of the two half band filters (11 and 14) and the compensator filter (20), the implementation requirement is optimized and the specifications are reached.

6. CONCLUSION

The target of this paper is to present the optimized structure for a decimation filter in ΣΔ converters. This filter reduces the sampling rate by a factor of 64 and guarantees a stop band attenuation of 80 dB. Three topologies are presented. Our work indicated that the third architecture is the most suitable structure in regards simplicity in implementation. Future work will include the hardware realization of this decimation filter and the full ΣΔ converter.

REFERENCES

[1] J. Candy and G. Temes, *Oversampling methods for A/D and D/A conversion*, IEEE Press, 1992.

[2] S.R. Norsworthy, R. Schreir and G. Themes, *Delta-Sigma data converters, theory, design and simulation*. IEEE Press, 1996.

[3] S. Chu and C.S. Burrs, "Multirate filter design using comb filters", *IEEE Trans. Circuits and systems*, Vol CAS-31, pp 913-924, 1984.

[4] J. Kaiser and R. Hamming, "Sharpening the response of symmetric non recursive filter by multiple use of the same filter", IEEE Trans. on acoustics, speech and signal processing, Vol 25, pp 415-422, 1997.

[5] A.Y. Kwentus, Z. Jiang, A.N. Wilson, "*Application of filter sharpening to cascaded integrator-comb decimation filters",* IEEE Trans. on signal processing, Vol 45, N° 2, pp 457-467, 1997.

[6] S.K. Mitra and J.F. Kaiser, *Handbook for digital signal processing,* John Wiley & Soon, 1993.

[7] V.K. Ingle and J.G. Proakis, *digital signal processing using MATLAB,* book ware companion series, 2000.