Residual Stress Measurement by the Hole-Drilling Strain-Gage Method: 
Influence of Hole Eccentricity

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Abstract - The accuracy of the hole-drilling strain-gage technique, the most widely used method for residual stress measurement, is directly related to: 1) the eccentricity between the hole center and rosette center, respectively, and 2) the accuracy of corrections for integration tendency of the strain gages. Although the drilling tool is optically aligned, the eccentricity always can not be zero and it is a source of error, affecting the accuracy of this technique. An equation for correction of the data affected by hole eccentricity has been proposed by A. Ajovalasit, but it is not precise enough, because it is founded on the wrong hypothesis that in the strain gage area there is a uni-axial state of stress, instead bi-axial one. The paper presents a new equation for data correction, which take account of the plane (bi-axial) state of stress in the strain gages area. After the hole-drilling and data acquisition, the hole eccentricity will be measured, using an optical microscope. Knowing the eccentricity, the data will be corrected using the proposed equation. This equation is more precise, because it is founded on a realistic hypothesis (bi-axial state of stress in the strain gages area).

I. Introduction

A predominant factor contributing to structural failure of machine parts may be the residual stresses. Hole drilling is the most widely used method for residual stress measurement [1], [2]. ASTM E837-01 is the accepted standard for residual stress measurement by the hole-drilling method [3]. A detailed explanation of this ASTM standard is presented in the Technical Note TN 503 by the Measurement Group [4]. The accuracy of this technique is directly related to the precision with which the hole is drilled through the center of strain gage rosette. In practice, the required alignment precision is to within 0,025 mm [3]. In order to obtain a so small eccentricity, the drilling tool is optically aligned so that its drilling axis is precisely positioned over the target at the center of the strain gage rosette. But, although the tool is optically aligned, the eccentricity can not be zero and eccentricity of the hole is a source of error, affecting the accuracy of this technique [5, 6, 7]. An equation for correction of the data affected by hole eccentricity has been proposed by Ajovalasit [8], but it is not precise enough, because it is founded on the wrong hypothesis: in the strain gage area there is a uni-axial state of stress, instead bi-axial one. The paper presents a new analytical method for experimental data correction, in order to diminish the errors introduced by the hole eccentricity and to improve the accuracy of the hole-drilling technique. This method take account of the plane (bi-axial) state of stress in the strain gages area. The case of the through hole is studied.

II. Description of the actual methods

Rendler and Vigness [5] observed for the first time that “the accuracy of the method… will be directly related to the operator’s ability to position the milling cutter precisely in the center of the strain gage rosette”. More recent works have quantified the error in calculated stress due to eccentricity of the hole [9, 10]. In the conditions of a non-uniform stresses distribution around the hole, the integration tendency of the strain gages is an important source of error [9]. This error can be corrected using the average theorem from the integral calculus.
The output of the strain gage is proportional with the average strain on the grid area \[11\].

In the non-uniform axial state of stress, the W. Soette and R. Van Crombrugge method \[11\] can be used in order to correct the experimental data affected by errors. The output of the strain gage will be

\[
\bar{\varepsilon} = \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} \varepsilon_s(x)dx
\]

where:
- \(x_2 - x_1 = GL\) is the length of the strain gage grid;
- \(\varepsilon_s(x)\) is the strain in the direction of the strain grid filaments.

In the non-uniform bi-axial state of stress, the R.G. Boiten and Ten Cate method \[11\] can be used in order to correct the experimental data

\[
\bar{\varepsilon} = \frac{1}{S} \int S \varepsilon_s(x, y) dS
\]

where
- \(S = \text{strain gage grid surface}\).

In the paper \[8\], Ajovalasit presented an analytical method that can be used in order to correct the experimental data affected by the hole eccentricity. Because the mathematical problem is complex one, he used the Equation (1) instead Equation (2). Using this way, we must do only a usual integral, instead a double one but, of course, the precision of obtained equations is affected. The present paper use the Equation (2) in order to obtain a better formula for experimental data correction (affected by the hole eccentricity).

### III. Presentation of a new method

In Fig. 1, a strain gage grid near a hole with eccentricity is presented. The wrong position of hole has the center in \(O'\), instead of \(O\). In Fig. 2 and Fig. 3, the gage grid reported at its own system of axis is presented. The average value of the strain gage output can be determined using the R.G. Boiten and Ten Cate method

\[
\bar{\varepsilon} = \frac{1}{S} \int dx \int_{-y_1}^{y_2} \varepsilon_s(x, y) dy
\]

\[
\bar{\varepsilon} = \frac{1}{S} \left[ \int_{x_1}^{x_2} dx \int_{-y_1}^{y_2} \varepsilon_s(x, y) dy + \int_{x_1}^{x_2} dx \int_{y_1}^{y_2} \varepsilon_s(x, y) dy \right]
\]

For a symmetric function (as \(\varepsilon_s\) in the area near a hole in a plate in tensile), we can write

\[
\int_{-y_1}^{y_2} \varepsilon_s(x, y) dy = \int_{0}^{y_2} \varepsilon_s(x, y) dy
\]

From Eq. (4) and (5) we can write

\[
\bar{\varepsilon} = \frac{1}{S} \left[ \int_{x_1}^{x_2} dx \int_{0}^{y_2} \varepsilon_s(x, y) dy + \int_{x_1}^{x_2} dx \int_{0}^{y_2} \varepsilon_s(x, y) dy \right]
\]

and

\[
\bar{\varepsilon} = \frac{1}{S} (S^* \bar{\varepsilon}^* + S' \bar{\varepsilon}')
\]

where the grid surface is

\[
S = S' + S^*
\]
The main residual stresses are calculated for a plate in a plane state of stress, with a through hole in it, function of $a$ and $b$ coefficients, which take account of integration tendency of strain gages. The two coefficients are well known from literature [10] and are determined for a centric hole (with the center in O). For this particular situation, we have (Fig. 2, Fig. 3)

\[ y_1 = y_2 = \frac{GW}{2} \]  
(9)

\[ S' = S'' = \frac{S}{2} \]  
(10)

\[ \varepsilon' = \varepsilon'' = \varepsilon \]  
(11)
Fig. 3. Strain gage grid in x”-y” system of reference

In order to calculate the residual stresses for a through centered hole, \( \alpha \) and \( \beta \) coefficients are used (fig. 4):

\[
\alpha = \frac{2}{w(r_2 - r_1)} \cdot \frac{1}{r^2} \cdot (\theta_1 - \theta_2) \tag{12}
\]

\[
\beta = \frac{2(1 + \nu)}{w(r_2 - r_1)} \cdot \frac{1}{r^2} \left[ \frac{2(1 - \nu)}{1 + \nu} \cdot (\theta_1 - \theta_2) + \left( \sin 2\theta_1 - \sin 2\theta_2 \right) - \frac{1}{r^2} \left( \sin 2\theta_1 \cdot \cos^2 \theta_1 - \sin 2\theta_2 \cdot \cos^2 \theta_2 \right) \right] \tag{13}
\]

where

\[
w = \frac{2GW}{D}
\]

\[
r_1 = \frac{2R_1}{D}
\]

\[
r_2 = \frac{2R_2}{D}
\]

\[
r = \frac{D}{D_0} = \frac{D}{2R_0}
\]

\[D = R_1 + R_2\]

In order to correct the errors introduced by hole eccentricity

For an eccentric hole, the correction can be made simply using Eq. (7) and coefficients \( \tilde{\alpha} \), \( \tilde{\beta} \), instead of \( \alpha \) and \( \beta \). The \( \tilde{\alpha} \), \( \tilde{\beta} \) coefficients are calculated for two strain gages near a centric hole, having the grid width \( 2(GW/2 + \Delta GW) \) and \( 2(GW/2 - \Delta GW) \) respectively

\[
\tilde{\sigma} = \frac{1}{GW} \left[ \left( \frac{GW}{2} + \Delta GW \right) \sigma' + \left( \frac{GW}{2} - \Delta GW \right) \sigma^* \right] \tag{15}
\]
\[ \vec{b}_e = \frac{1}{GW} \left[ \left( \frac{GW}{2} + \Delta GW \right) \vec{b}' + \left( \frac{GW}{2} - \Delta GW \right) \vec{b}'' \right] \]  

(16)

where \( \vec{a}', \vec{b}' \) and \( \vec{a}'', \vec{b}'' \) are determined using (12) and (13) equations, but applied for \( S' \) and \( S'' \) respectively surfaces.

Developing the equations we obtain

\[ \vec{a}_e = \frac{R_0^2}{GW \cdot GL} (\Delta \theta' + \Delta \theta'') \]  

(17)

\[ \vec{b}_e = 2(1-\nu)\vec{a}_e + \frac{R_0^2 (1+\nu)}{GW \cdot GL} \left( (\sin 2\theta' - \sin 2\theta''_1) + (\sin 2\theta'' - \sin 2\theta''_2) - \frac{R_0^2}{2} \left( \frac{1}{R_{1e}^2} \sin 2\theta'_1 \cos^2 \theta'_1 + \sin 2\theta''_1 \cos^2 \theta''_1 \right) \right) \]  

(18)

\[ \frac{1}{R_{2e}^2} \left( \sin 2\theta'_2 \cos^2 \theta'_2 + \sin 2\theta''_2 \cos^2 \theta''_2 \right) \right] \}

where

\[ \Delta \theta' = \theta'_1 - \theta'_2 \]

\[ \Delta \theta'' = \theta''_1 - \theta''_2 \]  

(19)

The method presented above is a very easy way to correct the errors introduced by eccentricity. It is in the same time more accurate, because the Eq. (2), good for a plane stress of state, was used in this demonstration, instead of Eq. (1), good for axial state of stress and used by the actual methods for error correction.

IV. Conclusions

The hole eccentricity of is an important source of error, which affect the accuracy of residual stress measurement by the hole-drilling method. Although in practice the optical alignment is used, the eccentricity can not be zero.

Fig. 4. Centered hole with a strain gage grid
A very easy and more accurate method for errors introduced by eccentricity correction has been presented in this paper. It takes account of the plane (bi-axial) state of stress in the strain gages area reduces the case of the eccentric hole at the well known case of the centric hole.

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References