G. Conclusion

At the end of this procedure we retrieve as the best model the `mbj2` model that uses a second-order plant model and a first order noise model. This model is slightly better than the initial `m3` model. This is mainly due to the more flexible model structure that allowed the plant and noise models to be decoupled. The retrieved model structure corresponds also with the exact models that were used to generate the simulation data.

4.7 FREQUENCY DOMAIN IDENTIFICATION USING THE TOOLBOX FDIDENT

The goal of this exercise is to illustrate more advanced aspects of the identification procedure using the frequency domain identification toolbox FDIDENT (Kollár, 1994). This toolbox is completely directed to the use of nonparametric noise models. A full identification run will be made, starting from the raw data to a final model. Some model selection tools and model tests will be discussed in more detail. Here we illustrate the use of periodic excitation signals. We refer the reader to Exercise 74 to deal with non-periodic excitations.

Exercise 76 (Using the frequency domain identification toolbox FDIDENT)
Goal: Make a complete identification run using the FDIDENT toolbox. The same system as in Exercise 75 is identified, but this time using a periodic excitation.

- Generate the system G_0:
  ```
  [b0,a0] = cheby1(2,10,2*0.25);b0(2) = b0(2)*1.3;
  ```
- Define the noise generating filter:
  ```
  [bNoise,aNoise] = butter(1,2*0.2);
  bNoise = bNoise+0.1*aNoise;
  ```
- Generate a zero mean random phase multisine with a flat amplitude exciting the spectral lines:
  ```
  Lines = [1:NPer/3].
  ```
  Scale the rms value to be equal to 1. Use a period length $N = 1024$, and generate $M = 7+1$ periods. The first period is used to eliminate all transient effects in the simulations ($N_{\text{Trans}} = 1024$).
- Generate y_0, $v(t)$: $y(t) = G_0(q)u_0 + v(t)$ with $v(t) = 0.1G_{\text{noise}}(q)e(t)$ filtered white noise $e(t) \sim N(0, 1)$. Eliminate the first N_{Trans} data points.
- Generate a time domain object that can be imported by FDIDENT, normalizing the sample frequency equal to 1:
  ```
  ExpData = tiddata(y(:),u0(:),1);
  ```
- Start the GUI of FDIDENT (type `fdident` in the command window) and follow the menu in the GUI as explained below.

Observations The successive windows of the GUI are shown and shortly discussed.

H. Main window

The main window allows the data to be imported in the GUI, using either time or frequency domain data. Double click the 'Read Time Domain Data' to open the data importing window.

I. Importing and preprocessing the data

`read time domain data`

In a series of successive steps (see Figure 4-31), the data are

- loaded into the GUI (`Get Data`),

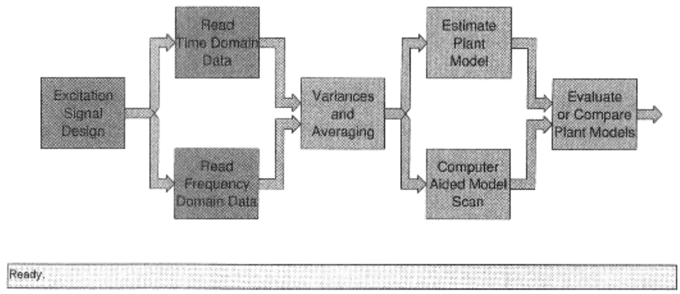

Figure 4-30 Opening window GUI-FDIDENT.

- the successive periods are separated (`Segmentation`),
- and converted to the frequency domain (`Con. to Freq`).
- A first possibility to select the frequencies of interest is offered (`Freq Select`).

The corresponding windows are shown.

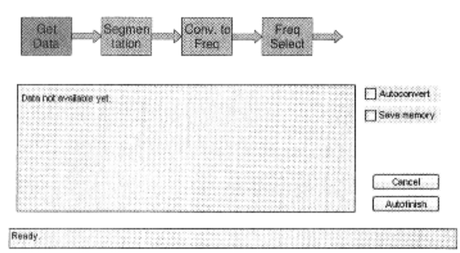

Figure 4-31 Opening window GUI-FDIDENT.

(i) Get the data: The data object `ExpData` that was created in the m-file is loaded.

(ii) Segmentation: Put `period length` to `1024` and click `Apply Periods`.

(iii) Convert to frequency domain: The frequency domain results are shown. The user can select, for example, the FRF, the input–output Fourier coefficients, etc., by making the appropriate choice under the `Type of Figure` instruction.

(iv) Frequency selection: In this window a first selection of the active frequencies to be used in the identification process can be used. We postpone in this exercise this choice, and `select all frequencies` in this step. Notice that the system was not excited above 0.33 Hz.

At the end of these 4 substeps, the Read Time Domain Data block is highlighted indicating that the data are ready to be processed in the next block of the main menu.

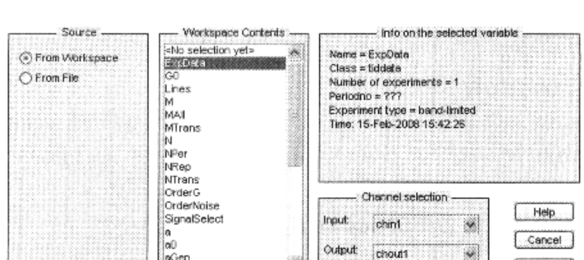

Figure 4-32 Load the time domain object `ExpData`.

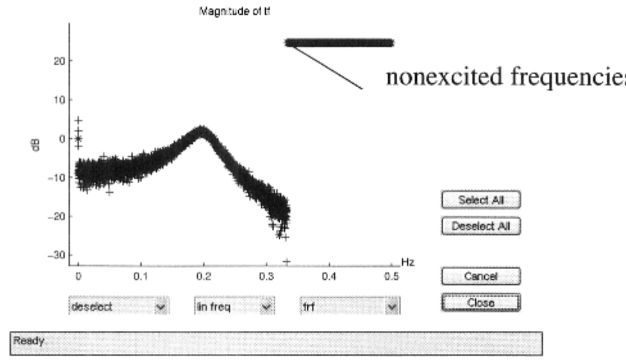

Figure 4-33 Frequency selection: all frequencies selected. The FRF is shown. Notice that above 3.3 Hz, no excitation was present which is indicated in the plot by the straight line.

J. Nonparametric noise analysis

`variances and averaging`

In this block, the data are averaged over the periods: the sample mean and sample variance are calculated. We advice to make the final frequency selection in this block in the window where the input and output data are shown.

In this window, it is very easy to select the excited frequency lines in the input window, using the `frequency selection` button. Selection of the not excited frequencies would not affect the MLE, but it would become more difficult to generate good initial estimates of the system parameters to start the nonlinear search.

Once the frequencies are selected, we are ready to start the identification step, as is visible in the main menu.

K. Parametric identification step

`Estimate plant Model`

or

`Computer Aided Model Scan`

All the information is now available to start the parametric model estimation step. In the toolbox, the sample maximum likelihood (SMLE) is used, minimizing the cost function (4-33). First a series of simplified cost functions is minimized to generate starting values (hidden for the user). The SMLE cost function for each of these parameters is calculated, and the best result (lowest cost function) is retained to start the nonlinear search. The user can select a single

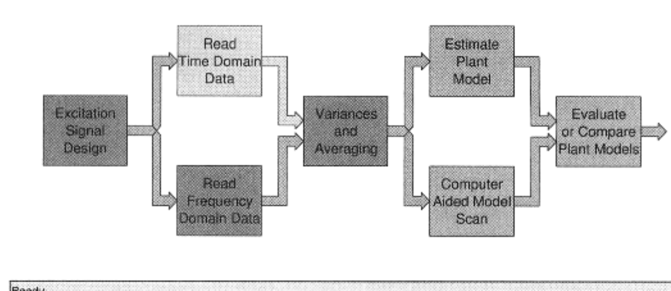

Figure 4-34 The time domain data are imported and transformed to the frequency domain.

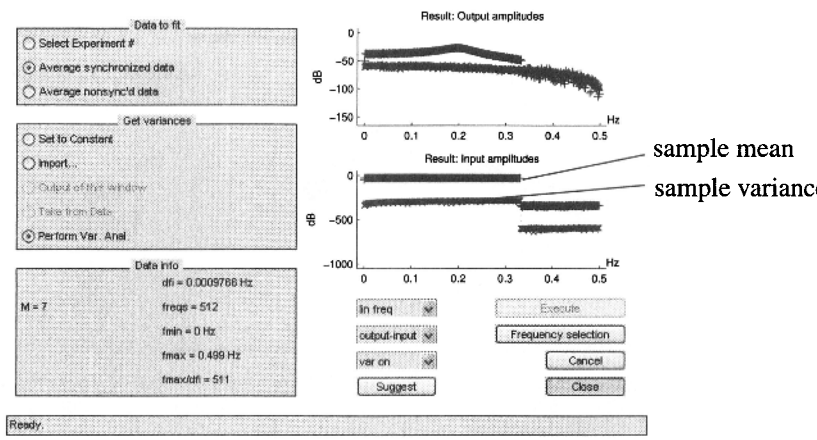

Figure 4-35 Sample mean and sample variances of the input and the output calculated in the nonparametric preprocessing. Notice that the noise level for the input is 300 dB below the actual input. This is the MATLAB® calculation precision, corresponding to 15 digits (20 dB/digit).

model, or a whole bunch of models with different orders can be scanned. The last option is chosen in this exercise because we will illustrate also the model selection procedures. Opening the `Computer Aided Model Scan` offers a number of user choices (see Figure 4-37). In this window, the user has to select the nature of the model (for example discrete or continuous time), the orders to be scanned (a selection - deselection tool is available). The discussion of the other optional choices is out of the scope of this book, and the user is referred to the `help` functions of the GUI. The results are accessible in the `Evaluate or Compare Plant Models window`.

Some of the available results are discussed in the next section.

L. Evaluation of the estimated models

`Evaluate or Compare Plant Models window`

(i) Comparing the estimated models

Once the estimates are available, it is tempting to select as "best" model the one corresponding to the lowest cost function value, but it will be shown that this is not the best or even a good strategy (see also Exercise 11 in Chapter 1).

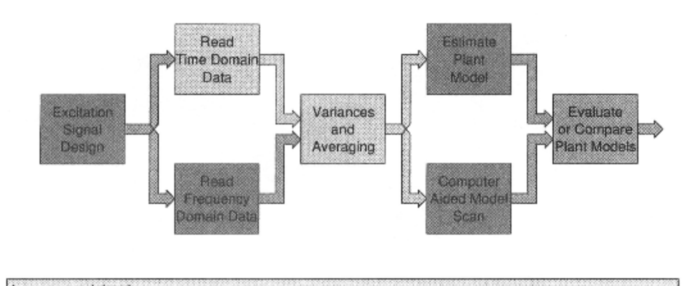

Figure 4-36 The nonparametric preprocessing step is finished. The data are ready to start the parametric identification step.

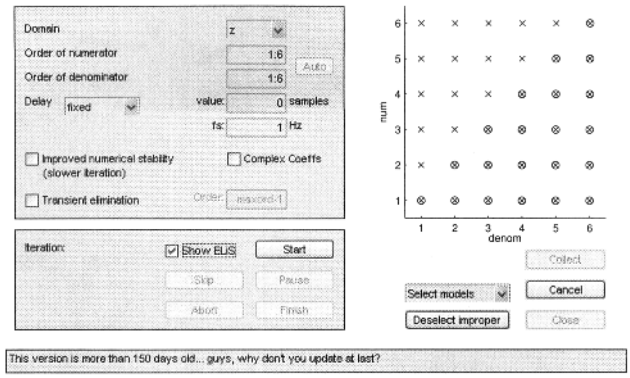

Figure 4-37 Preparation for the computer-aided model scan. The user has to select
- continuous or discrete time model,
- selected set of model orders to be scanned.

In this run, the lowest cost function was obtained for the model 5/5 (5 zeros, 5 poles). One could expect that the model 6/6 would do better than the 5/5 model, because the latter is a subset of the 6/6 class of models. Since the cost function for the 6/6 model is larger than that of the 5/5 model, it shows that the program got stuck in a local minimum for the 6/6.

Notice that the observed cost functions are close to the theoretical expected value (number of frequencies - $n_\theta/2$, with n_θ the number of free parameters in the model, for example for a 2/2 model, $n_\theta = 5$ (the model is invariant with respect to a scaling of all parameters). This is an indication that the models are reasonable, the remaining residuals can be explained by statistical properties of the noise. In the value of the cost function, there is no evidence of the presence of model errors. A cost function that is much larger than the theoretical value, is a strong indication of model errors. A cost function that is significantly smaller than the theoretic value is an indication for a wrong nonparametric noise model (e.g., the presence of a correlation of the noise over the frequencies).

In Figure 4-39, the 2/2 model is compared to the 5/5 one. The FRF and the amplitude of the complex errors $|G(j\omega_k) - G(j\omega_k, \hat{\theta})|$ is shown. These plots indicate that the behavior is quite similar. This points in the direction that 5/5 may be a too complex model for the data.

(ii) Selecting the best model using a model selection tool

In Chapter 1, Exercise 11, we learned that it is not always a good idea to choose the model corresponding to the lowest cost function. This can result in a higher model variability

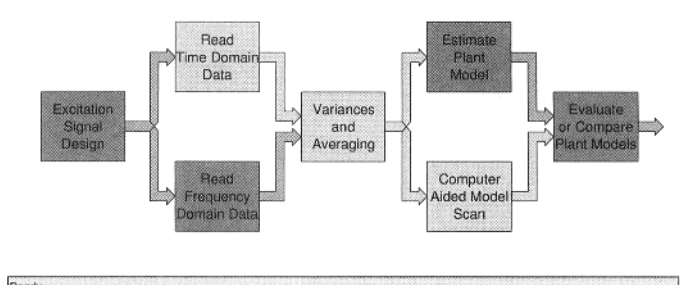

Figure 4-38 Evaluation and comparison of the estimated models: `Evaluate or Compare Plant Models` window.

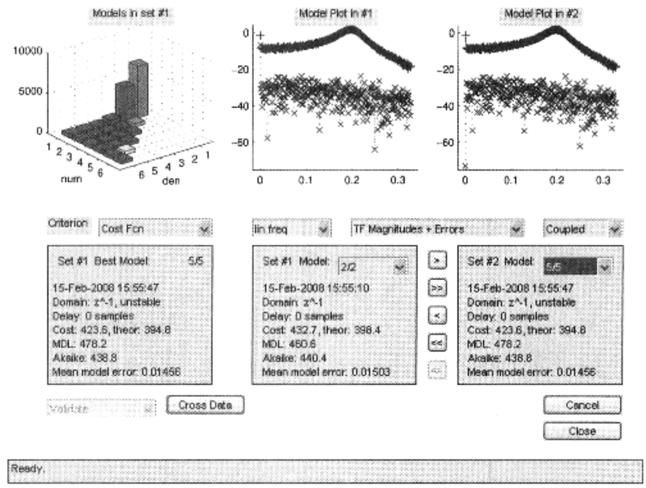

Figure 4-39 `Evaluate or Compare Plant Models` window
`TF Magnitude + Erros` is selected

due to an increased noise sensitivity of complex models (see Exercise 10). In the previous section it was observed that 5/5 may be too complex. In order to make a better choice, model selection tools are developed that balance the model complexity versus the model variability by adding a penalty factor for the complexity to the cost function (4-33) (see also Excercise 11). The Akaike information criterion (AIC) or the minimum description length (MDL) are two popular tools that start from the weighted least squares cost function V:

$$V_{\text{AIC}} = V\left(1 + \frac{2n_\theta}{N}\right), \quad V_{\text{MDL}} = V\left(1 + \frac{2n_\theta \log N}{N}\right). \tag{4-42}$$

It can be seen from (4-42) that for the same value of the cost function V, a more complex model results in a higher AIC or MDL criterion. In Figure 4-40, the MDL criterion selects the 2/2 model as the best one.

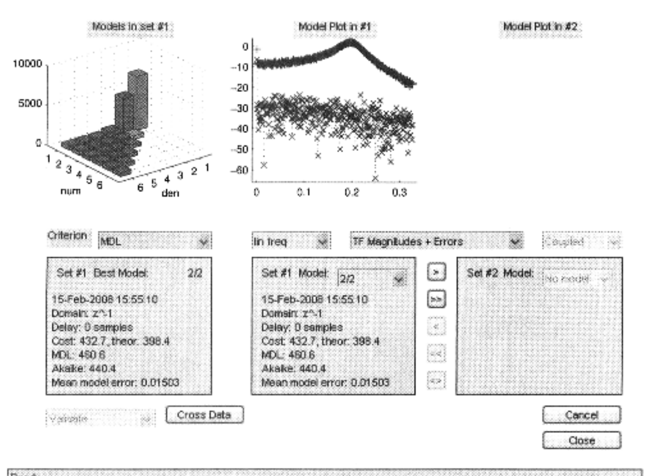

Figure 4-40 `Evaluate or Compare Plant Models` window
`TF Magnitude + Errors` is selected; MDL criterion is selected

(iii) Residual analysis

The residuals are that part of the data that the model could not reproduce. Since we have access in the frequency domain to good estimates of the FRF $G(j\omega_k)$, a lot of information can be gained by analyzing the residuals between the measured and modeled FRF:

$$\varepsilon_F(k) = \frac{G(j\omega_k) - G(j\omega_k, \hat{\theta})}{\sigma_G(k)}. \quad (4\text{-}43)$$

If no model errors are left, ε_F should be white (Gaussian) noise. This is no longer so if some dynamics are missed (under modeling). Since these model errors have a smooth behavior, a correlation becomes visible that can be detected in a correlation test. Notice that this test does not protect against overmodeling, only under-modelling is detected. In Figure 4-41, the correlation analysis is shown for the 2/2 model where no statistical significant correlation is visible. Also the 1/2 model is analyzed, and here it is obvious that the residuals are strongly correlated, which is a very strong indication for model errors. This is also confirmed by the much larger cost function of this model.

(iv) Pole-zero cancellation

What happens with the extra poles-zeros of the 5/5 model if a 2/2 model does fit the data well? Figure 4-42 shows the answer to that question: the 3 additional poles are canceled by the 3 additional zeros in pole-zero pairs that almost completely coincide. In order to be sure that within the uncertainty bounds the pole and zero coincide, a statistical test would be needed, keeping also in mind that often a strong correlation between these poles and zeros is present. However, if the pole-zero plot shows very close pole/zero pairs, then these are good candidates to be eliminated in an order reduction step without affecting the quality of the estimated model.

The presence of these pole/zero pairs affects also the behavior of the uncertainty of the model, which is visualized in the next section.

(v) Cloud of models

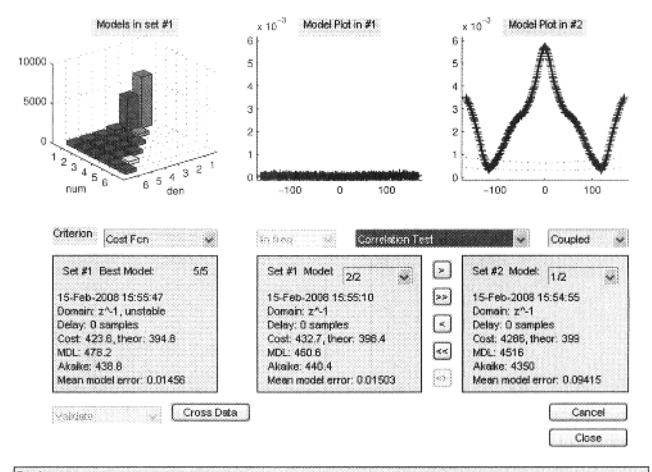

Figure 4-41 Evaluation and comparison of the estimated models: `Evaluate or Compare Plant Models` window; `correlation analysis` is selected.

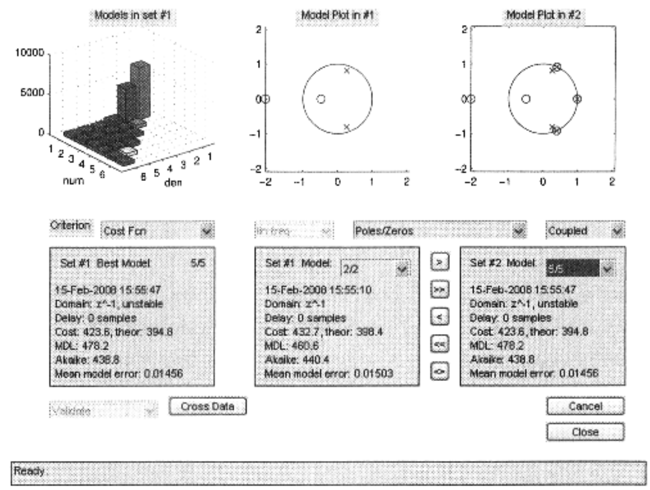

Figure 4-42 Evaluation and comparison of the estimated models: `Evaluate or Compare Plant Models` window `pole-zero cancellation` is selected.

To get an idea of the variability of the model, many possibilities exist. Before we have seen that uncertainty bounds can be generated (see Exercise 69). An alternative is to draw a cloud of models. From the estimated model parameters and covariance matrix, a series of model parameters are generated within the 95% uncertainty bounds, and the corresponding FRF is drawn for each of these. This results in a "cloud of models" that gives a very good visual impression about the noise sensitivity of the estimated FRF (or the poles and zeros). Because this approach relies less on intermediate realizations, it can give a more realistic impression of the system properties. In Figure 4-43, the cloud of models is shown for the 2/2 and the 6/6 model. It can be seen that the cloud of the 6/6 model is thicker than that of the 2/2 cloud. Also spikes can be seen in the 6/6 cloud, which is a very typical phenomenon that indicates the presence of coinciding pole/zero pairs, and hence a particular indication of overmodeling.

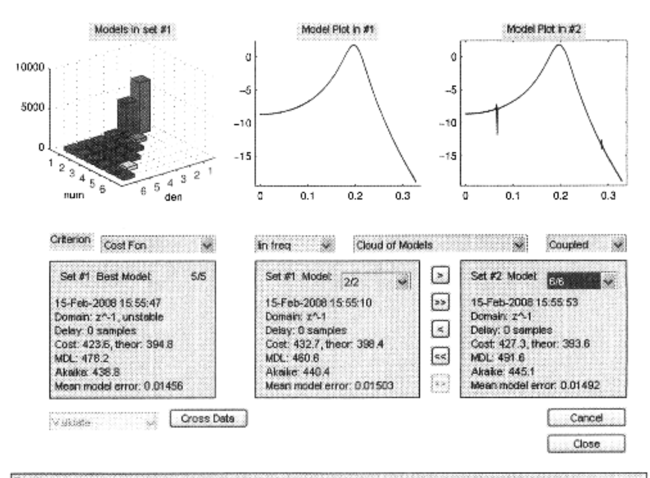

Figure 4-43 Evaluation and comparison of the estimated models: `Evaluate or Compare Plant Models` window; `Cloud of Models` is selected.