

# Correspondence

## Another Step Towards an Ideal Data Acquisition Channel

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**Abstract**—In a recent paper [1] a method has been presented for the design of amplitude- and phase-compensating IIR filters. The procedure is based on inversion of the transfer function, identified in the  $z$ -domain. Unstable poles are reflected into the unit circle, and the spoiled phase response is equalized again using allpass sections. This paper points to the fact that this method is not optimal in the sense that it does not reach the minimum error for a given order (or provides the desired error compensation by using a too high order). By refitting the compensating filter, a lower order filter is obtained with the same performance. In the investigated case the order is reduced from 34/34 to 26/26.

### I. INTRODUCTION

In a recent paper [1] a method has been presented for the design of amplitude- and phase-compensating IIR filters. The procedure is based on inversion of the transfer function of the anti-aliasing filter, identified in the  $z$ -domain. Unstable poles are reflected into the unit circle, and the spoiled phase response is equalized again using allpass sections. The given anti-aliasing filter had originally about 0.2 dB amplitude error in the passband and more than 50° phase error near the center of it (Fig. 1). The complex error has been reduced to be less than 0.01 dB, using a 14/14 IIR filter for the amplitude compensation and a 20/20 one for the phase equalization. Thus the resulting compensation filter has the order 34/34.

Obviously, the compensation procedure should consist of two steps. First, we have to determine the transfer function of the anti-aliasing filter from noisy measurements. This is an *identification problem*. Second, a digital filter is to be designed for compensation of the identified transfer function. This is a *filter design problem*.

The correct solution of the identification task is to fit the measured data by an appropriate order  $s$ -domain rational form. Since the noise may be assumed to be Gaussian, this fitting should be done in LS sense (see, e.g., [2], [3]). On the other hand, the filter is to be designed in the  $z$ -domain, preferably in minimax sense. The design can also be done in LS sense, but the maximal complex error of the compensation may be large. Consequently, the method presented in [1] can be refined, using an  $s$ -domain initial fitting and minimax filter design. However, in the investigated case this will not result in a dramatic change. Since the filter has a steep rolloff, aliasing will cause no problems, and having a sufficient number of measured complex frequency amplitudes (about 50 points, that is, considering both the real and imaginary parts, about 100 equations), the estimated  $z$ -domain transfer function with  $2 \times 15 - 1 = 29$  parameters does not have too much freedom to follow the noise: The result is practically the same in the passband as that of

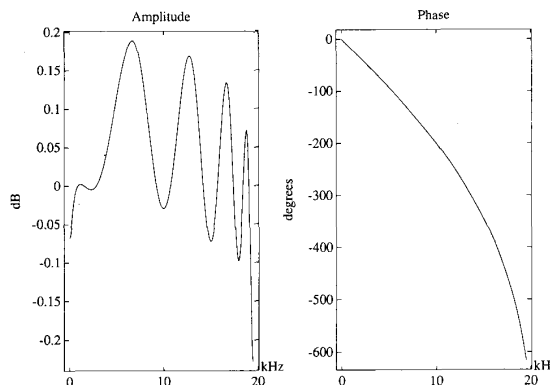


Fig. 1. Passband of the anti-aliasing filter.

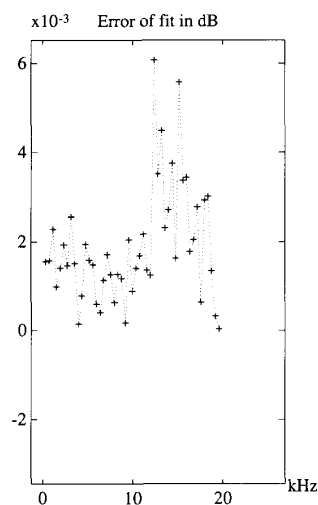


Fig. 2. Complex relative error of the 14/14 fit.

the  $s$ -domain identification. The compensation error can be somewhat decreased if designing the phase equalizer in minimax sense, but to our experience, the decrease of the error is, in the investigated case, not more than 20–30%.

Once an identified transfer function is obtained, the residual error can be studied (see Fig. 2). This error has a random component and a systematic one (it is the sum of the measurement noise and a modeling error). The detailed analysis of the modeling error needs a more thorough analysis. For the purpose of this study we assumed that the modeling error of the 14/14 fit is less than 3 mdB (see [1]), and demanded from the further fittings not to introduce more additional complex error than 7 mdB, with respect to the 14/14 fit. The worst-case total compensation error (10 mdB) corresponds to 0.12% amplitude error or 0.066° phase error.

It is clear from the description of the above procedure that it is suboptimal only, since the two steps (amplitude compensation and phase equalization) are done separately, and the goal of the second step is different from that of the first one. Moreover, both fits are

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made in LS sense, thus the error of the fit may be larger at some part of the passband than elsewhere. These facts lead to the conclusion that the above results can certainly be improved: It is possible to decrease the error using the same order, or the same error can be achieved with a lower order.

## II. POSSIBILITIES OF LOWERING THE ORDER

To decrease the error of the first fit, the easiest way is to take the parameters of the designed 34/34 filter as starting values, and iterate from this towards a LS complex error or towards the minimax complex error, maintaining the stability of the filter during the whole procedure. With some luck, the algorithm given in [1]–[3] will reach a stable solution, otherwise it can be modified to maintain stability. Using the reweighted LS technique (see, e.g., [4]), where for the FIR case a method is given, which can be extended also for IIR filters) a minimax fit can also be achieved. Another possibility is to use the minimum  $p$ -error method of Deczky ([5]).

In this correspondence we investigate the second possibility: to decrease the order, maintaining the same maximal error. This is of practical importance, since by this the load of the arithmetic processor can be decreased.

An obvious solution is to start with a fit of smaller order than 14/14. However, since this order follows from the structure of the input channel (see [1]), a lower order might result in bad modeling: one can never be sure which zeros and poles will be eliminated from the ones in the 14/14 fit. We may also allow larger phase error in the phase equalization step, and iterate later towards a smaller error, but to our experience the amplitude compensation stage is less likely to take over the task of a part of the phase equalizer, than vice versa. Thus we chose a different approach: We tried to directly design a new IIR filter, which compensated both the amplitude and the phase, with a lower order than 34/34.

## III. DESIGN OF THE LOWER-ORDER IIR FILTER

For the design of the lower-order IIR filter we again have a few options. A possibility would be to fit directly the measured passband data, similarly to the first step in [1]. However, our attempts to find such a fit using the available passband information failed, because even by using the same delay as for the phase corrector, some of the poles were always unstable (usually near to the edge of the passband). We also observed very different stopband curves, which is not surprising since these are extrapolated from the noisy passband data. Thus, we decided to provide *stopband information*. We extended the set of complex amplitudes to be fitted by stopband zeros with small fitting weight, but the result was unstable. Then we created a new data set by taking samples of the frequency response of the well-fitted (but unstable) 14th-order compensation filter (see Fig. 3), and tried to fit this by filters of order less than 34/34, but with the same delay, that is, by the form  $z^7 H(z)$ . By appropriate choice of the weights (the stopband need not be fitted very exactly, moreover, with uniform weighting the error around zero was somewhat larger than elsewhere), a 26/26 filter was obtained with practically the same fitting error as the 34/34 one. Even a 24/24 filter could be designed with less than 10 dB fitting error, and a 22/22 with a 25 dB fitting error.

It may be interesting to throw a glance onto the pole/zero patterns (Fig. 4): The poles far from the unit circle and two of the three pole/zero pairs (almost coinciding pole and zero) disappeared, and their effect was substituted by a slight rearrangement of the rest. The pole/zero pairs were also present already in the original 14/14 filter, but there the order could not be decreased without a significant increase of the error: This filter has not enough freedom to "simulate" the effect of these poles and zeros, if they

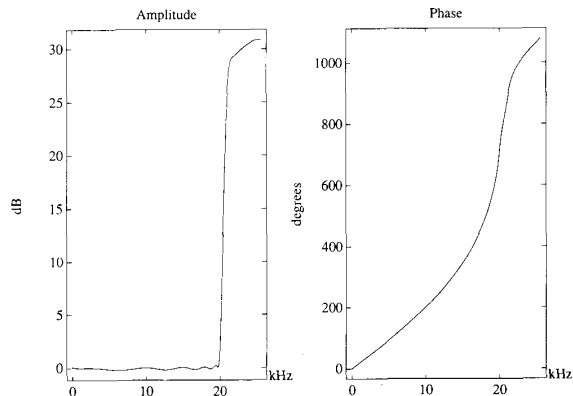


Fig. 3. Magnitude and phase response of the inverted  $z$ -domain IIR filter, order: 14/14.

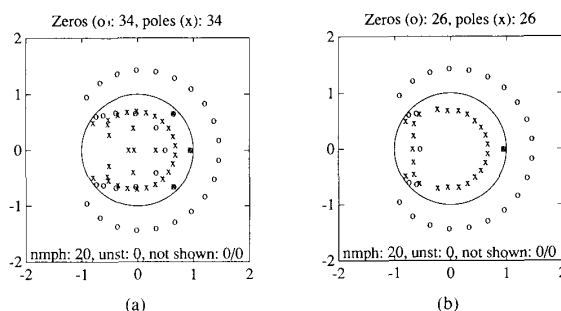


Fig. 4. Pole-zero patterns of the compensating filters: (a) the 34/34 filter (stabilized 14/14 fit, cascaded with a 20/20 phase equalizer); (b) the reduced-order filter (26/26).

are missing. In the 24/24 fit the last pole/zero pair also disappears, but this is already at the cost of a slight increase of the error.

With the above technique we did not succeed in further reducing the order without a significant increase in the error. However, it still remains an open question whether even lower order stable filters exist with similar performance.

## IV. CONCLUSIONS

It has been illustrated in the example of the compensation of the input channel of a dynamic signal analyzer that while the straightforward method (fitting—stabilization—phase correction) works well, its result is not optimal: The same performance can be achieved also by using lower order IIR filters. The order was decreased from 34/34 to 26/26.

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