Processing of bidirectional exponential stimulus in ADC testing

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ADC histogram test methods with exponential stimulus are ADC test methods alternative to sine wave testing. Exponential stimulus test methods published until now were based on simple, single-component exponential pulses. This can sometimes partially mask non-linearity of ADC transfer characteristics. Moreover, estimation of pulse parameters in time domain requires memorizing and processing long records. The new approach is based on the exponential ADC stimulus with two or more different exponential components, e.g., rising and falling slopes of exponential pulses that can be generated very simply and with low costs. Moreover, the different way of signal processing using histograms instead of time record is introduced. Unknown parameters needed for estimation of INL and DNL are calculated by LS fitting using simplified Newton method. The new test method was verified by simulations and experiment.

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1. Introduction

The deviation of real ADC transfer characteristic from the ideal one is described by differential and integral non-linearity (DNL and INL). These parameters can be tested by standardized methods [12,13], which are generally categorized into two groups – static and histogram methods. The static method requires high accuracy DC calibrator and voltmeter and test procedure consumes a lot of time. The standardized histogram method based on sine wave histogram analysis is faster and for this reason it is more frequently applied. The bottleneck of the standardized method is requirement on extreme sine wave purity especially for ADC with high resolution. Generation of such sine wave requires expensive instrumentation and therefore it is not convenient for low cost applications, e.g., ADC self test in embedded systems.

This fact has been a challenge for many researchers to suggest various improvements and alternative ADC test methods [1–6]. One of those alternative nonstandardized methods is the method based on simple exponential stimulus [7–9] which can be generated very simply by discharging of capacitor across a resistance on ADC input. The advantages of such an exponential signal are:

– Simple generation of high quality stimulus, close to the ideal one.
– Simple and cheap generator circuitry in comparison with harmonic signal generator.
– Easy implementation convenient for the implementation in ADC chip or for embedded systems.

According to [7] the acquired digital samples on the output of ADC under test are memorized and processed by the least squares (LS) fitting in time domain. The fit allows estimation of input exponential waveform parameters. Following, DNL[k] and INL[k] are calculated from the comparison of real measured code histogram and cumulative histogram with the ideal ones calculated from the estimated parameters of stimulus.

The exponential stimulus ADC histogram test method [7] seems to be very promising not only because of the simplicity of stimulus generating circuitry [10] but also because of its good noise robustness [11]. The main disadvantage of the method in [7] is a threat of masking some
nonlinearities in ADC transfer characteristics as it can be demonstrated by a simple simulation. (Figs. 1–3). Fig. 1 shows the modelled INL of a simulated ADC. Fig. 2 shows the INL achieved by simulated test of the ADC according to [7] using simple monotonic exponential test signal. The difference between the modelled INL and the measured one (error of testing) is shown in Fig. 3.

The measured INL differs from the origin modelled INL – here the maximal error is about 0.5 LSB. The source of the error is in the LS fitting of distorted data. Its cause is inherently the LS cost function. In general LS fitting does not calculate the original input exponential stimulus but yields a stimulus that has the smallest LS fit error for non-linearly distorted samples taken from ADC output. The core problem is that the method cannot distinguish between changed time constant and erroneous INL if the effect of the INL is similar for the given signal. This could also be eliminated by a few measurements with changed amplitude: e.g., fitting full, 50% low, 50% middle, 50% high. If fit to the lower and higher part of the excitation is different from that of the full fit, this phenomenon appears.

The other bottleneck of the method [7] is the need to process long records of samples by the LS fitting algorithm that requires a deep memory and a calculation power.

2. Models of bidirectional exponential signal

The disadvantages of basic method in [7] led the authors to a new approach presented in this paper. This new approach is based on the use of exponential signals in the form of pulses with at least two different components (slopes) that differs at least in one parameter of their mathematical representation. The most simple and common practical example of such a pulse with rising and falling component is shown in Fig. 4.

The generated exponential waveform must overload ADC input full scale \([F_1, F_2]\) (Fig. 4). The INL and DNL of ADC under test are estimated only from samples acquired by the ADC inside of its full scale, i.e., the samples with the minimal and maximal codes are excluded from data processing. This condition enables that all switching effects in the control square signals are always out of ADC input range and do not affect results of measurement.

In general the test signal may be in the form of one pulse, periodical or non-periodical train of pulses. In case of train of pulses the generating circuit has to produce all pulses with the same form in each run (time constants and final voltages given by voltage levels of control square waveform). Any time jitter of control square waveform does not affect the test because any time shift of starting point of exponential slope does not affect shape of the slope. The record must cover an integer number of pulses, e.g. the record must be started and finished only when signal is either above or below ADC input range. In the case of periodical or quasi-periodical train of exponential pulses the sampling frequency must be chosen as relatively prime to the pulse rate analogously as in standardized sine wave test [12,13]. For any form – one shot or pulse train the stimulus parts, e.g., rising and falling slope can be simply recognized, separated and regrouped in parallel with ADC output code train processing. Moreover, the segments of signal when the testing signal is out of the ADC input range (code bins 0 and \(2^N - 1\)), i.e., they do not contribute for ADC nonlinearity testing, can be simply recognized and excluded from data processing.

Likewise, as for any other histogram method, the quality of the exponential stimulus must be adequate for testing ADC with a given resolution. As it was studied and proved in [10], the main source of distortion is the capacitor in the pulse shaping circuit that must be carefully chosen. Study [10] also showed that some common types of capacitors used in shaping circuit enable generating exponential stimulus for testing ADC with resolution up to 16 bits. Distortion caused by other components is usually negligible (parasitic properties of common resistors and drift of common nowadays reference sources that may be applied in control square generator). According to [11], noise of reference voltage sources in square generator can be effectively suppressed by a convenient choice of the final voltage values \(B_r\) and \(B_f\) in relation to ADC full range (Fig. 4).

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Therefore the generalized formula without indexes

generates in following for modelling any part of the
ADC input range borders (Fig. 4).

Normalization constants $D_I$ and $D_r$ given by parameters of
stimulus and ADC under test, ensure that functions $P_I(x)$
and $P_r(x)$ fulfill the mathematical condition for distribution
function, i.e., function values of $P_I(x)$ and $P_r(x)$ have to be
within interval $[0, 1]$ for input signal $x$ within ADC input
full-scale range (interval $[F_1, F_2]$).

The formulas are formally identical for rising and falling
exponential stimulus. The only difference is the value
(position) of parameter $B$ with regard to ADC input range.
Therefore the generalized formula without indexes $j$ and $r$ will be used in following for modelling any part of the
complex exponential stimulus.

The general formula (3) for the normalized cumulative
code histograms $H_{cn}(k, B)$ and simple code histograms
$H_{n}(k, B)$ for ideal ADC with resolution $N$ bits and for any
slope of exponential stimulus can be simply derived using
(2) by substitution $x = F_1 + k \cdot Q$, where $Q$ is the nominal
quantization step of ADC.

Here $H(i, B)$ is measured code histogram for a chosen part of
exponential stimulus with parameter $B$. $H_{cn}(k, B)$ and
$H_{n}(k, B)$ are normalized measured cumulative and simple
code histograms, respectively, and $M(B)$ is total number of
hits acquired in code bins $k = 1, 2, \ldots, 2^N - 2$ for chosen part
of exponential signal with parameter $B$. Histograms build-
ing process can be performed immediately during acquiring
samples from ADC so that no long record is needed, e.g., for
two-component exponential stimulus and $N$-bits ADC
only memory for $2(2^N - 2)$ numbers is needed. Histogram
values for the terminal code bins $k = 0$ and $k = 2^N - 1$
are excluded from model as well as from all measurement
because they do not contain any information of ADC nonlin-
erarity. Moreover, an unpredictable number of hits can be
registered in these bins due to unknown and unimportant
time while the stimulus is being out of ADC input range, e.g.,
time interval $[t_2, t_3]$ in Fig. 4. This restriction does not
matter in practical testing and it is equal to correction of
ADC gain and offset error by straight line connecting these
terminal code transition levels, i.e., $\text{INL}(0) = \text{INL}(2^N - 1) = 0$.

ADC DNL and INL for rising or falling part of exponential
stimulus can be calculated from the next expression which
is valid generally for all ADC histogram test methods using
a convenient signal with known distribution function $P(k)$:

$$DNL(k) = \frac{Q(k) - Q}{Q} = \frac{H_u(k) - H_{ud}(k)}{H_{ud}(k)} = \frac{H_u(k) - (P(k) - P(k - 1))}{P(k) - P(k - 1)},$$

$$\text{INL}(k) = \frac{T(k) - T_{ud}(k)}{Q} = \sum_{i=0}^{k} DNL(i) = \text{INL}(k - 1) + DNL(k),$$

for $k = 0, 1, \ldots, 2^N - 1$.  

where $T(k)$ and $T_{ud}(k)$ are real (measured) and
nominal transition code levels, respectively, and $Q(k)$ and $Q$ are real
(measured) quantization step for code $k$ and nominal
quantization steps of ADC, respectively. The models of his-
tograms in (3) do not depend on time constant of exponential
stimulus. It means that parameter $B$ is the only
unknown parameter needed for analysis of histograms
and calculation of DNL and INL using formula (4). More-
over, this parameters can be estimated from measured
normalized histogram $H_{u}(k)$ instead of time record fitting
as it was done in [7]. This fact greatly simplifies DNL
and INL calculation in comparison with [7], where two
unknown parameters \( B \) and \( \tau \) of the exponential stimulus were required to determine ADC INL using simple exponential stimulus.

3. Histogram models for simple exponential stimulus

The basic method how to estimate \( B \) from measured histograms is least squares fit. To simplify the fitting, Eq. (3) can be rewritten into a more simple and general form replacing formally the real transition code levels and the real quantization step \( Q \) in voltage by their formalized values equivalent to ADC binary codes: \( F_1 \sim 0 \), \( F_2 \sim 2^N - 1 \) and \( Q \sim 1 \). Using this formalization, Eq. (3) can be rewritten into following form:

\[
H_{\text{mid}}(k, B) = H_{\text{mid}}(k, b) = D \ln \left( 1 - \frac{k}{B} \right),
\]

\[
H_{\text{mid}}(k, b) = \frac{B}{Q}, \quad D = 1 / \ln \left( 1 - \frac{2^N - 1}{B} \right), \quad (5)
\]

where \( b \) represents normalized parameter of exponential pulse (voltage \( B \)) – the final value of each slope of the bidirectional exponential stimulus (Fig. 4). If needed, the real transition code levels can be calculated from known DNL or INL using at least two real transition code levels measured by another method, e.g., by the static test method [12,13].

The model can be further simplified using the idea published in [14]: the probability of code \( k \) can be approximated by simple rectangular rule (Fig. 5) for numerical calculation of integral instead of analytical integration of density function as follows:

\[
H_{\text{mid}}(k) = \int_{x_{k-1}}^{x_k} p(x)dx \approx a \cdot p \left( \frac{x_k + x_{k-1}}{2} \right) \cdot (x_k - x_{k-1}) = \bar{H}_{\text{mid}}(k),
\]

\[
\bar{H}_{\text{mid}}(k) = p \left( \frac{k}{2^N - 1} \right) (k - (k - 1)) = p(k - 0, 5) = \frac{a_0}{(k - 0, 5)^{2}}, \quad (6)
\]

where \( a \) is a constant ensuring that the normalized cumulative histogram is equal to 1 for code \( 2^N - 2 \). Using this normalization condition the Eq. (6) may be rewritten into the following form:

\[
\bar{H}_{\text{mid}}(k) = \frac{\widehat{S}}{k - \frac{1}{2} - b}, \quad \widehat{S} = \frac{1}{\sum_{i=1}^{2^N-2} \frac{1}{x_i - b}},
\]

\[
\tilde{H}_{\text{mid}}(k) = \sum_{i=1}^{k} \tilde{H}_{\text{mid}}(i) = \sum_{i=1}^{k} \frac{\widehat{S}}{i - \frac{1}{2} - b}. \quad (7)
\]

This model is more convenient for calculation especially for implementation on chip or in embedded self-calibrating systems because it contains only arithmetical operations with negligible calculation error as it was shown in [14].

4. DNL and INL estimation

The novelty of this algorithm is that DNL and INL of ADC under test is not calculated only from simple exponential stimulus [7] but from a multi-component exponential stimulus consisting in general case from \( L \) simple parts having its own various values of constant \( D_l \) (\( l = 1, 2, \ldots, L \)). The most common, simple, and practical case is the stimulus according Fig. 4 consisting from two parts – rising and falling. The calculation of INL can be performed by minimizing of following cost function CF:

\[
\text{min}(CF) = \text{min} \left( \sum_{i=1}^{n-2} (\text{INL}(i, b_r) - \text{INL}(i, b_f))^2 \right), \quad (8)
\]

where \( \phi(b_r, b_f) \) is the cost function of least square fit for INL and \( b_r \) and \( b_f \) are the unknown parameters (the normalized voltage values of rising and falling part of bidirectional exponential stimulus for infinity time). \( \text{INL}(i, b_r) \) and \( \text{INL}(i, b_f) \) are INLs calculated for rising and falling part of bidirectional exponential stimulus (Fig. 4). All those functions can be calculated according to the following expressions:

\[
\text{DNL}(i, b_r) = \frac{H_{\text{mid}}(i) - H_{\text{mid}}(i, b_r)}{H_{\text{mid}}(i, b_r)} = \frac{H_{\text{mid}}(i) - 1}{H_{\text{mid}}(i, b_r)}, \quad \text{DNL}(i, b_f) = \frac{H_{\text{mid}}(i) - H_{\text{mid}}(i, b_f)}{H_{\text{mid}}(i, b_f)} = \frac{H_{\text{mid}}(i) - 1}{H_{\text{mid}}(i, b_f)}, \quad (9)
\]

where \( H_{\text{mid}}(i) \) and \( H_{\text{mid}}(i, b_r) \) are measured normalized histograms calculated for rising and falling slope of stimulus, respectively, \( b_r \) and \( b_f \) are normalized values of parameters \( B_r \) and \( B_f \) according (5). The cost function (8) can be also defined alternatively using DNLs instead of INLs but because of “noisy” nature of ADC DNLs this alternative can lead to less accurate estimation of unknown parameters \( b_r \) and \( b_f \).

Parameters \( b_r \) and \( b_f \) can be estimated from two pairs of measured and normalized histograms, each built for rising and falling part of stimulus using models given by Eqs. (7), (5).

The local minimum of CF is given by system of conditions:

\[
\frac{\partial \phi(b_r, b_f)}{\partial b_r} = 0, \quad \frac{\partial \phi(b_r, b_f)}{\partial b_f} = 0. \quad (10)
\]

The system (10) is a nonlinear system and therefore can be solved only numerically by an iteration method. We chose Newton method because it gave the best results for similar task in [14]. Newton iteration process for (10) is given by (11). At each iteration step the increments \( h_r \) and \( h_f \) have to be determined by solving linear system with partial derivatives calculated at point \((b_r^{n-1}, b_f^{n-1})\):

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The approximation errors are of order $O(h^2)$ and $O(h)$ (see (12)). If we suppose that the values $\phi$ are calculated in signal processing system with the precision $\varepsilon (\varepsilon \approx 10^{-8} - 10^{-7}$ and $\varepsilon \approx 10^{-15} - 10^{-14}$ for ordinary and double precision, respectively), then the reasonable choice of the value $h$ is if we take the same orders of errors in the first and in the second term of right sides of (12):

$$\frac{\varepsilon}{h} \approx h^2, \quad \frac{\varepsilon}{h} \approx h.$$  

For both cases we get $\varepsilon \approx h^3$, hence $h \approx \varepsilon^{1/3}$. For ordinary precision we may take $h \approx 10^{-3} - 10^{-2}$ for double precision the reasonable choice is $h \approx 10^{-5} - 10^{-4}$. The error estimates in (12) will be then $O(\varepsilon^{2/3})$ and $O(\varepsilon^{1/3})$ for the first and second order derivatives, respectively.

Finally the required parameters $b_r$ and $b_f$ are computed in the iteration process (Newton) as follows:

$$b_r^{[n+1]} = b_r^{[n]} - \frac{\partial \phi}{\partial b_r} \frac{\partial^2 \phi}{\partial b_r \partial b_f} \frac{\partial \phi}{\partial b_f} \frac{\partial^2 \phi}{\partial b_r \partial b_f} A,$$

$$b_f^{[n+1]} = b_f^{[n]} - \frac{\partial \phi}{\partial b_f} \frac{\partial^2 \phi}{\partial b_r \partial b_f} \frac{\partial \phi}{\partial b_f} \frac{\partial^2 \phi}{\partial b_r \partial b_f} A,$$

where determinant $A$ is equal to:

$$A = \begin{vmatrix} \frac{\partial^2 \phi}{\partial b_r^2} & \frac{\partial^2 \phi}{\partial b_r \partial b_f} & \frac{\partial^2 \phi}{\partial b_r \partial b_f} & \frac{\partial^2 \phi}{\partial b_r \partial b_f} \\
\frac{\partial^2 \phi}{\partial b_r \partial b_f} & \frac{\partial^2 \phi}{\partial b_f^2} & \frac{\partial^2 \phi}{\partial b_r \partial b_f} & \frac{\partial^2 \phi}{\partial b_r \partial b_f} \\
\frac{\partial^2 \phi}{\partial b_r \partial b_f} & \frac{\partial^2 \phi}{\partial b_r \partial b_f} & \frac{\partial^2 \phi}{\partial b_f^2} & \frac{\partial^2 \phi}{\partial b_r \partial b_f} \\
\frac{\partial^2 \phi}{\partial b_r \partial b_f} & \frac{\partial^2 \phi}{\partial b_r \partial b_f} & \frac{\partial^2 \phi}{\partial b_r \partial b_f} & \frac{\partial^2 \phi}{\partial b_f^2} \end{vmatrix}.$$

The iteration stop condition is $|b_r^{[n+1]} - b_r^{[n]}| \leq \varepsilon_i$, where $\varepsilon_i$ is chosen residual uncondition of approximation. The initial values for this iteration algorithm $(b_r^{[0]}, b_f^{[0]})$ have to be chosen properly (above or below ADC input range) to ensure good convergence of the iteration process. They may be taken from roughly known voltage of the control square pulses in stimulus generator (Fig. 4) or calculated from histograms built for rising and falling part of exponential stimulus using method published in [7] or [14]. The resulting $b_r^{[n+1]}$ and $b_f^{[n+1]}$ are used for calculation of $\text{DNL}_r(k)$, $\text{DNL}_f(k)$, $\text{INL}_r(k)$, and $\text{INL}_f(k)$ of ADC under test according the procedure described hereinabove – Eq. (4) and any model for simple exponential stimulus ((5) or (7)).

In general the calculated INLs and DNLs for rising and falling slope can a bit differ each to other because of residual estimation errors of $b_r$ and $b_f$ and because of a finite number of samples in code histograms. The uncertainty given by the finite number of samples can be partially decreased by the averaging of INLs and DNLs calculated for rising and falling slope of exponential stimulus as follows:

$$\text{DNL}_r(k) = \frac{\text{DNL}_r(k) + \text{DNL}_f(k)}{2},$$

$$\text{INL}_r(k) = \frac{\text{INL}_r(k) + \text{INL}_f(k)}{2}.$$  

Performed experiments described herein below indicate that averaging can also partially decreased error of INL and DNL caused by residual uncertainty of estimation of parameters $b$.

5. Experimental results

The proposed algorithm was verified at first by the simulated measurement on simulated ADC. Nonlinearity of the simulated ADC was given by a modelled INL. INL was chosen to be the input and resulting parameter of simulated measurement because differences of INLs are much better readable than differences of DNLs. Fig. 6. shows measured INL and DNL of ideal linear 8-bit ADC ($\text{INL}_r(k) = 0$ for $k = 0, \ldots, 255$). The other parameters used for simulation were: input range of simulated unipolar 8-bit ADC: 0–255, $M = 2 \times 10^6$ (106 samples for each slope), $b_r = -30$, $b_f = 290$, $\varepsilon = 10^{-6}$, $h = 10^{-3}$. The iteration process started from values $b$ calculated for each slope individually using simple histogram method [14] and finished at values $b_r = -29.996407$, $b_f = 289.996020$.

Fig. 7 shows INL and DNL of the same ideal simulated ADC calculated by averaging of DNLs and INLs from Fig. 6. Simple comparison of Figs. 6 and 7 indicates that the averaging decreased uncertainty of measurement.
given by finite number of samples as well as by residual estimation error of $b$.

The next simulation used simulated nonlinear 8-bit ADC with modelled INL shown in Fig. 8. All other conditions of the measurement were the same as in the previous simulated measurement. Averaged measured DNL and INL are shown in Fig. 9. Fig. 10 shows the difference of modelled and measured averaged DNLs and INLs. The error of measured INL and DNL are caused by the finite number of samples in histograms and by a residual error of estimation of parameters $b$ in fitting.

The proposed algorithm was also verified by the real experimental measurement. The ADC under test was

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14-bit ADC implemented in USB6009 device by National Instruments [15]. The device was tested by the standardized sine wave histogram method [12,13], by the simple unidirectional exponential histogram test method [7], and by the new bidirectional exponential histogram test method.

Fig. 11 shows INL measured by the standardized sine wave histogram test method [12,13]. The sine wave with amplitude 1.2 V and frequency 1.1111 Hz was generated by ultra-low distortion synthesized function generator Stanford Research DS360 [16]. Sampling frequency of USB 6009 was 40 kHz, full scale input range ±1 V and length of record $10^7$ samples.

Fig. 12 shows INL measured using exponential stimulus with time constants $\tau_r = \tau_f = 40$ ms, voltages $B_r = 5$ V, $B_f = -5$ V and at the same other conditions as those for the sine wave test. Fig. 12 shows also difference of INL measured by standard sine wave test from INL measured by the new bidirectional exponential method. The residual differences may be caused by distortion of both signals – sine wave as well as exponential signal and residual estimation errors of parameters $b$.

To compare improvement brought by the new test method, another test using simple exponential stimulus method was performed. Fig. 13 shows INL of USB6009 measured by simple unidirectional (falling) exponential stimulus [7] with parameters $B \equiv -5$ V, $\tau \equiv 40$ ms, and achieved at the same conditions as those in previous tests.
Maximal difference of INL obtained by simple exponential test from INL by sine wave test is about twice bigger than difference between INLs by the new bidirectional exponential test and by sine wave test.

6. Conclusions

The paper presents a new approach to the ADC DNL and INL testing methodology using bidirectional exponential stimulus. Data processing algorithm was proposed with respect to simple implementation in ADC on-board self-testing systems. The method was verified by simulations and experimental measurements in comparison with standardized sine wave histogram test. All performed tests confirmed applicability of the new method for alternative simple and low cost testing of ADC at least up to 14–16 bits. The results achieved by the new proposed methods are much closer to the modelled INL and DNL in simulated test and to INL achieved by standardized histogram test than results achieved by simple exponential method. The new test method can be used in applications, where generation of high quality sine wave is a technology problem, e.g., in embedded systems with self test ADC.

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