REQUIREMENTS OF TIMING AND FREQUENCY SYNCHRONIZATIONS FOR MULTI–USER OFDM ON SATELLITE MOBILE CHANNEL

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Abstract: The performance of Multi–user Orthogonal Frequency Division Multiplexing (OFDM) with raised cosine time domain pulse shaping is studied on the Satellite Mobile channel (SMC). The relationship between the requirement of the synchronization and the roll–off rate (β) of the pulse shaping is explored. In Gaussian and typical Rician (K=10) channels Multi–user OFDM with β=0.2 requires timing and carrier frequency synchronization accuracies of 4% and 2%, respectively, in order to avoid severe degradation due to multi–user adjacent channel interference, while on Rayleigh channels these values must be better than 2% and 1%.

I INTRODUCTION

In a classical frequency division multiplexing (FDM) system, the total frequency bandwidth is divided into I non–overlapping frequency channels. Each channel is used by one user and modulated with a separate symbol sequence. One of the most bandwidth efficient schemes employs Root Nyquist filtering (RNF) where each user occupies one of the I carriers from the set

$$\mathbf{f} = \{f_{ci} = f_o + i\Delta f_c \ : i \in \mathbf{I}\ \} \tag{1}$$

with $\mathbf{I} = \{-\frac{I-1}{2}, -\frac{I-3}{2}, ..., \frac{I-1}{2} \ : I \text{ odd}\}$ and $\Delta f_c = (1+\beta)/T$. In (1), T is the symbol duration and β is the roll–off factor, $0 < \beta \le 1$. The frequency response of each user's signal is given by $G_N(fT, \beta)$ with

$$G_N(x,y) = \begin{cases} 1 & |x| \le 0.5(1-y) \\ \cos\left[\frac{2|x|-1+y}{4y}\right] & \frac{1-y}{2} < |x| < \frac{1+y}{2} \\ 0 & |x| \ge 0.5(1+y) \end{cases} \tag{2}$$

We refer to this scheme as FDM/RNF. A large value of β is not bandwidth efficient, but the smaller the value of β, the more difficult is the filter design. Efficient use of bandwidth can also be obtained without RNF if the frequency spectra of the different users are permitted to overlap. This leads to orthogonal frequency division multiplexing (OFDM).

OFDM is well known as a bandwidth efficient modulation scheme for data communication since the 1960's [1–4]. In classical OFDM a *single* channel is subdivided into many subchannels, so several parallel streams of data are transmitted simultaneously. The spectra of the subchannels are permitted to overlap in order to achieve a high bandwidth efficiency, but the spectra of the different users do not overlap. For a large number of subchannels the Nyquist rate can be approached in the limit. In [1] [2] OFDM with bandlimitted pulse shaping was analyzed in a bandlimitted Gaussian Channel. In [3] [4] OFDM with rectangular pulses was studied for the Cellular mobile fading channel. OFDM can achieve a similar bandwidth efficiency as FDM/RNF, but does not need RNF for each user and an FFT operation or chirp transformation can be applied to simplify the transmitter and receiver. In [3] the duration of each data element is spread out to a value much larger than the average fade duration, making it robust to fading. But this spreading not only causes large time delays, but also requires accurate carrier recover techniques due to the small frequency separation between the subcarriers. In [3], for example, a 7.5kHz channel is divided into 512 subchannels, giving a pulse duration of 68ms at a transmission rate of 7.5k baud and the separation between subcarriers is only about 15Hz.

In this paper we extend the OFDM technique from the single user to the multi–user case, that is, the spectra of different users are permitted to overlap and each user occupies one or several of the parallel frequencies. It is obvious that the multi–user OFDM scheme requires accurate timing and carrier frequency synchronization for all users to avoid adjacent channel interference (ACI). On the down–link (satellite or base station to mobile station), this synchronization is easy to achieve, because the different signals originate from the same source (the satellite or base station). However, for the up–link (mobile station to satellite or base station) synchronization is more difficult to

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realize because the signals come from different sources (mobile stations) and suffer different degrees of multi–path time delay, Doppler frequency shift and Doppler broadening. In this paper we study how much accuracy in the synchronization is required in order to maintain an acceptable level of performance in the SMC. We select MDPSK as the modulation scheme for both OFDM and FDM/RNF because of its simplicity and robustness to fading. The pulse shaping of the new OFDM system is the time raised cosine pulse $p(t) = G_N[t(1+\beta)/T, \beta]$ and the carrier frequency set is the same as (1). In FDM/RNF β is the frequency roll–off factor, while in OFDM it is the time roll–off factor.

The Satellite Mobile Channel (SMC) [5] is approximated well by a Rician fading channel and can be characterized by a parameter ($K=P_s/P_d$) which is the ratio of powers in the specular and diffuse components. When $K=\infty$, the SMC reduces to the Gaussian channel (GC). When K=0, it is a single–path Rayleigh channel (SPRC). In this paper formulas are derived to estimate the bit error probability (BEP) as a function of various system parameters for OFDM over the SMC which includes the GC and the SPRC as a special cases.
In Section II, the signal and system are described. In Section III the formulas to estimate the BEP are derived. In Section IV some numerical results are presented. Finally in Section V we present the conclusion.

II SIGNAL AND SYSTEM

Fig. 1 shows the baseband equivalent model of a system for two of the I users. In what follows the subscript to a parameter denotes a particular channel. Let i=0 be the main user and the others the interfering users, df_i and dt_i be the frequency and timing inaccuracies of user #i, n(t) is zero–mean, white Gaussian noise with power spectral density N_o and $\mathbf{a}_i = \{a_{i,k}\}$ is a sequence of independent, equiprobable, M–ary symbols from the set $\{\pm 1, \pm 3, \ldots, \pm (M-1) : M \text{ even}\}$.

The i–th transmitted signal is

$$s_i(t) = \sum_{k=-\infty}^{\infty} \exp[j2\pi(df_i + f_{ci} - f_o)(t - dt_i) + j\phi_{i,k}]\, p(t - dt_i - kT) \quad (3)$$

where the transmitted phase is

$$\phi_{i,k} = \phi_{i,k-1} + a_{i,k}\pi/M. \quad (4)$$

The output of the channel_i is

$$r_i(t) = \sqrt{2P_{si}}\, s_i(t) + \sqrt{P_{di}}\, s_i(t - t_{di}) \exp(-j2\pi f_{Di} t) \xi_i(t) \quad (5)$$

where P_{si} and P_{di} are the average power of the direct and diffuse components, t_{di} is the time delay between the specular and diffuse component, and $\xi_i(t)$ is a zero mean, complex Gaussian process with autocorrelation $R_{\xi i}(\tau)$ which for an omnidirectional antenna is given by [5][6]

$$R_{\xi i}(\tau) = 0.5\overline{\xi_i(t)\xi_i^*(t-\tau)} = J_0(2\pi f_{Dmi}\tau), \quad (6)$$

where $J_0(.)$ is the zero order Bessel function of the first kind, f_{Dmi} is the maximum Doppler frequency, the overbar denotes averaging and the superscript * denotes complex conjugation.

Let us now consider user #0 (main user) whose signal, after matched filtering, is given by

$$r(t) = \sum_{i\in I} \sqrt{2P_{si}}\; G_{si}(t) + \sum_{i\in I} \sqrt{P_{di}}\; G_{di}(t)\; \xi_i(t) + n_0(t), \quad (7)$$

where $G_{si}(t)$ is the overall transmitter and receiver filter impulse response, i.e.,

$$G_{si}(t) = \sum_{k=-\infty}^{\infty} \exp(j\phi_{i,k})\; g_{si}(t - kT), \quad (8)$$

$$G_{di}(t) = \sum_{k=-\infty}^{\infty} \exp(j\phi_{i,k})\; g_{di}(t - kT), \quad (9)$$

$$g_{si}(t) = \{p(t - dt_i)\; \exp[j2\pi(f_{ci} - f_o + df_i)(t - dt_i)]\} \otimes p^*(-t), \quad (10)$$

$$g_{di}(t) = \{p(t - dt_i - t_{di})\exp[j2\pi(f_{ci} - f_o + df_i - f_{Di})(t - dt_i - t_{di})]\} \otimes p^*(-t), \quad (11)$$

and $\otimes$ denotes convolution.
In (7) $n_0(t)$ is zero mean Gaussian noise with autocorrelation

$$R_{n0}(\tau) = N_o \int_{-\infty}^{\infty} |P(f)|^2 \exp(j2\pi f\tau) df \quad (12)$$

where P(f) is the Fourier Transform of p(t), P_{n0} is the noise power and $P_{n0} = R_{n0}(0)$.

The signal to noise ratio of user #i is related to the energy to noise ratio per bit by

$$SNR_i = \frac{P_{si} + P_{di}}{P_{n0}} = \left(\frac{E_{bi}\log_2 M}{N_o}\right) \Big/ \left[T \int_{-\infty}^{\infty} |P(f)|^2\, df\right]. \quad (13)$$

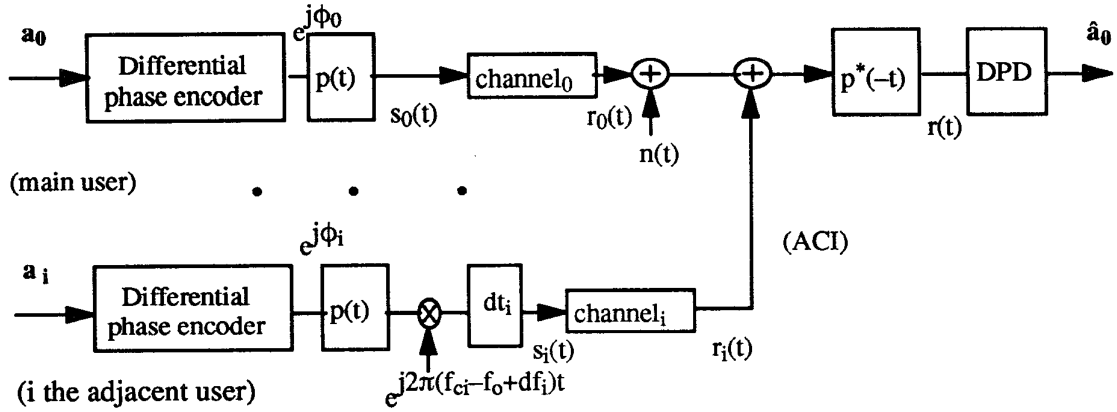

FIGURE 1 –Baseband equivalent model of system with differential phase detector (DPD)

III BIT ERROR PROBABILITY ESTIMATION

We assume that Nf(i) future and Np(i) past symbols from the i–th user affect the decision of $\hat{a}_{0,k}$. Let

$$\mathcal{A}_k = \{a_{i,k-Np(i)}, \ldots, a_{i,k+Nf(i)} : i \in I\} \tag{14}$$

be the set of symbols which interfere with each other. The cardinality of this set is

$$L = M^N , \tag{15}$$

where

$$N = I + \sum_{i \in I}(Np(i) + Nf(i)) . \tag{16}$$

Further, let $\Psi \equiv \{\psi_i = \phi_{i,k-Np(i)-1} : i \in I\}$ be the set of I independent random initial carrier phases which are assumed to be uniformly distributed with probability density function (pdf) $f_\Psi(\psi)$.

The symbol error probability of the main user is then given by

$$P(e) = \frac{1}{L} \sum_{\alpha \in \mathcal{A}_k} \int_{-\pi}^{\pi} P(e \mid \alpha, \psi)\ f_\Psi(\psi)\ d\psi , \tag{17}$$

where $P(e|\alpha,\psi)$ is the conditional error probability which can be computed by the formulas in [5] and [7].

For a small number of interfering symbols, we can compute the exact value of P(e). But for a large number of interfering symbols, this becomes practically impossible. For example, if I=10, M=4 and only two symbols from each interfering user affect the decision $\hat{a}_{0,k}$, then N=20 and $L=1.1 \times 10^{12}$. This is too large to run on any computer. We resort to a technique well suited for this problem and described in [9], which is essentially an optimized variant of importance sampling.

According to [9] P(e) can be estimated by

$$P'(e) = \frac{1}{S} \sum_{s=1}^{S} P(e \mid \alpha_s,\ \psi_s) , \tag{18}$$

where S is the number of sample trials, α_s and ψ_s are independent realizations of $\mathcal{A}$ and Ψ which are generated according to the pdf's of $\mathcal{A}$ and Ψ, $f_{\mathcal{A}}(\alpha)$ and $f_\Psi(\psi)$, respectively.

Because $\overline{P'(e)} = P(e)$, the estimator of the error probability in (18) is unbiased and its normalized error is [9]

$$\varepsilon = \frac{\sqrt{var[P'(e)]}}{P(e)} = \frac{\sqrt{\overline{W} - [P(e)]^2}}{\sqrt{S}\, P(e)} , \tag{19}$$

where

$$\overline{W} = \frac{1}{L} \sum_{\alpha \in \mathcal{A}_k} \int_{-\pi}^{\pi} P^2(e \mid \alpha, \psi)\ f_\Psi(\psi)\ d\psi . \tag{20}$$

The evaluation of (20) has the same complexity as (17) and can not be calculated exactly. Therefore we estimate it also by an unbiased estimator, given by

$$\varepsilon' = \frac{\sqrt{\overline{W}' - [P'(e)]^2}}{\sqrt{S}\, P'(e)} \tag{21}$$

where

$$\overline{W}' = \frac{1}{S} \sum_{s=1}^{S} P^2(e \mid \alpha_s, \psi_s) . \tag{22}$$

The results of [9] show that P′(e) is very close to the exact value P(e) if $\varepsilon' \leq 10\%$.

Now for a small number of interference symbols we can use equation (17) to compute P(e). For a large number of interference symbols we use equation (18) to estimate P(e) and use ε' in (21) to monitor the accuracy of this estimator.

Using a Gray code, the bit error probability can be approximated by

$$P_b'(e) \approx P'(e)/\log_2 M\,. \qquad (23)$$

IV NUMERICAL RESULTS

In this section let $\{P_{si}, P_{di}, t_{di}, f_{Di}, f_{Dmi}\}$ be identical for all $i \in I$. For different $i \in I$, $\{\xi_i(t)\}$ are all independent. Parameters without subscripts are valid for all users and parameters with subscripts refer only to the specific user indicated by the subscript. We assume that $df_0 = dt_0 = 0$ and M=4, i.e., QDPSK for all cases. Let

$$dF_i = df_i T,\ dT_i = dt_i/T,\ F_{Di} = f_{Di}T,\ F_{Dmi} = f_{Dmi}T,\ T_{di} = t_{di}/T \qquad (24)$$

be the normalized frequency offset and timing offset, the normalized Doppler frequency, the normalized maximum Doppler frequency and multipath time delay for the i–th user.

During the computation, the values of Np(i) and Nf(i) are selected such that

$$\max\{|g_{si}(\tau_o - kT)|,\ |g_{di}(\tau_o - kT)|\} < 10^{-3}|g_{si}(\tau_o)|$$

$$\text{for } k < -Nf(i) \text{ or } k > Np(i) \qquad (25)$$

where τ_o is the sampling time of user #0.

We are mainly interested in the performance of the ill–synchronized user and we assume all other users to be perfectly synchronized. We select S=4096, because during the computation we find this value suitable for computing time and $\varepsilon' < 10\%$ at most points of interest.

In the computation we have to find that I=30 is minimally required for $\beta=0$ and sufficient for other values of β. But because selecting a larger value of I will only slightly increase computing time, we select I=40+1 (40 interfering users and the main user) for all values of β. During the computation we concentrate on $\beta \le 0.2$ because of its excellent bandwidth efficiency. Also for Gaussian and Rician channels we select E_b/N_o =12 and 20dB, respectively, since these correspond to an ACI–free bit error rate of about 10^{-5}. For the Gaussian and Rician channels τ_o=0.

Fig 2 and 3 show the BEP as a function of dT_i for Gaussian ($K=\infty$) and Rician (K=10) respectively. In both figures $\varepsilon' < 10\%$. Fig. 4 shows the BEP as a function of dT_i for single path Rayleigh channels. In Fig. 4 $\varepsilon' < 1\%$. The results show that if $dF_i \le 2\%$ and $dT_i \le 0.1\%$ for β=0, $dT_i \le 2\%$ for β=0.1, $dT_i \le 4\%$ for β=0.2, on Gaussian and Rician channels, and if $dF_i \le 1\%$ and $dT_i \le 1\%$ for β=0.1, $dT_i \le 2\%$ for β=0.2 on single path Rayleigh channels, the performance is only minimally affected by ACI. It is also found that the ill–synchronized user is more severely affected by the other users than visa versa.

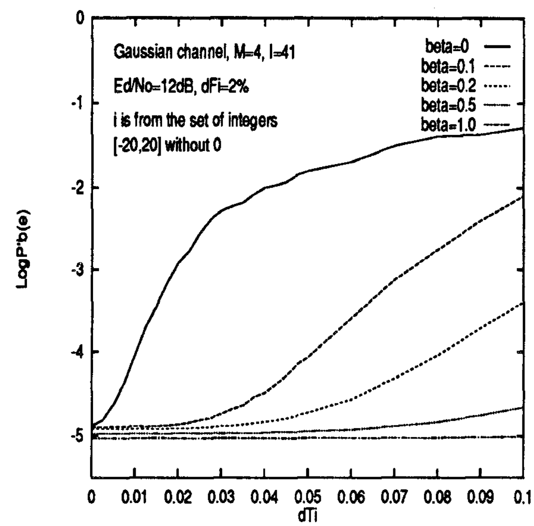

FIGURE 2. $P'_b(e)$ as a function of dT_i for a Gaussian channel, I=41, E_b/N_o=12dB and dF_i=2%, with $i \in I \backslash \{0\}$.

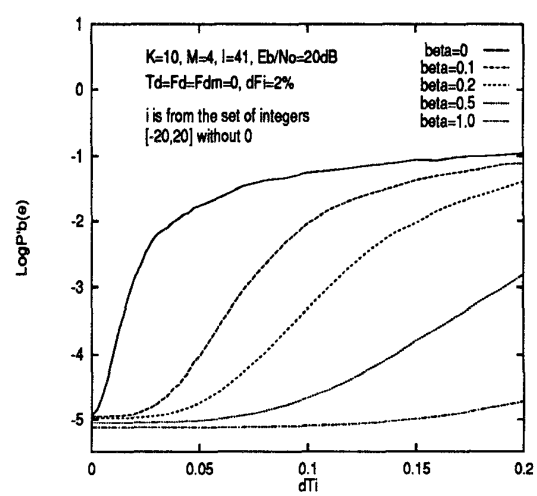

FIGURE 3. $-P'_b(e)$ as a function of dT_i with K=10, I=41, E_b/N_o=20dB, $F_D = F_{Dm} = T_d$=0 and dF_i=2% with $i \in I \backslash \{0\}$.

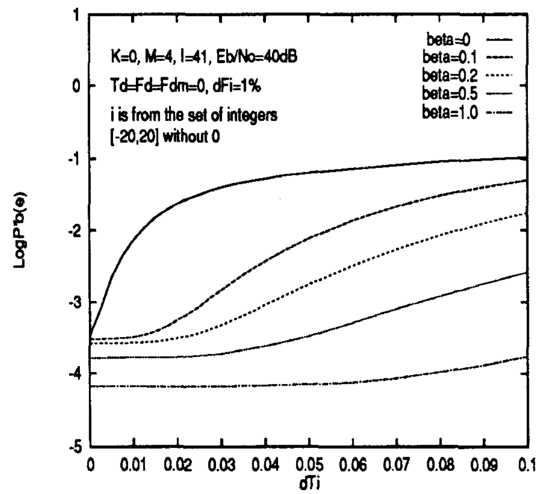

FIGURE 4. $P'_b(e)$ as a function of dT_i with M=4, K=0, $F_D=F_{Dm}=T_d=0$, I=41, E_b/N_o=40dB, dFi=1% with $i \in I \setminus \{0\}$.

V Discussion and Conclusion

The requirements of the timing and frequency synchronizations in a new multiple access modulation concept, multi–user OFDM with time raised cosine pulse shaping, has been analyzed on the SMC. The results show that with estimation accuracies of timing and frequency, $dT_i<4\%$ and $dF_i<2\%$, on Gaussian and typical Rician channels the effects of multi–user ACI on the performance of the system with $\beta=0.2$ is minor, while on single path Rayleigh channels these values must be better than 2% and 1%.

We conclude that although the multiuser OFDM system requires multi–user time and carrier frequency synchronization, the requirements are not overly restrictive. If we can achieve accurate multi–user timing and carrier frequency synchronizations, multiuser OFDM may achieve higher capacity than FDM/RNF, because $0 \leq \beta \leq 0.2$ can be selected for OFDM and not for FDM/RNF. Furthermore, frequency hopping CDMA technique with minimum frequency distance 1/T may also be applied to OFDM/QPSK. In [9] the performance of multiuser OFDM has been studied in detail over the Satellite Mobile and two–path Rayleigh Fading channels. The results were compared with the traditional FDMA system using the Root Nyquist filter.

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