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NUMERICAL CORRECTION AND DECONVOLUTION OF NOISY HV IMPULSES BY MEANS OF KALMAN FILTERING

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Abstract - Deconvolution of noisy transient signals is an important task in several fields of science, as in HV engineering. Due to the limited bandwidth of impulse voltage measurement systems, the measured signal is often a more or less deformed version of the original waveform, and what is worse, it is usually corrupted by noise, which makes deconvolution rather difficult.

The paper presents an optimized filtering method for deconvolution, based on Kalman filtering. The results are significantly better than that of formerly published algorithms.

After a brief survey of the literature, the new approach is described and its performance is illustrated.

INTRODUCTION

In many fields of science we are interested in waveforms that can only be observed after passing through a linear, time invariant system. Examples include HV engineering (e.g. [1...7], reflection seismology, astronomy, communication systems etc. [8]. The effect of an LTI system is described by the convolution (or Duhamel's) integral:

$$y(t) = \int_{-\infty}^{t} x(\tau)h(t-\tau)d\tau, \qquad (1)$$

where $y(t)$ is the measured waveform, $x(\tau)$ denotes the input signal we are interested in, and $h(t)$ is the system impulse response. The aim in such cases is to find $x(\tau)$ if $y(t)$ and $h(t)$ are given.

A similar problem arises if $x(\tau)$ and $y(t)$ can be measured and $h(t)$ is to be determined, or in general, if the distortion of measurement results can be described by a formula like (1). Applications include quite diverse tasks as X-ray diffractometry [9], ultrasound tissue characterisation [10] etc.

Though Equ. (1) shows the problem at the first glance to be solvable, there are some practical aspects causing difficulties:

a) $y(t)$ can never be exactly measured because of the noise produced by electromagnetic interference and electronic components. Quantization has the same effect.
b) Finite observation time causes edge effects.
c) Data are often obtained in digital form [3], which has advantages, but proper discretization of wide-band signals is not easy, and roundoff errors of the arithmetics spoil precision of the calculations, especially in the case of very selective system transfer functions.
d) Equ. (1) means filtering, i.e. multiplication in the frequency domain by

$$H(f) = \mathcal{F}\{h(t)\}.$$

Deconvolution can be similarly described as division by $H(f)$. If $H(f)$ has zeros or very small values, the solution becomes indefinite, and noise is amplified unacceptably. In other terms, e.g. the solution of the following set of equations for $\{x_i\}$

$$y_k = \sum_{i=0}^{k} x_i h_{k-i} \qquad (2)$$

is an ill-posed problem, which means that small uncertainties (or noise) result in large errors.

Due to these difficulties deconvolution is far from being trivial.

If both $x(\tau)$ and $y(t)$ can be modelled by (in wide sense) stationary stochastic processes, optimal (minimum variance) solutions can be obtained by means of Wiener or Kalman filtering, or other similar related algorithms [8] . However, the problem of impulse deconvolution, where $x(\tau)$ is a transient waveform as it often arises in HV engineering, is substantially different from the stationary case.

Apart from the above principal difference this optimal criterion for the restoration of $x(\tau)$ does not correspond entirely to our demand. In HV impulse tests the slope and peak value of the original impulse are needed, and the least squares criterion has no practical meaning.

PREVIOUS RESULTS IN HV IMPULSE DECONVOLUTION

Kiersztyn [1] suggested a numerical method for deconvolution. This is an attempt to find the inverse of (2). As it was already stated in the discussions, his method was theoretically correct, the "ill-posedness" of the problem, however, was not treated.

Malewski [3] suggested smoothing of $y(t)$ before deconvolution, depending on noise and system characteristics. He was in principle right, but unfortunately an elaborated method or criterion was not provided.

Schon and Gitt [2] proposed an iterative, piecewise linear restoration to improve the shortcomings (noise amplification) of Kiersztyn's method. Though this rather heuristic approach seems to work much better than simple direct deconvolution, it is applicable first of all for piecewise linear waveforms.

Charrat et al. [4] presented another iterative algorithm to restore $x(\tau)$. Their approach, i.e. minimizing

$$\sum \hat{x}_i^2 \qquad \text{while} \qquad \sum \left(y_k - \sum_{i=0}^{k} \hat{x}_i h_{k-i} \right)^2$$

is kept constant, is a systematic one; however, the criterion cannot be explained in heuristic terms.

Nikolopoulos and Topalis [5] proposed practically the same method as Malewski [3], but with the difference that after smoothing deconvolution is performed step by step, formulated on the basis of Laplace transfer functions of elements of the measuring system.

Wei and Shee-kong [6] treated HV impulse deconvolution on the basis of the

$$X(f) = Y(f)/H(f) \tag{3}$$

expression, taking apparently no care of noise problems.

McKnight and Lagnese [7] presented the application of a statistical method for obtaining confidence intervals for $\{x_i\}$. The results seem to be convincing, but computational efforts are high.

OTHER PUBLICATIONS RELATED TO THE TOPIC

A series of papers deals with the problem of zeros. Guillaume and Nahman [12], further Parruck and Riad [13] deal with regularization of (3) by the replacement of $1/H(f)$, viz.

$$\frac{H^*(f)}{|H(f)|^2+\gamma f^4} \quad \text{or} \quad \frac{H^*(f)}{|H(f)|^2+\lambda}, \quad \gamma, \lambda > 0, \tag{4}$$

respectively. The choice of γ or λ determines the maximum gain applied to the noise spectrum. The method incorporates an interesting way to decrease the difficulties with small values of H(f); systematic theoretical foundation, however, is lacking.

Grimble [14], Candy and Zicker [15] proposed the use of Kalman and Wiener filters modified for uncertain system models. The measurement system with observation noise is modelled in the usual way, and the input waveform is taken into consideration as unknown disturbance which is produced by a subsystem, excited by white noise. The method for uncertain systems was originally proposed by Schmidt, and is described by Jazwinski [16].

This last approach is based on the minimum variance criterion. Though, as noted above, this criterion does not directly describe our target, a "best fit" on the input signal in any sense may meet our demand. Moreover, noise amplifi-cation through deconvolution (small H(f)-values)seems to be the main trouble, and Wiener and Kalman filters do more or less what one would expect from an optimal algorithm: compensate the measurement system in such a way that where the signal dominates in the frequency domain, the resulting transfer function equals approximately 1; the resulting transfer function decreases.

THE PROPOSED METHOD

The measurement system is to be modelled by the state equation:

$$x_{k+1} = \Phi x_k + \psi u_k, \tag{5}$$

where x_k is the state vector and u_k is the input waveform. The observation equation is:

$$y_k = M x_k + v_k, \tag{6}$$

where v_k is the observation noise (EMI, quantization etc.), which is assumed to be white and to have a known variance σ.

From Equ. (5) an *augmented system* is formed:

$$\begin{pmatrix} x_{k+1} \\ u_{k+1} \end{pmatrix} = \begin{pmatrix} \Phi & \Psi \\ O^T & A_u \end{pmatrix} \begin{pmatrix} x_k \\ u_k \end{pmatrix} + \begin{pmatrix} O \\ e_k \end{pmatrix}, \tag{7}$$

where e_k is again assumed to be white (which is a rather strange assumption, as the Kalman estimation of $\{e_k\}$ will usually be not white, since $\{u_k\}$ is a deterministic pulse). σ_e is a parameter we can adjust to find a good compromise between signal restoration and noise amplification. A_u represents the dynamic behaviour of $\{u_k\}$, it can be chosen e.g. on the basis of the energy density spectrum of $\{u_k\}$, or most simply as A_u = 1(integrator). In the following simulations, A_u is chosen to be equal to 1, since not otherwise stated.

For the model described in Eqs. (6) and (7) the Kalan filter equations are well-known, and they can be straightforwardly programmed. In our simulations we have recalculated the Kalman gain in every step (instationary Kalman filter). By solving the Riccati matrix equation, the much simpler stationary Kalman filter can be programmed as well, since - according to our experience - the Kalman gain converges already in some steps. When estimating the augmented state vector (7) from $\{y_k\}$, the last element is our estimate for the input series.

SIMULATION RESULTS

First we have compared the performance of the Kalman filter for a well-documented case presented in [2]. Fig. 1 shows the result of a direct deconvolution based on the solution of (2). In this simulation the observation noise has uniform distribution and zero mean, its maximum amplitude is 0.5% of the maximum of the input waveform. The sampling interval is 5ns, the point number is 200.

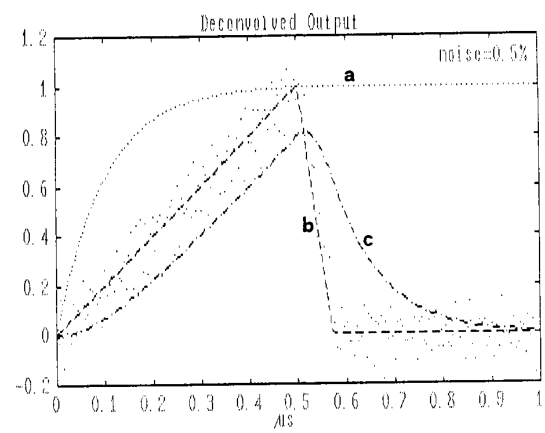

Figure 1 Deconvolution of the noisy output of a lowpass filter.

a = step response
b = input waveform
c = noisy output
. . . = result of deconvolution

In Fig. 2 the results of our method are presented (continuous line). The fit with the original signal is good, though some noise can still be observed. To understand how the filter works, the Bode diagrams of the original and the compensated systems are shown in Fig. 3. It is clearly visible that the transfer function of the system is partly compensated, but a decrease shows up beyond approx. 10MHz. Above this frequency full compensation would amplify the noise too much with no significant gain in the signal.

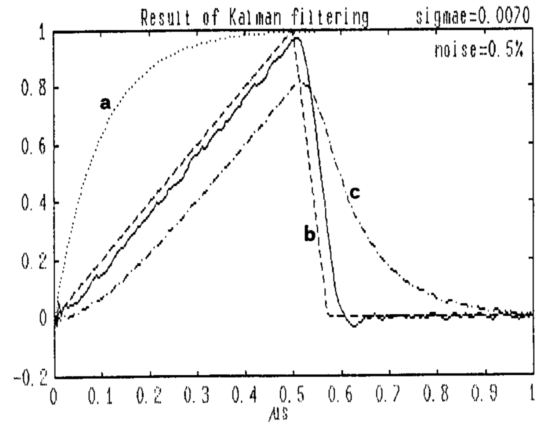

Figure 2 Signal restoration using the Kalman filter. a, b, c see Fig. 1.

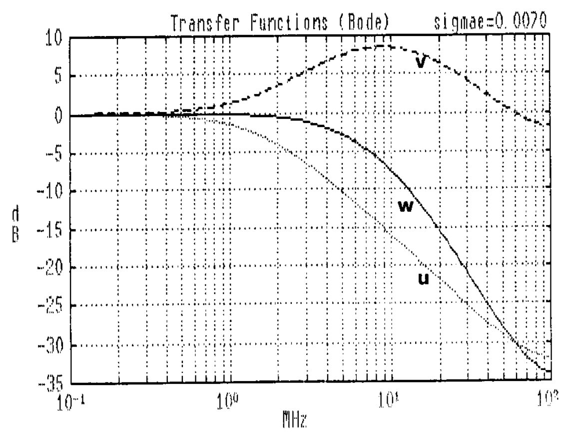

Figure 3 Compensation of H(f)

u = H(f)
v = Transfer function of the Kalman filter
w = Compensated transfer function

The form of the transfer function of the *compensated system* can well be explained if we plot the power density spectrum (PSD) of the noises e and v (see Eqs. (6) and (7), reduced to the input of the system (Fig. 4). The compensated bandlimit is about at the frequency, where the reduced observation noise starts to dominate.

On the basis of these results, the following questions may arise:

- Is the performance only restricted to first-order systems?
- What happens if the state-space model of the system is not completely correct?
- Is the performance influenced by the waveform of the pulses?
- What happens when the observation noise is increased?
- Can also high quantization noise strongly be reduced?
- Is the algorithm not too time-consuming?

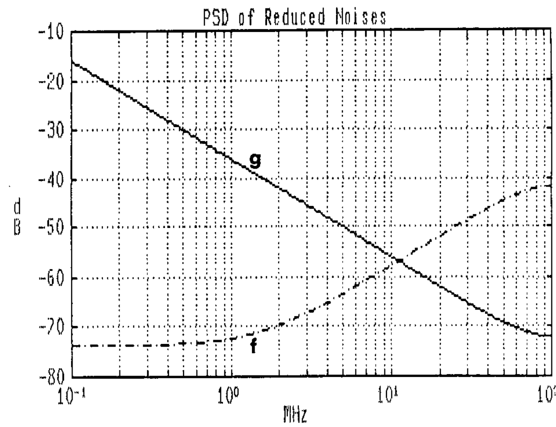

Figure 4 Power spectral density of the noises reduced to the input of the system.

f = PSD of the reduced obs. noise
g = PSD of the reduced (hypothetical) noise e

Let us examine the above questions one by one.

Results on a second-order system

Fig. 5 shows an example for the same input ramp function as assumed before but with a second-order step response and an output function distorted by the same noise magnitude as for Fig. 2: The performance is worse than that in the first-order case, though the result is much better than the restored input signal (i.e. the dotted "curve") produced by 9-point smoothing and deconvolution. In this simulation, A_u (see equ. 7) was chosen to describe approximately the spectral behaviour of $\{u_k\}$ - see also Fig. 9: a second-order system with two poles of f = 1,05 MHz.

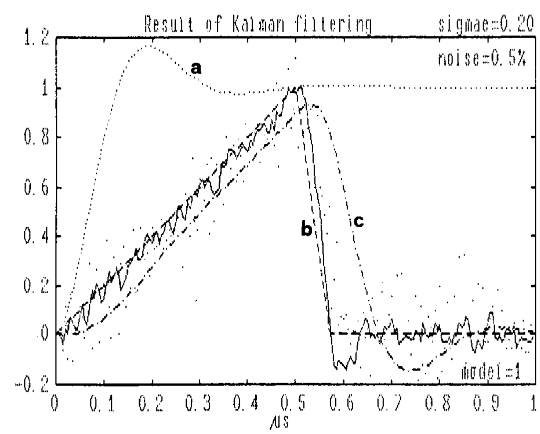

Figure 5 Deconvolution for a second-order system.
a, b, c see Fig. 1.

Deconvolution with imperfect system model

To have an impression about the sensitivity of the algorithm on modelling imperfections, the output of the formerly used first-order system has been processed supposing an increased time constant (Fig. 6) , $T_2 = 1.1\ T_1$. This change did not deteriorate the performance significantly, if compared with Fig. 2.

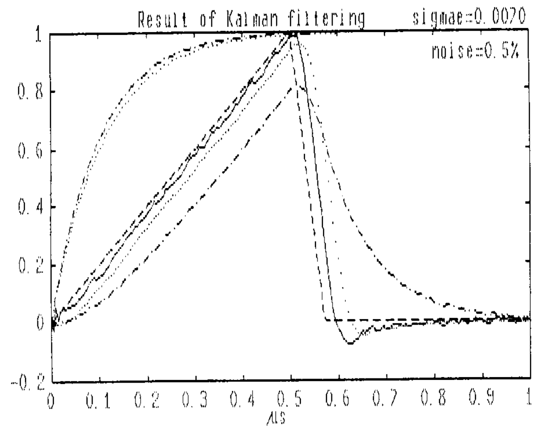

Figure 6 Deconvolution with imperfect system model

A direct deconvolution has been performed too, but after a 15-point smoothing (dotted curve), with the same system model. The results are obviously similar, though the peak value of the restored input is less. The different behaviour may be explained on the basis of the transfer functions (Fig. 7): smoothing only roughly approximates deconvolution.

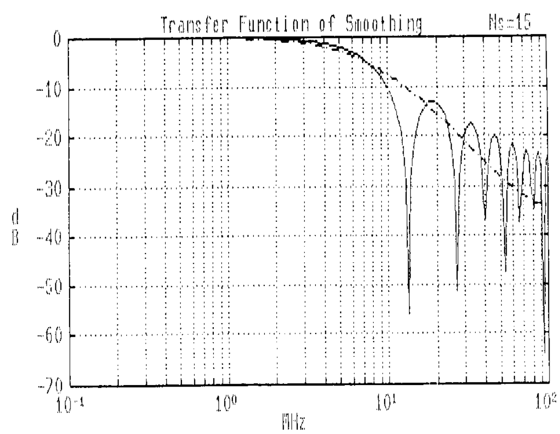

Figure 7 Transfer functions for the compensated system (dashed line) and for the output signal with 15-point smoothing (full line; one sample per 5 ns).

Deconvolution of an aritifcial waveform

To check whether the performance relies on the form of the used *chopped impulse* only , an artificial waveform has been created (exponential plus sine) and processed with the algorithm. Fig. 8 shows the results for the first-order system and the 0.5% observation noise for the output signal. The reconstructed input signal fits again well with the original one.

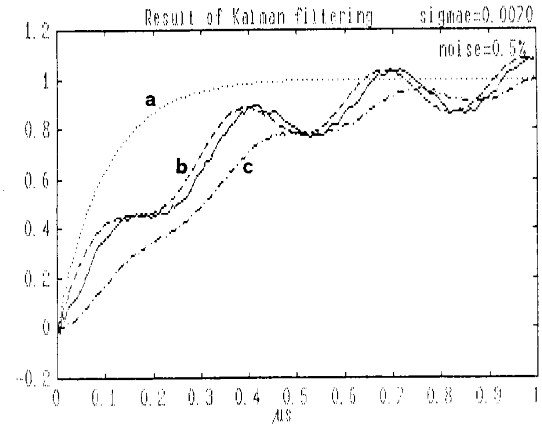

Figure 8 Reconstruction of an oszillating waveform. a, b, c see Fig. 1.

To explain at this point the rather good performance of the Kalman filter, the energy density spectra of the two input signals used are displayed in Fig. 9, together with the energy of the reduced observation noise, calculated for the finite measurement time. It can be seen that nearly the whole energy of the signals is to be found within the passband of the compensated system, for which the energy of the noise remains almost negligible.

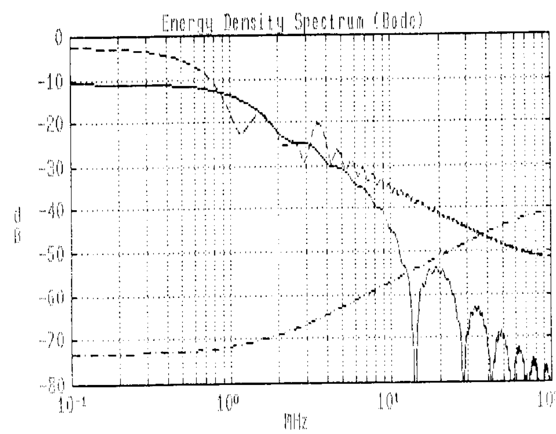

Figure 9 Energy density spectra for the two input signals used and for the observation noise (0.5%).

High-level observation noise

The observation noise applied up to now may be assumed to be very low. Therefore, a heavily distorted output signal with 5% noise amplitudes was reconstructed for conditions otherwise used in Fig. 2. The results are shown in Fig. 10. Though the reconstructed input becomes rather noisy, the result resembles to the original input pulse.

Deconvolution of roughly quantized waveforms

An output signal has been digitized with $2^B = 64$ quantum levels in the (0,1) interval. Reconstruction is reasonably good (see Fig. 11), i.e. that the quantization error can well be modelled with the white noise model.

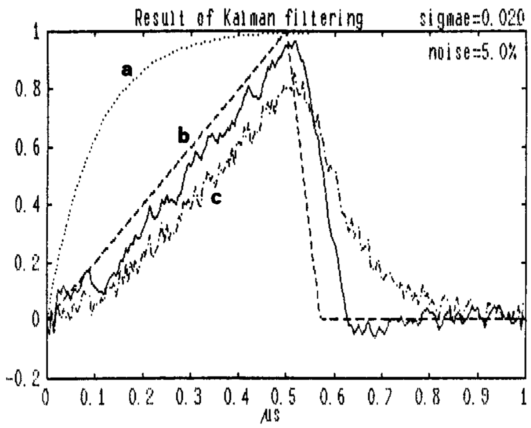

Figure 10 Deconvolution of a very noisy output signal. a, b, c see Fig. 1.

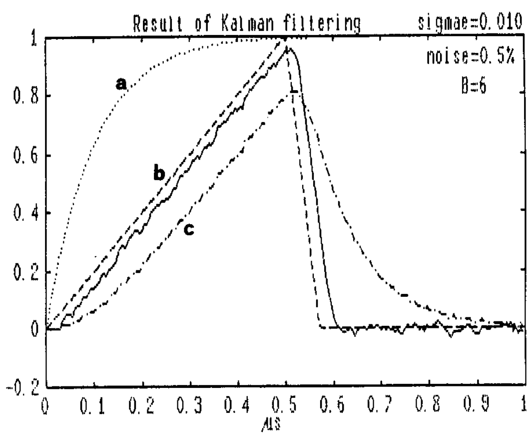

Figure 11 Reconstruction for a digitized output signal with low rated resolution. a, b, c see Fig. 1.

Calculation speed

All calculations for the 200 samples taken for each example did not take more time than 1 min when the Kalman gain was recalculated at every step, and 30 s using the stationary Kalmann filter (with pre-calculated gain) on an IBM-PC/AT. By using the simplifications proposed by Jazwinski [16], these calculation times can be further reduced.

CONCLUSIONS

A Kalman filter based optimized deconvolution method [15] has been presented and investigated for the reconstruction of linearly rising front-chopped pulses. On the basis if simulations it seems to be well suited for the use in HV measurements.

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