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FREQUENCY DOMAIN SYSTEM IDENTIFICATION TOOLBOX FOR MATLAB: IMPROVEMENTS AND NEW POSSIBILITIES

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Abstract. Frequency domain system identification has been rapidly developing during the past years. The comprehensive Frequency Domain System Identification (FDIDENT) Matlab toolbox is a successful implementation of the methods of the whole identification procedure. Its core is a weighted least squares estimate of parametric transfer function models, maybe with a delay, but also offers help for nonparametric transfer function identification. It contains tools for excitation signal design, data preprocessing, parameter estimation, model and error presentation, model validation, simulation, archivation and documentation.
This paper describes the latest developments of the toolbox, and illustrates them on a few examples. Finally, the future plans are briefly summarized.

Key Words. System identification; frequency domain; linear system; parameter estimation; maximum likelihood; weighted least squares; transfer function; excitation signal; multisine; pseudorandom noise; computer software; software tool; Matlab toolbox.

1. INTRODUCTION

The Frequency Domain System Identification Toolbox (FDIDENT) for Matlab was first presented at Sysid'91 (Kollár et al, 1991). By 1994, distribution of a professional version was started by the MathWorks (Kollár, 1994). At Sysid'94 an elaborated application example was presented, based on this package (Kollár et al, 1994).

Since then, several new theoretical results have been achieved, and many of them will appear in the next version of the toolbox. The purpose of this paper is to summarize the novelties, and to illustrate some of them.

2. A SHORT OVERVIEW

The toolbox offers now a wide spectrum of services for all the important steps of system identification, as:

- excitation signal design (multisine, pseudorandom bit sequence, discrete interval binary sequence, optimal power spectrum design)
- data preprocessing (proper calculation of Fourier coefficients also for non-integer periods, variance analysis, precompensation with known terms),
- parameter estimation (different numerical solvers with numerous setting possibilities),
- model and error presentation (transfer function plots along with errors, pole/zero plots with confidence ellipses),
- model validation (cost function, calculation of residuals, Akaike criterion)
- simulation (time or frequency domain, steady-state or transient),
- archivation and documentation (automatic session diary, save results to files).

The present version is based on Matlab 4.2. It also runs smoothly under the new Matlab 5, but still does not make use of its new possibilities, like 3-dimensional arrays, data structures, objects, and so on.

3. NOVELTIES

Since 1994, several new features have been implemented, and will become parts of the new version. Here is an enumeration of the already implemented functionalities.

- orthogonal polynomials which allow a much wider application range than before, like mechanical systems of high number of resonances (e.g. numerator: order 100, denominator: order 100)

- significantly improved starting values
- dealing with transient signals and arbitrary excitations without suffering from leakage errors
- elimination of system transients from input signals
- residue analysis allowing to detect undermodeling, overmodeling and nonlinear distortions
- data preprocessing for non-integer periods
- possibility of stability maintaining during iteration
- approximate Chebyshev fit via weighted least squares
- Warburg impedance (transfer function estimation with numerator and denominator polynomials in terms of $\sqrt{s}$),
- Graphical User Interface (GUI) utility for easy interactions into lengthy iterations (proper finishing without fulfillment of termination criterion)
- new utilities, e.g. for transfer function and cost function evaluation
- line search during iteration
- simple calls for common cases, e.g. elisml (maximum likelihood estimator), elistper (estimator for periodic excitation)

4. TWO EXAMPLES OF NEW FEATURES

Orthogonal Polynomials

One of the basic difficulties of s-domain transfer function identification is that the transfer function itself is an inherently badly conditioned system model. The main cause is that the numerator and denominator are polynomials of s, and the values of the powers of s are strongly correlated (the scalar product is large) on the frequency grid. A solution to the problem is the use of an alternative representation of the polynomials, and combinations of *orthogonal polynomials*, see (Rolain et al, 1995). The original polynomials need not be calculated at all, both the transfer function values and the poles/zeros can be directly calculated from the orthogonal polynomial representation. This representation and all internal calculations will be transparent for the user, he/she should only notice improved numerical capabilities.

Figures 1-5 illustrate the difference between the classical s-polynomial approach and the new one. The computer used in these simulations was a Sun Sparc4. The example is a Wilkinson-type system. Both polynomials have roots approximately at $\pm j, \pm 2j, \pm 3j, \ldots$, the poles and zeros are slightly shifted from each other, and a little negative real part is added to them, so that the shape of the root loci form a turned flat 'V' in the s-domain, easy to recognize on a pole/zero plot. Figure 1 illustrates the result of a classical identification of a 40/40 system. The fit is ap-

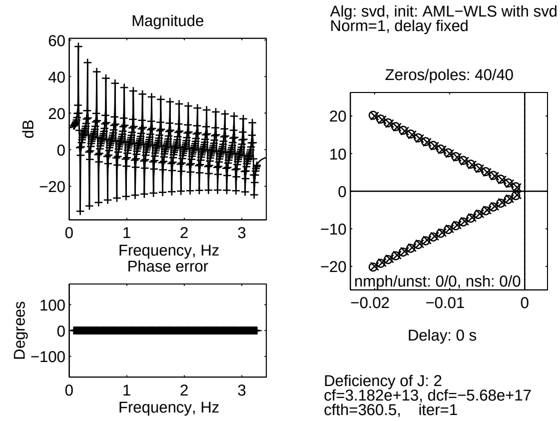

Fig. 1: Identification of a Wilkinson-type system, order 40/40, s-polynomials

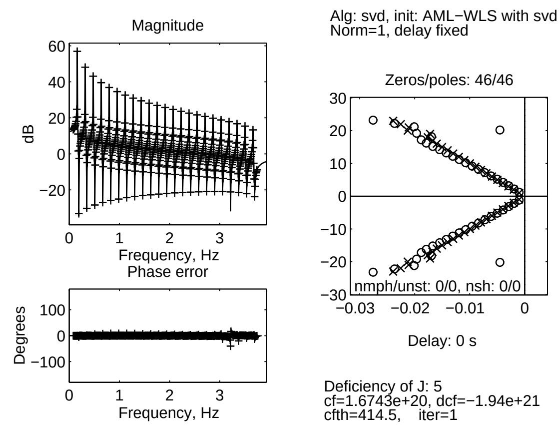

Fig. 2: Identification of a Wilkinson-type system, order 46/46, s-polynomials

parently good, but the solution is already badly conditioned: 2 singular values of the Jacobian $\mathbf{J}$ are too small. The *condition number* is not given in the plot, because deficiency means that conditioning is worse than machine precision. Bad conditioning problem is clearly visible for a 46/46 system: here some zeros are clearly wrongly positioned, and there are 5 too small singular values (Fig. 2).

The algorithm based on orthogonal polynomials is obviously much better conditioned. The condition number for the same setting is only 10^7 (Fig. 3), while the machine precision is $2 \cdot 10^{-16}$.

The solution is still well conditioned when the order is increased to 68/68 (Fig. 4).

When the order is increased to 76/76, the solution is still reasonable (Fig. 5), although there are a few too small singular values.

The above system, since it is based on Wilkinson-type polynomials, is very difficult to identify. For true systems, like mechanical ones, even systems of orders above 100/100 can usually be identified.

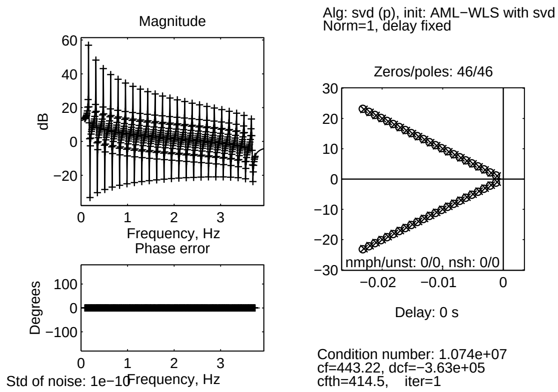

Fig. 3: Identification of a Wilkinson-type system, order 46/46, orthogonal polynomials s-domain

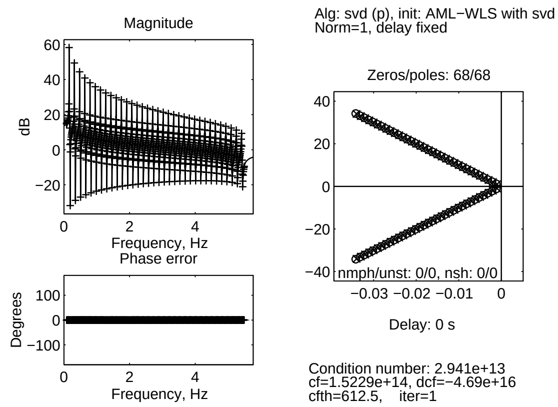

Fig. 4: Identification of a Wilkinson-type system, order 68/68, orthogonal polynomials

Improved Starting Values

In maybe the simplest case of elis (Estimation of Linear Systems), the following cost function is minimized:

$$C(\mathbf{P}) = \frac{1}{2}\sum_{k=1}^{F} \frac{|N(j\omega_k,\mathbf{P})X_{mk} - D(j\omega_k,\mathbf{P})Y_{mk}|^2}{\sigma_{yk}^2|D(j\omega_k,\mathbf{P})|^2 + \sigma_{xk}^2|N(j\omega_k,\mathbf{P})|^2},$$

where $N(j\omega,\mathbf{P})$ and $D(j\omega,\mathbf{P})$ are the numerator and the denominator of the transfer function,

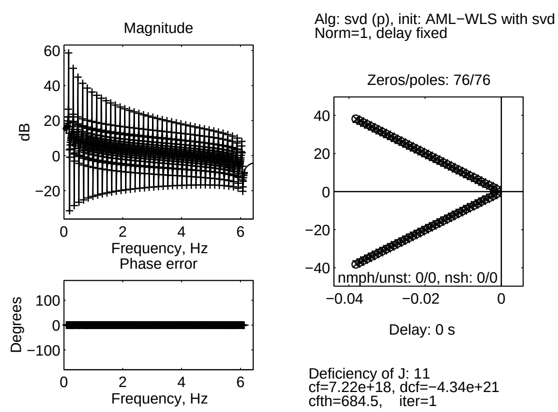

Fig. 5: Identification of a Wilkinson-type system, order 76/76, orthogonal polynomials

respectively. Measurements are made at angular frequencies ω_k, $k = 1, 2 \ldots F$. X_{mk} and Y_{mk} are corresponding input and output spectra, contaminated by noise. σ_{xk}^2 and σ_{yk}^2 are corresponding variances — halves of the total variances —, defined by:

$$\mathrm{var}\{N_{xk}\} = 2\sigma_{xk}^2 \quad \text{and} \quad \mathrm{var}\{N_{yk}\} = 2\sigma_{yk}^2,$$

where N_{xk} and N_{yk} are the complex input and output noises, respectively (errors-in-variables model). The unknown parameters are the coefficients of the transfer function, collected in $\mathbf{P}$.

The starting values are determined by setting the numerators to one, and solving the remaining linear least squares problem. This works more or less fine. However, in some difficult-to-identify cases, the observation is that this algorithm tends to overemphasize high frequencies. Therefore, it is reasonable to modify initial weighting by compensating for this phenomenon (Rolain, 1996). The above Wilkinson-type examples were generated using this improvement. Figure 6 illustrates what happens to this example when the "classical" initial value setting algorithm is used.

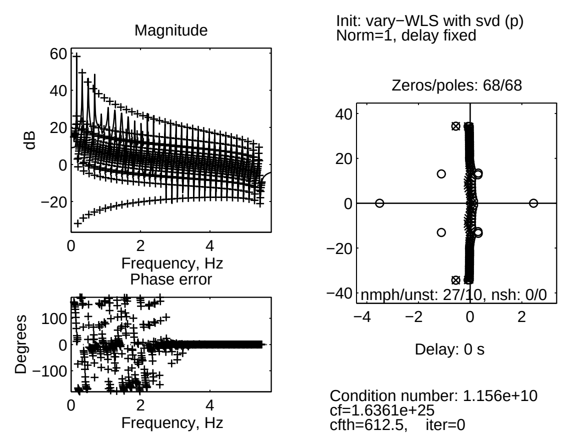

Fig. 6: "Classical" starting value setting in identification of a Wilkinson-type system, order 68/68, orthogonal polynomials

5. DEVELOPMENT PLANS

The above new possibilities already significantly extend the usability of the toolbox. Therefore we plan in the near future to concentrate on easy, user-friendly use. We foresee two solutions for that.

- Introduction of data structures. One of the major novelties of Matlab 5 is that it is possible now to handle groups of data as one entity. We plan to merge interrelated data into a few structures. Data parameters will be stored together in a consistent way. This will significantly decrease the number of parameters in each command line call. The user will only need to remember the names of his data structures. Exchange between this

toolbox and the time domain System Identification Toolbox will also be facilitated by this.
- Implementation of a graphical user interface. We are working on the implementation of an easy-to-use interface which allows the user to access the basic functionalities through windows and menus, based on mouse clicks.

With the above improvements, the toolbox will become a really powerful, user-friendly tool which allows users to only concentrate on their identification tasks, instead of fighting Matlab and the toolbox functions.

In the future, we also plan to add further functionalities to the toolbox. Such possibilities:

- Automatic selection of the orders – this will make it possible to run "automatic" identification programs (Rolain et al, 1997).
- Extension of frequency domain system identification to MIMO systems (Guillaume, 1992).
- Further iterative algorithms (bootstrapped elis, logarithmic cost function minimization, etc.).
- Improved method for crest factor minimization (Guillaume et al, 1991).
- Elimination of linear and quadratic terms from the inputs.
- Stable filter design with automatic delay selection (Vuerinckx et al, 1996).

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