Dynamic Range of Digital Spectrum Analyzers*

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1 Introduction

Digital spectrum analyzers belong to the most difficult-to-understand instruments. Their readouts and especially the possible errors present in their readouts cause a lot of headache for most users. The situation may be even worse, if an appropriate type is to be chosen on the basis of the available data sheets. Neither is it easy to calculate the performance of the usually available computer programs for spectral analysis: the non-specialist user is often helpless even with the best data processing package.

This paper deals with one of the key feature of spectrum analyzers – their dynamic range. Even its definition is very often not quite clear – not to speak of the nature of the phenomena, which limit its value. In the paper these phenomena are briefly summarized, and expressions are derived to estimate the effect of two of them: Input quantization and the often used block-float FFT.

2 The Meaning of the Dynamic Range

The most frequently used definition is as follows: it is the ration of the largest signal to the smallest signal that can be displayed simultaneously with no analyzer distortion products (Hewlett–Packard, 1974), that is, it is possible to detect both signals unambiguously, with sufficiently exact readout. The auxiliary verb 'can' means that this must hold true only if the signal levels are adjusted appropriately – the greater one is generally to be adjusted to the full-scale level. However, the dynamic range exhibits no direct relationship with the input range, which usually can be much greater.

In the resulting spectrum two different types of components can be distinguished, which put a limit to the dynamic range. First, there are spectral components depending on the greater signal itself, as e.g. the results of spectral leakage, or of nonlinear distortions. They are of deterministic nature, and thus they cannot be modified by averaging. Second, there are 'spurious responses', consisting of the spectrum estimates of the noises produced by the analyzer itself. Snce the periodogram of such noises has great (≈ 100%) variance, averaging can help in decreasing their effect.

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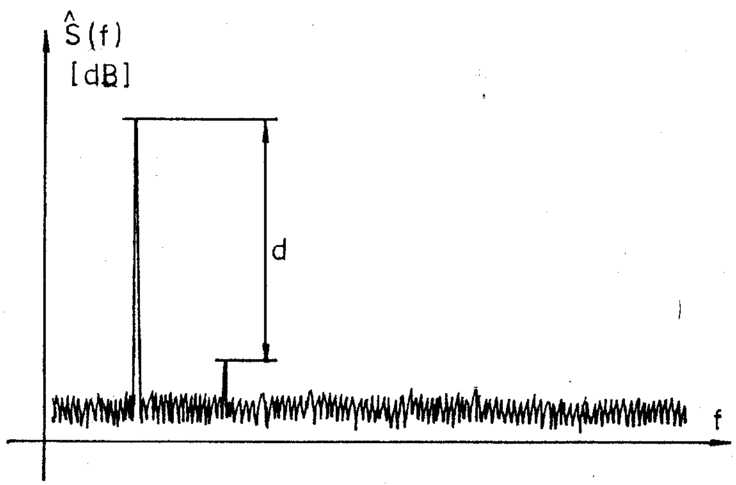

Figure 1: Definition of the dynamic range

3 Error Sources in the Measurement of Spectra

Considering the achitecture and usual operating principles of Fourier analyzers, the effects of spectral error sources, affecting the dynamic range, are as follows:

- Consequence of digital computations: bias and variance due to
  - the uniform input quantizer;
  - the window function coefficients, which are stored with limited accuracy;
  - the finite wordlength FFT;
  - averaging with finite wordlength;
  - the finite wordlength complex demodulator (if there is any).
- Consequence of the use of the (modified) periodogram: spectral leakage.
- The effect of non-ideal components in the analyzer:
  - nonlinear distortion of the input amplifier and the anti-aliasing filter;
  - finite rejection of the anti-aliasing filter;
  - time jitter of sampling, resulting in slight modulation;
  - differential and integral nonlinearities of the A/D converter;
  - statistical error of quantum levels (internal noise of the ADC);
  - finite rejection of the low-pass filter in the complex demodulator.

The effect of some of the above error sources is rather obvious, thus the errors can be effectively reduced by careful design e.g. a window can be chosen with very good sidelobe behaviour (see Harris, 1978; Nuttall, 1981; Cox, 1978), reducing spectral leakage, etc. However, some others have not so trivial an effect. In the following lines we are going to deal with two error sources of the latter type: the input quantizer and the finite wordlength FFT, using the commonly used block-float number representation.

4 The Effect of the Input Quantizer

4.1 Nonlinearity

The most awkward feature of A/D converters is their (differential and integral) nonlinearity. A nonlinearity of ± 0.5 LSB may result in an additive harmonious distortion component of amplitude 0.64 LSB (see *Fig. 2*), which means, using a B-bit A/D converter and a $(2^{B-1} - 1)$ LSB amplitude sine wave as an input signal that the dynamic range is not more than

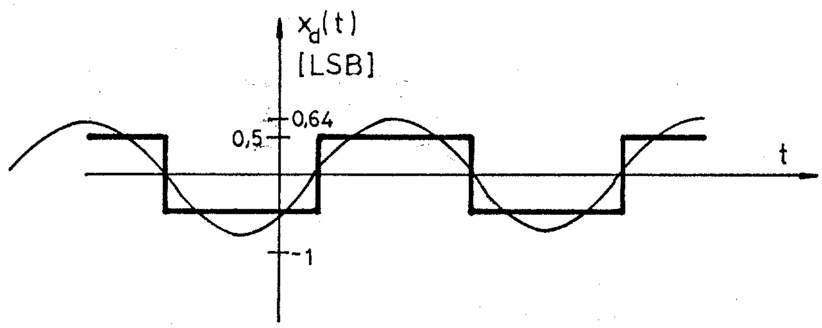

Figure 2: Harmonious distortion component caused by nonlinearity

$$d = 20 \cdot \lg \left(\frac{2^{B-1} - 1}{0.64} \right) \approx (20 \cdot \lg 2)B - 20 \cdot \lg 1.3 \approx 6B - 2.1 \,, \tag{1}$$

that is, 70.1 dB for $B = 12$, and 58.1 dB for $B = 10$. Thus, very linear ADC-s must be used, and a dither signal with amplitude of several quantum sizes (LSB-s) is to be applied. This dither, 'moving' the measured signal along the ADC characteristic, 'averages' differential nonlinearities.

4.2 Quantization Noise

In addition to the above distortions, quantization noise is always present in the processed signal. This is the quantization error of an ideal uniform quantizer, and it is usually can be modelled by means of a uniformly distributed, independent, additive white noise (WIDROW, 1961). This noise results at every estimated point of the spectrum a component of second-order chi-square (that is, exponential) distribution:

$$\hat{S}_{n_q}(-k\Delta f) = \hat{S}_{n_q}(k\Delta f) = \frac{q^2}{12}\frac{1}{2}\chi_2^2 \,, \quad k = 1, 2 \ldots \frac{n}{2} \,, \tag{2}$$

assumed that the length of the sampling interval, Δt, is chosen as time unit, and the spectral estimator is

$$\hat{S}(k\Delta f) = \frac{1}{N_e} |X_w(k\Delta f)|^2 \,, \tag{3}$$

where

$$N_e = \sum_{i=0}^{N-1} w^2(i\Delta t) = \frac{1}{N} \sum_{k=0}^{N-1} W^2(k\Delta f) \,, \tag{4}$$

$w(t)$ is the linear window function, and

$$X + w(k\Delta f) = \sum_{i=0}^{N-1} w(i\Delta t)x(i\Delta t)e^{-j2\pi\frac{ki}{N}} \,, \qquad (5)$$

that is, for smooth spectra $\hat{s}(k\Delta f)$ is approximately unbiased.

The components shown in *Eq.* (2) are independent of each other. We assume a peak in the spectrum to be detectable, if it is significantly (with 95% confidence) larger than the spurious peaks of the spectrum estimate of the quantization noise. This means in an N-point spectrum ($\frac{n}{2}$ independent spectral components) that

$$\frac{X^2}{4N_e}W^2(0) > S_{n_q}\left[-\ln\left(1 - \sqrt[N/2]{0.95}\right)\right] \approx S_{n_q}\ln\left(\frac{N}{2\cdot 0.05}\right) = \frac{q^2}{12}\ln\left(\frac{N}{0.1}\right) \,, \qquad (6)$$

where X is the amplitude of the sine wave, and the picket fence effect is not taken into account. To consider the effect of this phenomenon, on the left

$$\frac{X^2}{4N_e}W^2\left(\frac{\Delta f}{2}\right)$$

should be written.

From *Eq.* (6), using $X_{\max} \approx 2^{B-1}LSB = 2^{B-1}q$, we obtain for the dynamic range:

$$\begin{aligned} d_1 &= 20\lg\frac{2^B}{2\sqrt{\frac{N_e}{3W^2(0)}\ln\left(\frac{n}{0.1}\right)}} = 20\lg\frac{2^B}{\sqrt{\frac{4K_w}{3N}\ln\left(\frac{N}{0.1}\right)}} \approx \\ &\approx 6B - 10\lg\left[\frac{4K_w}{3N}\ln\left(\frac{N}{0.1}\right)\right] \,, \end{aligned} \qquad (7)$$

where K_w depends on the window shape only:

$$K_w = \frac{N_e}{W^2(0)}N = \frac{\sum_{k=0}^{N-1} W^2(k\Delta f)}{W^2(0)} \,. \qquad (8)$$

If the spectra are averaged, because of the central limit theorem normal distribution may be assumed instead of χ_2^2:

$$d_m \approx 20\lg\frac{2^B}{\sqrt{\frac{4K_w}{3N}1.65\frac{1}{\sqrt{m}}}} \approx 6B - 10\lg\left(\frac{4K_w}{3N}1.65\frac{1}{\sqrt{m}}\right) \,, \qquad m \gg 1 \,. \qquad (9)$$

In the case of $N = 256$, $K_w = 3.6$ (Flat Top Window, see Cox, 1978; Kollár–Nagy, 1982):

	$B = 10$	$B = 12$
d_1	68.5 dB	80.6 dB
d_{16}	81.3 dB	93.4 dB
d_{64}	83.3 dB	96.4 dB

5 The Effect of the Block-Float FFT

In his classical paper WELCH (1969) suggests the following formula:

$$\frac{\text{rms}\,\{\text{error}\}}{\text{rms}\,\{\text{result}\}} < \frac{C \cdot N \cdot 2^{B_{FFT}}}{\text{rms}\,\{\text{initial sequence}\}}\,, \tag{10}$$

where B_{FFT} is the number of bits (*without* sign bit): the number representation is B_{FFT} bits plus a sign; the initial sequence is adjusted in such a way that its maximal value is approximately 1, and C is a slightly waveform-dependent constant, which may be chosen in our case for $C = 1.1$. Using a similar noise model as in the previous section (generally no better model can be suggested),

$$d_1 = 20\lg \frac{2^{B_{FFT}}}{\sqrt{5 \cdot \ln \frac{N}{0.1}}} \approx 6B_{FFT} - 10\lg \left(5 \cdot \ln \frac{N}{0.1}\right)\,, \tag{11}$$

and in the case of averaging

$$d_m = 20\lg \frac{2^{B_{FFT}}}{\sqrt{8.25 \frac{1}{\sqrt{m}}}} \approx 6B_{FFT} - 10\lg \frac{8.25}{\sqrt{m}}\,, \qquad m \gg 1 \tag{12}$$

are obtained. With $B_{FFT} = 15$ and $N = 256$ we get

d_1	74.4 dB
d_{16}	87.2 dB
d_{64}	90.2 dB

6 Combination of the Above Limits of the Dynamic Range

Since the input quantization noise and the FFT roundoff noise are independent and approximately normal, in the result of the FFT their combination appears again as an additive normal noise with a variance

$$\text{VAR}_c = \text{VAR}_q + \text{VAR}_{FFT}\,. \tag{13}$$

Thus, an additive noise can be taken into account with the above variance. Concerning the values of the dynamic range, they should be transformed back to variances, and then summed:

$$d_{mc} = -10\lg \left(10^{-\frac{d_{mq}}{10}} + 10^{-\frac{d_{mFFT}}{10}}\right)\,. \tag{14}$$

E.g. from $d_1 = 80.6$ dB and $d_1 = 74.4$ dB the result is $d_1 = 73.5$ dB.

On the basis of the above results it is understandable, why in modern Fourier analyzers usually 12-bit ADC-s and 16-bit block-float FFT processors are used, and why the dynamic range is commonly given as being 70–75 dB. Also, the performance of a spectral analysis program can be easily checked using the above formulae.

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