EVALUATION OF SINE WAVE TESTS OF ADC'S FROM WINDOWED DATA

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Abstract - Recently, much excitement has been caused in the EUPAS group by the evaluation of the IEEE 1241 draft standard. This draft contains a lot of useful information and several well established facts. However, there are also some details which are not fully clarified, or need further examination. This paper deals with one of these: the possibility of data processing, based on windowed data.

Keywords: IEEE-STD-1241, windowing, windowed data, sine wave method, EUPAS.

1. INTRODUCTION

Finite record length effects may deteriorate the results of measurements with sinusoidals significantly. The draft standard recognizes this fact, and at several places advocates coherent sampling. This means that in a test, an integer number of periods needs to be taken, that is,

$$f_i = \frac{m_i}{M} f_s \,, \tag{1}$$

where f_i is the frequency of the sine wave, m_i is an integer less than $M/2$, M is the number of samples in the record, and f_s is the sampling frequency (cf. 4.1.5.1).

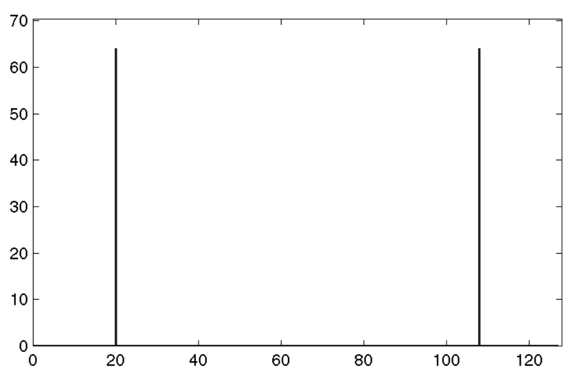

Fig. 1. DFT result of a coherently sampled sinusoidal. M=128, f_i=20

It is right indeed that whenever possible, coherent sampling is the proper choice (Fig. 1).

However, when incoherent sampling may occur (see (i) and (ii) above Eq. (4.1.5.1.3)), windowing is almost unavoidable. In the following sections we are going to discuss the consequences of applying windows in detail.

2. MODELING A SINE WAVE

The cause of many problems is the fact that theoretically, the Fourier transform of a sine wave consists of two Dirac delta functions at the appropriate frequencies:

$$F\{A\cos(2\pi f_i t+\phi)\} = \frac{A}{2}e^{j\phi}\delta(f-f_i)+\frac{A}{2}e^{-j\phi}\delta(f+f_i). \tag{2}$$

In strict sense, this does not exist, and in measurements we obtain only a secondary function based on this. We collect samples from a *finite-length record*. This operation can be modeled by cutting out the finite-length record from the infinite-length sinusoidal applying multiplication by a window function. The equivalent of this multiplication is a convolution in the frequency domain:

$$F\{A\cos(2\pi f_i t+\phi)\cdot w(t)\} = \frac{A}{2}e^{j\phi}W(f-f_i)+\frac{A}{2}e^{-j\phi}W(f+f_i). \tag{3}$$

The consequence of discrete processing is first of all that the shape of the above-described window function slightly changes because of aliasing, so the results for continuous-time windows only approximately hold. In other words, the window shape becomes slightly dependent on the number of samples, M. Second, in the DFT we see only the samples of the discrete-time window, taken at the frequency bins of the discrete Fourier transform:

$$f_k = k\frac{f_s}{M} = k\cdot\Delta f\,. \tag{4}$$

Now the problem we try to solve is the following: we have M samples in the time or in the frequency domain, corrupted by some noise and by some distortion components. From these samples, we would like to determine the parameters of the sine wave as precisely as possible, then subtract it from the samples, and evaluate the rest for SINAD, THD, etc., maybe using also the amplitude of the sinusoidal.

In the case of no extra windowing, we have the so-called rectangular window. For the continuous-time case:

$$w_{\text{rect}}(t) = \begin{cases} 1 & \text{if } 0 \le t \le T \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

$$W_{\text{rect}}(f) = e^{j\pi f T} T \frac{\sin(\pi f T)}{\pi f T} . \tag{6}$$

When we apply the DFT to discrete points, we have the following:

$$w_{\text{rect}}(i) = \begin{cases} 1 & \text{for } 0 \le i \le M-1 \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

The DFT is clearly a delta function at zero, and zero elsewhere. However, when writing the N-point DFT, we see more:

$$W_{\text{rect},M/N}(k) = e^{j\pi k} \frac{\sin\left(\pi \frac{k}{N} M\right)}{\sin\left(\pi \frac{k}{N}\right)} . \tag{8}$$

The absolute value of $W_{\text{rect},M/N}(k)$ is shown in Fig. 2.

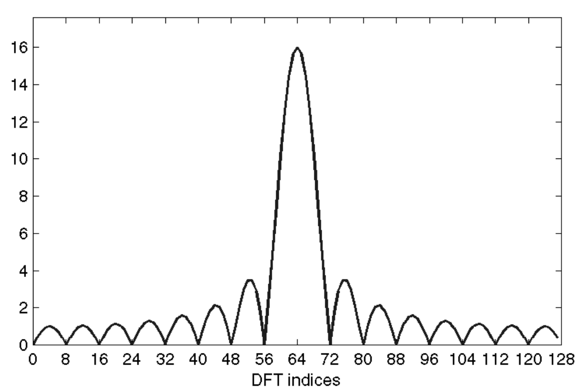

Fig. 2. DFT of the rectangular window, N=128, M=16.

The records obtained by coherent sampling are easy to use. As long as the time domain window consists of at most a few low-frequency sinusoids whose frequencies correspond to the DFT bins, the frequency domain equivalent of the window contains zeros at distances $n \cdot \Delta f$ from the center. As an example, let us consider the Hanning window.

$$w_{\text{H}}(t) = \left(1 - \cos\left(2\pi \frac{1}{T} t\right)\right) w_{\text{rect}}(t) \tag{9}$$

$$\begin{aligned} W_{\text{H}}(f) &= \left(\delta(f) - 0.5\delta\left(f - \frac{1}{T}\right) - 0.5\delta\left(f + \frac{1}{T}\right)\right) * W_{\text{rect}}(f) \\ &= W_{\text{rect}}(f) - 0.5W_{\text{rect}}\left(f - \frac{1}{T}\right) - 0.5W_{\text{rect}}\left(f + \frac{1}{T}\right). \end{aligned} \tag{10}$$

The formulae are similar for the discrete case.

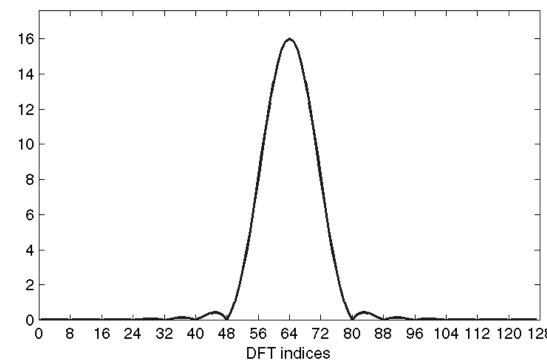

Fig. 3. Hanning window in the frequency domain, N=128, M=16.

Apart from the main lobes, the repeated rectangular windows have zeros at the same places. This means that in the Fourier transform there is only a large central peak, while the sinusoid has no effect on the other bins. Therefore, coherent sampling is usually insensitive to windowing. We will come back to this later. Let us discuss now what we are interested in, and how this is obtained in the case of incoherent sampling.

3. DISCUSSION

Usually, we have one or maximum 2-3 sinusoidal signals. We are interested primarily not in them, but rather in the rest of the spectrum. We usually wish to remove them as profoundly as it is possible, since the remaining part is the error we want to measure. The basic steps are as follows:

- determine the sine parameters from the record,
- subtract (suppress) the sine(s),
- analyze the residuals.

Determination of the sine parameters

Let us look first at a typical DFT result calculated from incoherently sampled data.

In Fig. 4 we see in the individual bins the *samples of the window function*, positioned around the frequency of the sine wave. The exact frequency of the sine wave is somewhere between the adjacent large peak pairs. The true frequency of the sine wave is at most $\Delta f/2$ from the place of the maximum peak, and the maximum peak is also down from the theoretical value (64).

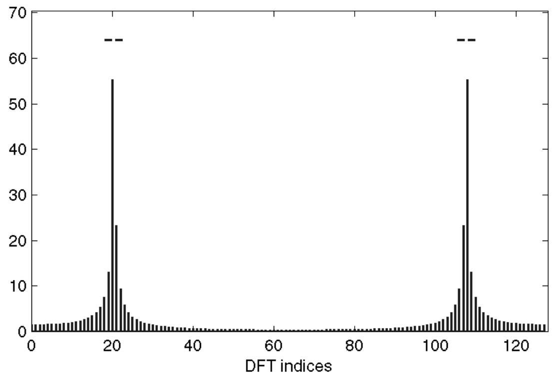

Fig. 4. Typical DFT of an incoherently sampled sine wave, f_i=20.3

This inaccuracy is far too much for our purposes, especially because the value of the maximum also may vary down to about 64% of the true value. There are basically two approaches to tackle these problems.

By the *nonparametric* approach we can significantly improve the approximation of the maximum value by applying the so-called *flat top* window [2]. By this, we modify the shape of the window in order to achieve that the value of the window is essentially constant in the frequency domain between [$-\Delta f/2, \Delta f/2$]. In Fig. 5. the maximum amplitude error is about 0.2% (the difference between heights of the lines, being at a distance Δf from each other, is much larger). However, the determination of the frequency is still not improved – we can only use this method for our purposes when the frequency of the sine wave is known. If we know both the amplitude and the frequency, the determination of the phase is straightforward.

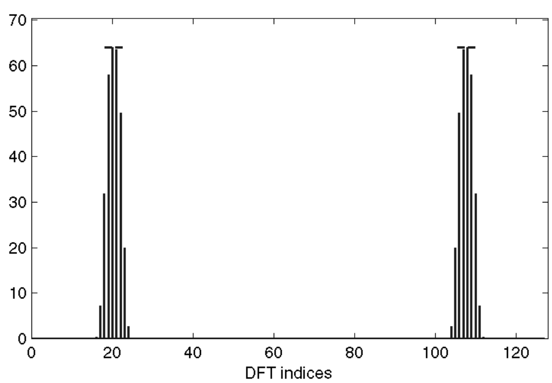

Fig. 5. Incoherent sampling processed with flat top window

Parametric model fitting

A more useful approach is *parametric model fitting*. This can be performed in the time domain or in the frequency domain. Intuitively it is clear that when performed properly, the procedures in the two domains are essentially equivalent. The basic idea in any case is that we assume a certain model of the signal (sine wave with noise, etc), and determine the parameters from the model which fits the best the measured data.

There is however a fundamental problem. We can solve model fitting properly if the model is valid. When any non-modeled phenomena occur, even the best method may yield wrong results. For example, in the case of incoherent sampling, any other harmonics (e.g. harmonic distortion) may disturb the fitting procedure. Then the consequence is that the parameters are imprecise, and therefore the removal of the sine wave is inaccurate. The error we commit may be in the same order of magnitude as the residuals we want to evaluate. This can be a serious problem in the four parameter method [1, 4.1.4.3].

A possibility to circumvent the above difficulties is to *model every important harmonic* which may be present, and after the fit, remove the fundamental sine wave only. The difficulty is that in general it is very difficult to select the frequencies where such harmonics are present. Selecting too many harmonics can make the algorithm slow and sensitive to local minima. The only reasonable way is to extend the four parameter method by a mechanism which takes into account that most such components are harmonically related to each other. Therefore, while there are several components, their frequencies are described by one single parameter ω_i and the (fixed) harmonic numbers only. This approach is not yet described in the draft, maybe because of complications difficult to tackle in advance.

The equivalent of the four parameter method, or of its several-component extension, is to fit the DFT result by a scaled version of the frequency domain form of the discrete window, or by a set of such windows. This is in general more complex than in the time domain, so we do not discuss it further.

When we want to avoid the above-described too complex modeling, and still want to determine the parameters of the sine wave properly, the best way is to avoid that different harmonics disturb the estimates of each other. This is the basic idea of *windowing*.

Windowing

From Fig. 4 it is straightforward that each incoherently sampled sine wave results in components at the other frequency bins (leakage). The cause is the form of the rectangular window: its sidelobes are too large. The idea is then to modify the shape of the window function to have as small sidelobes as it is possible.

Here, there are two approaches again. One is the use of so-called *harmonic windows*: windows which are the sum of a few low-frequency sinusoidal functions, which therefore have a series of zeros for coherent sampling (see Eq. (10)). This gives the so-called Blackman-Harris windows [3,4].

However, we can realize that the requirement of the zeros (for coherent sampling) is not necessary. We can speculate that it is enough to prescribe that the sidelobes do not surpass a certain level. This leads to the designed low-ripple windows, like the Kaiser (Kaiser-Bessel) window [3,4,9] or the Dolph-Chebyshev one [3,4,8].

In general, we may observe when we prescribe the frequency domain behavior, and wish to design a symmetric time series to this, we just design a linear phase FIR filter. Consequently, any good FIR filter design algorithm, e.g. the well-known REMEZ algorithm can be used to obtain the best window we can have at all.

It is true for each window that sidelobe suppression is at the cost of widening the main lobe. Therefore, windowing somewhat decreases selectivity: non-overlapping components may not be closer than a few bins (a few times Δf). Therefore, the frequency of the sine wave must be larger than a few times $\Delta f = f_s/M$ for proper processing.

In the light of the above discussion, we can probably refine the statement in the draft ([1, Section 4.1.5]: "The window functions are chosen in a trade off between the effective noise bandwidth (ENBW), or resultant DFT bins, and minimum stopband response of the window filter function as discussed in the following clause." In the above light, ENBW is not very important. The proper selection of the window function can be as follows: select the one with the allowed stopband response, with as small mainlobe bandwidth[1] as it is possible. Now the value of the ENBW will be determined by the window: it can be appropriately calculated from the window samples. Its definition may however deserve a little explanation. First of all, let us observe that it is amplitude scaling independent, and its value (see Eq. 4.1.5.1.4) is exactly 1 for the rectangular window.

Let us first discuss the scaling of the windows. There are a few strategies for this. We can

a) keep the scaling of the frequency domain peaks of sinusoidals constant (that is, maintain the same value of the time domain integral), or
b) keep the variance of a white noise sequence constant.

In this paper we follow the first strategy. By this the height of the peak of the Fourier transform of a sine wave is the same as without windowing[2].

For random signals, we cannot give a measure in terms of the amplitude, only in terms of the standard deviation or the variance. The integral of the window function remains constant (the denominator in the expression of the ENBW), while the variance changes for white noise in the following way:

$$\mathrm{var}_{\mathrm{ch}} = \frac{\frac{1}{M}\sum_{n=0}^{M-1} w^2(n)}{\frac{1}{M}\sum_{n=0}^{M-1} w_{\mathrm{rect}}^2(n)} = \frac{1}{M}\sum_{n=0}^{M-1} w^2(n) = ENBW. \quad (11)$$

Because of Parseval's theorem, this is also true in the frequency domain:

$$\mathrm{var}_{\mathrm{ch}} = \frac{\sum_{k=0}^{M-1} W^2(k)}{\sum_{k=0}^{M-1} W_{\mathrm{rect}}^2(k)} = \frac{1}{M}\sum_{n=0}^{M-1} w^2(n) = ENBW. \quad (12)$$

Therefore, the ENBW accounts for the change in the noise variance. It is equal to the multiplicative term, and its square root corresponds to the multiplicative term in the standard deviation.

We know now how the amplitude of the transform of a windowed sine wave changes, and we also know how the variance or the standard deviation of the noise peaks change. The last thing we have to discuss is whether the power of a sine wave can also be calculated from the spectrum, without the necessity of calculating the amplitudes. Here the precise answer is unfortunately no. The sum of squares of the frequency domain samples gives the power of the sine *in the window*, and this can be different from the power of the continuous-time signal. However, the deviation is usually not large. If we need the approximate power of the harmonic (as for the total harmonic distortion), the sum of the squares of the frequency domain samples, divided by ENBW and by M, gives an approximate value of the power. The result is not fully accurate (see Fig. 6), since it slightly depends on the frequency of the sine, but will still yield a good enough approximate value of the power we are looking for.

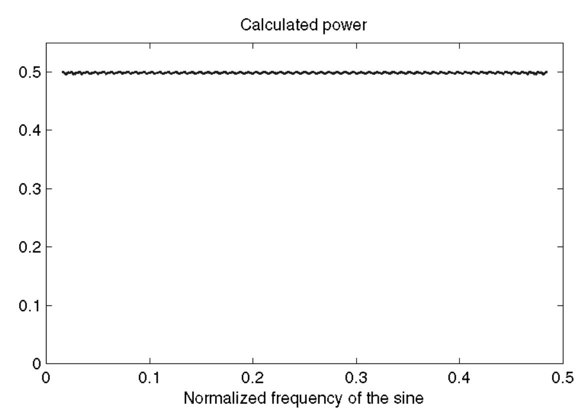

Fig. 6. Value of the power calculated from the adjacent lines, as a function of the frequency. Kaiser window with beta=3, M=128.

Now that we successfully separated the effect of the different harmonics, we can turn back again to the determination of the parameters. Now our problem is simplified to the following one: given a few frequency domain samples (around the sine frequency), determine the parameters.

The standard way for this is the so-called Interpolated FFT [5]. This is a set of algorithms which, based on some approximation, determine the parameters of the sine wave from the samples around the maximum. This gives a good estimate of the amplitude and of the frequency.

Another possibility is as follows. In testing we usually have some time for off-line calculations. There is a simple algorithm for this purpose. If we select all the samples from

[1] Keeping sine waves in mind, let us define the width of the mainlobe as the distance between the two stopbands, because this determines the minimum necessary distance between non-overlapping harmonics.

[2] This is at least theoretically true – for non-coherent sampling the maximum value of the spectrum can be smaller, depending on the frequency mismatch and the shape of the window. The smaller the frequency mismatch, the closer the maximum to the theoretical value is.

the main lobe, and set the rest to zero, we have with good approximation the Fourier transform of a windowed sine wave. Taking the IDFT of the two selected line groups, we obtain the time samples of the windowed sine wave. When dividing by the window function (and discarding those values which are divided by a very small number where the window function is close to zero), we can use the four parameter method to determine the desired parameters. This fairly simple procedure has not been described in the draft yet.

Proper removal of the sine components

As we already mentioned above, removal of the sine is only effective if the sine parameters are precisely known. Inaccurate estimation of the parameters is usually a problem because the sides of the sidelobes in Fig. 2 are steep, so a small error in the removal may cause a large residual. This is the main issue for windowing: we have to accept that the values of the sine parameters are not exact, but if correction is necessary which is comparable to the quantities to measure (harmonic distortion components, noise), accurate characterization of the ADC is illusory.

The removal is fairly easy when there are no significant sidelobes. We simply discard the group of the few large samples. We can even subtract the sine from the samples, take the ordinary DFT without windowing (or with some windowing again for the determination of the largest peaks, like needed for SFDR), and evaluate the result.

Analysis of the residuals

As we have seen above, we can have both the windowed and the non-windowed version of the residuals, as we like. The only question is which method is sensitive to incoherent sampling, and what is the result of windowing.

THD [1, Section 4.1.5] The draft standard says about the Total Harmonic Distortion test:

"The test described above is based on DFT analysis (e.g., via DFT) on *unwindowed* sample sets." … "Windowing is *not recommended* for these procedures due to the widening of resolution bandwidth."

Based on the above discussions, we can state the following. If we talk about the effect of periodic components, like in the case of the THD, it is logical to use a close-to-flat-top window after the removal of the sine wave. The precision of the flatness depends on the requirement for the measurement of the amplitudes: this is usually not very strict. Read the amplitudes, and sum up the power accordingly. The price we pay is that the noise floor increases, and we *must not have harmonics too close* to each other. These disadvantages are usually tolerable.

Another possibility is to select the GROUPS of lines belonging to the harmonic peaks (the number depends on the window), add the squared absolute values together, and divide by the ENBW and M^2. This will give a good estimate of the power of the harmonic, with lower noise floor.

SFDR [1, Section 4.4.5.3] The standard does not discuss the effect of windowing to the Spurious-Free Dynamic Range. However, the calculation is straightforward: take the average of the windowed DFT's calculated with an approximate flat top window (with possibly small bandwidth to prevent aliasing), and look for the largest peak.

SINAD [1, Section 4.5.1] Here we want to calculate the power of the residuals to have the Signal to Noise and Distortion Ratio. This is straightforward again: use the non-windowed residuals, and add the squared absolute values, or do the same with windowed data, and divide by the ENBW value.

4. SUGGESTIONS

In the previous sections, we have formulated suggestions to extend and improve the draft standard IEEE 1421. We feel however that the design of window functions, displaying their properties, evaluating complex algorithms with no programming bugs etc. is a difficult task for most users. On the other hand, nobody can undertake the job for developing all algorithms for all important computers, ready-to-use but also flexible enough for special purposes. The suggestion is that EUPAS, maybe jointly with the draft committee, make model algorithms publicly available via a WEB site, in order to allow that people can compare their algorithms to standard ones, have quick solutions for a few cases. The author is ready to work on such an undertaking based on Matlab itself and on function M-files.

5. ACKNOWLEDGMENT

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