Experimental modal analysis of a cavity using a calibrated acoustic actuator

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Abstract
In this paper, the acoustical modal analysis of a rectangular shallow cavity is performed. It is shown that a suitable choice for the acoustical excitation is the volume acceleration and, for the acoustical response, the sound pressure. When using a Finite Element model and an analytical model, it is shown that the computed Frequency Response Functions must be multiplied by the mass density of the air to yield units of Pascal per unit volume acceleration (\( \rho \)), which are straightforward to obtain experimentally. In the experiments, two types of excitation devices were used. The first utilizes a shaker-driven piston which thrusts against a thin rubber membrane stretched flush to one of the cavity side walls, covering a cylindrical hole. The other actuator was built based upon a research report developed in an EEC project (Brite-EuRam II: PIANO). This acoustic actuator has a high impedance (higher than any practical surrounding impedance) so that the impedance of the cavity does not need to be considered in the calibration factor relating the microphone signal and the source strength. A good agreement was obtained in the comparisons between experimental, analytical and numerical Frequency Response Functions and modes.

1. Introduction
The practical importance of acoustical modal analysis has increased in recent years. In the experimental domain, some difficulties are still to be solved and the present work addresses some of them, namely the unit corrections which are necessary to allow the comparison between analytical, numerical and experimental results, the acoustic excitation realization, and the mode shape visualization.

In the current literature on acoustical systems, little attention is paid to defining excitation and response acoustic variables such that an experimental modal analysis is feasible. Augusztinovicz and Sas [5] have addressed this problem. They have proposed a formulation where volume acceleration is the input variable and pressure the response variable in the dynamic equations of the acoustical system. Pressure may be easily measured with microphone, while volume acceleration can be produced by calibrated sound sources such as loudspeakers in specially designed configurations.

Nieter and Singh in [6] developed a methodology for acoustical modal analysis where the same tools applied in solid mechanics (Fourier analyzers, modal parameter extraction methods, etc) are used. The technique faces no problem with the pressure measurement, which can be done with microphones or very sensitive pressure transducers, but has to deal with the acoustic input (volume acceleration of the fluid) that has no direct measurement, due to the lack of a particle velocity transducer. The solution found was the use of a piston driven by a shaker with an accelerometer attached. Later work by Singh and Kung [7] proposes another solution, based on a horn-drive loudspeaker, where the volume velocity is monitored by a microphone mounted on a small enclosure of known volume at the back of the driver. The acoustic driver was mounted directly at the point of excitation in the acoustic system being tested.

The problem related to the acoustic mode visualization is treated by Whear and Morrey in [8], where a probe with three aligned microphones gives, using a second-order finite difference calculation, the second-order derivative of the pressure relative to the space. Given that the first-order derivative is related (Euler’s equation) to the particle acceleration, the second-order derivative, while being directional, will still exhibit the same nodes and antinodes as the pressure distribution. The disadvantage of this method is the noise amplification effect of differentiating twice the measured pressure field. Another approach is given
by Byrne [9] who uses the pressure measured at an array of points in the acoustic experimental domain to extract approximating functions formed by polynomials to calculate the pressure gradient in one specific direction. Given the pressure gradient, the particle acceleration is readily calculated by the Euler’s relation.

In the present work, a simple geometry (rectangular shallow cavity) is used to investigate the acoustical modal analysis methodology. Analytical and numerical models are developed for the comparison with the experimental results. Two types of excitation devices were constructed. One based on a shaker-driven piston and another using a horn-drive loudspeaker. The latter is similar to a model developed by an EEC project [2] and has an advantage over the direct use of the driver located at the excitation point. This advantage is due to the use of a tube conducting the air volume acceleration, which allows the actuator to be easily placed at different locations in a confined space. The experimental modal analysis is performed, and a method using multi-dimensional spatial Fourier transforms of the array of pressure measurements is proposed for the construction of the particle displacement field for acoustic mode visualization.

2. Analytical model

The general solution for the acoustic modes in a rectangular cavity is a linear combination between the solutions for each mode \( r \) \( (l,m,n) \)

\[ p_r = \sum_r A_r \phi_r \]  

(1)

The wave-vectors \( (k_{xl}, k_{ym}, k_{zn}) \) are calculated from the boundary conditions of zero velocity in the walls \( \nabla \cdot \mathbf{v} = 0 \), and the mode \( \phi_r \) is written

\[ \phi_r = \cos(k_{xl}x) \cos(k_{ym}y) \cos(k_{zn}z) \]  

(2)

Inserting the modal solution for the pressure (1) in the homogeneous wave equation

\[ \nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0 \]  

(3)

the circular natural frequency for each mode is given by

\[ \omega_{l,m,n} = \sqrt{\left(\frac{l \pi}{L_x}\right)^2 + \left(\frac{m \pi}{L_y}\right)^2 + \left(\frac{n \pi}{L_z}\right)^2} \]  

(4)

Applying the modal superposition in the non-homogeneous wave equation

\[ \nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = -\frac{\partial G}{\partial t} \left[ \frac{kg}{m^3a^2} \right] \]  

(5)

the Frequency Response Function (FRF) is calculated

\[ FRF (x, y, z, \omega) = \sum_{r=1}^{\infty} \frac{\phi_r (x_0, y_0, z_0) \phi_r (x, y, z)}{\mu_r (\omega_r^2 - \omega^2)} \]  

(6)

where

\[ \mu_r = \frac{I_r}{c^2} \]  

(7)

and

\[ I_r = \int_{x_0}^{x_0 + L_x} \int_{y_0}^{y_0 + L_y} \int_{z_0}^{z_0 + L_z} \phi_r^2 dx dy dz \]  

(8)

This FRF equation has the unit \( \left[ \frac{m^2}{s^2} \right] \), which is not suitable for the comparison with the experimental data. The experimental FRF unit is given by the pressure response taken at the microphone unit divided by the volume acceleration being injected in the domain, that is, \( \left[ \frac{m^2}{s^2} \right] \). For this comparison to be made, it is necessary to multiply the analytical FRF by the air density

\[ FRF (x, y, z, \omega) = \rho_{air} \sum_{r=1}^{\infty} \frac{\phi_r (x_0, y_0, z_0) \phi_r (x, y, z)}{\mu_r (\omega_r^2 - \omega^2)} \]  

(9)

3. Finite element model

The two-dimensional wave equation

\[ \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = d(x, y) \]  

(10)

will be solved using the Finite Element Method (FEM). The choice of the 2D numerical analysis is related to the fact that one of the dimensions of the rectangular cross-section cavity chosen is much smaller than the other two. Due to this, the modes in the frequency range of interest (0 – 2000Hz) will be in the directions of the two longer dimensions.

The FEM model [3] used in the analysis is based on linear two-dimensional triangular elements (Figure 1), and the dynamic matrix system is written
4. Actuator development

The development of this acoustic actuator was based on [2], which presents a solution to some requirements to be met by the sound source, such as: the sound level, the frequency range of the produced sound, the directivity pattern (omnidirectional), and the relation between the volume velocity produced and the surroundings (i.e., the source acoustic impedance must be high). The proposed solution is the use of a horn-drive loudspeaker (high impedance loudspeaker) connected to a plastic tube with a microphone mounted on a metal socket at the other end. The high power of this type of loudspeaker assures that the impedance of the actuator is higher than any practical acoustic surrounding impedance, and the small diameter of the tube (12.7 mm) will smooth the peaks and dips generated by stationary waves. It is very important that the acoustic impedance of the actuator is higher than the surrounding’s acoustic impedance in a large frequency range, in a way that this actuator practically does not depend on the acoustic system volume it acts.

The calibration of this actuator will be based on its monopole-like behavior, which will be verified in an anechoic room. The pressure field generated by a monopole in a free field [1] is

\[ p(r, t) = \frac{i \rho_0 c k}{4 \pi r} \frac{Q}{\rho_0 c k} e^{i(\omega t - kr)} \]  

where \( Q \) is the source strength (volume velocity)

\[ Q = \frac{\dot{\rho}(r)}{\rho_0} \frac{4 \pi r}{i \rho_0 c k} \]  

Supposing a monopole-like behavior (omnidirectional), a complex transfer function \( \hat{H} \) can be written as the ratio between the pressures at some radius \( r \) from the source and the point source \( \dot{\rho}(0) \)

\[ \hat{H} = \frac{\dot{\rho}(r)}{\dot{\rho}(0)} \]  

The source strength is rewritten

\[ Q = \dot{\rho}(0) \frac{4 \pi r}{i \rho_0 c k} \]  

and the volume acceleration is calculated

\[ \dot{\mathbf{q}} = \frac{\partial Q}{\partial t} = i \omega Q = 4 \pi r \frac{m^3}{s^2} \]  

The power spectral densities \( G_{xx} = \langle \dot{\rho}(0)^* \cdot \dot{\rho}(0) \rangle \) and \( G_{rr} = \langle \dot{\rho}(r)^* \cdot \dot{\rho}(r) \rangle \), and

\[ \begin{bmatrix} E \{ \ddot{p} \} + [H] \{ p \} = \{ D \} \end{bmatrix} \]  

where \([E]\) is the compressibility matrix and \([H]\) the volumetric matrix. An ad hoc routine implemented in MATLAB was used to generate these matrices. The homogeneous system \((D = 0)\) gives the eigenvalues \( \omega_0^2 \) and the eigenvectors \( \{ \psi_r \} \) by solving the eigenvalue problem

\[ [H] - \omega_0^2 [E] \{ \psi_r \} = 0 \]  

Applying harmonic oscillations of the pressure response and the source volume acceleration

\[ \{ p \} = \left\{ \ddot{P} \right\} e^{i \omega t} \]  

\[ \{ D \} = \left\{ \ddot{D} \right\} e^{i \omega t} \]  

into the non-homogeneous system (11), the FRF \( \hat{F}(\hat{D}) \) is calculated

\[ \text{FRF}_{ij} = \left( [H] - \omega_0^2 [E] \right)^{-1}_{ij} \]  

where \( i \) and \( j \) are the response and the excitation nodes, respectively.

The source matrix \( \{ D \} \) results from an integration over the element volume

\[ \int_V d \left[ \frac{kg}{m^3/s^2} \right] dV = \{ D \} \left[ \frac{kg}{s^2} \right] \]  

The FRF unit becomes \( \left[ \frac{Pa}{kg/s^2} \right] \), needing to be multiplied by the air density to equalize the units of the FRF constructed experimentally

\[ \text{FRF}_{ij} = \rho_{air} \left( [H] - \omega_0^2 [E] \right)^{-1}_{ij} \left[ \frac{Pa}{m^3/s^2} \right] \]
the cross spectral density $G_{0r} = \langle \hat{p}(0)^* \cdot \hat{p}(r) \rangle$ will be useful in verifying the monopole-like behavior of the source. The “$r$” and the “0” subscripts represent the distance from the source and the point source, respectively.

The transfer function $\tilde{H}$ can be written as

$$\tilde{H} = \frac{G_{0r}}{G_{00}}$$

(23)

The pressure at “$r$” is given by

$$\hat{\rho}(r) = \tilde{H} \hat{\rho}(0)$$

(24)

and the power spectral density

$$G_{rr} = \langle \hat{\rho}(r)^* \cdot \hat{\rho}(r) \rangle = \left| \tilde{H} \right|^2 G_{00}$$

(25)

will be compared with the direct measurement at the microphone at a distance “$r$” from the source.

5. Experimental settings

The box containing the cavity was built in wood, except for its upper wall, made of plexi-glass to facilitate the microphone positioning. One concern of this construction was the stiffness of the walls. A small cavity (254mm × 199mm × 30mm) with relative high width walls avoided the vibroacoustic coupling in the range of frequencies analyzed, leading to the study of the uncoupled acoustic modes only. The experimental scheme of the cavity excited by the loudspeaker actuator is presented in figure 2, where a 1/4” electret microphone (nominal sensitivity of 25mV/Pa) has its positioning made by thin nylon strings.

The shaker-driven piston actuator (Figure 3) was composed by a small shaker, a 21.5mm diameter PVC piston, a thin rubber membrane stretched flush to one of the cavity side walls covering a cylindrical hole, and a piezoelectric accelerometer (nominal sensitivity of 100mV/g). The air volume acceleration is given by the accelerometer signal times the piston area.

The loudspeaker actuator uses a horn-drive loudspeaker with a nominal impedance of 8Ω at 1200Hz and a power of 100W. Its range of operation is 330 – 7600Hz. The connection tube has a diameter of 12.7mm, and the microphone on the socket at the end of this tube is equal to the one used inside the cavity.

6. Results

The monopole-like behavior of the loudspeaker actuator is verified by comparing the power spectral densities calculated, with equation (25), and measured. Figure 4 shows the results for the response microphone positioned 200mm far from the source in an anechoic chamber (environment simulating a free field). The curves are very coincident after 400Hz, which is the chamber cut-off frequency. Another six points 200mm far from the source where investigated, and their calculated and measured results showed the same similarity. The monopole-like behavior is finally validated by the calculated power spectral density comparison in the six points (Figure 5), which show good agreement.

The transfer function $\tilde{H}$, equation (23), is generated for a distance of 200mm from the source and a frequency range of 0 to 2000Hz. The source volume
acceleration is readily calculated from equation (22). Figures 6 and 7 present the comparison between analytical, equation (9), numerical, equation (17), and experimental (using the loudspeaker and the shaker-driven piston actuators) results at nodes 103 and 54 for the excitation at node 112 (see figure 1).

For the modal parameter extraction was used the Exponential Complex Method [4], and table 2 shows the error when the two experiments are compared with the analytical model. The results using the shaker-driven actuator are extremely close to the analytical and numerical results and it is important to notice its great efficiency at low frequencies. The loudspeaker actuator also demonstrated good agreement with the analytical and numerical results. The bad representation at low frequencies is explained by its low efficiency below 330 Hz, and its larger modal damping ($\xi$) estimates (Table 1) may be explained by the air volume of the actuator tube. In the analytical and numerical models, an artificial damping of 0.011 was inserted to account for the air friction in the walls, the small gaps between the plates composing the box and the apertures made for the nylon strings and microphone cable, which were the most important sources of damping in the experiment with the shaker-driven piston.

The errors found in the comparison between numerical and analytical results (Table 3), mainly in the higher frequencies, are given by the rough discretization of the acoustic domain. The use of more elements tends to decrease the errors in this comparison.
Table 1: Experimental modal parameters extracted

<table>
<thead>
<tr>
<th>mode</th>
<th>ξ</th>
<th>( f_n ) (Hz)</th>
<th>ξ</th>
<th>( f_n ) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shaker-driven piston</td>
<td>Loudspeaker</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0138</td>
<td>676.8</td>
<td>0.0172</td>
<td>695.2</td>
</tr>
<tr>
<td>2</td>
<td>0.0111</td>
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<td>0.0161</td>
<td>875.0</td>
</tr>
<tr>
<td>3</td>
<td>0.0169</td>
<td>1094.3</td>
<td>0.0172</td>
<td>1128.1</td>
</tr>
<tr>
<td>4</td>
<td>0.0100</td>
<td>1359.4</td>
<td>0.0130</td>
<td>1368.9</td>
</tr>
<tr>
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<td>0.0101</td>
<td>1608.7</td>
<td>0.0109</td>
<td>1623.5</td>
</tr>
<tr>
<td>6</td>
<td>0.0081</td>
<td>1719.0</td>
<td>0.0088</td>
<td>1731.3</td>
</tr>
<tr>
<td>7</td>
<td>0.0110</td>
<td>1858.2</td>
<td>0.0095</td>
<td>1875.9</td>
</tr>
</tbody>
</table>

Table 2: Errors in the experimental natural frequencies with respect to the analytical calculation

<table>
<thead>
<tr>
<th>mode</th>
<th>analytical/loudspeaker (%)</th>
<th>analytical/shaker-driven piston (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.96</td>
<td>0.24</td>
</tr>
<tr>
<td>2</td>
<td>1.53</td>
<td>0.55</td>
</tr>
<tr>
<td>3</td>
<td>3.04</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>1.37</td>
<td>0.67</td>
</tr>
<tr>
<td>5</td>
<td>1.34</td>
<td>0.42</td>
</tr>
<tr>
<td>6</td>
<td>0.45</td>
<td>0.27</td>
</tr>
<tr>
<td>7</td>
<td>1.34</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Table 3: Errors in the numerical natural frequencies with respect to the analytical calculation

<table>
<thead>
<tr>
<th>mode</th>
<th>( f_n ) (Hz)</th>
<th>( f_n ) (Hz)</th>
<th>error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FEM</td>
<td>analytical</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>675.2</td>
<td>675.9</td>
<td>0.10</td>
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<tr>
<td>2</td>
<td>861.8</td>
<td>865.3</td>
<td>0.41</td>
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<tr>
<td>3</td>
<td>1094.8</td>
<td>1102.8</td>
<td>0.73</td>
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<tr>
<td>4</td>
<td>1350.4</td>
<td>1355.9</td>
<td>0.41</td>
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<tr>
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<td>1602.0</td>
<td>1621.3</td>
<td>1.20</td>
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<td>1723.6</td>
<td>1751.9</td>
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<tr>
<td>7</td>
<td>1851.1</td>
<td>1889.1</td>
<td>2.05</td>
</tr>
</tbody>
</table>

Figure 7: Comparison between analytical [- - -], numerical [- -] and experimental [-] results at node 54
(a) Shaker-driven piston (b) Loudspeaker

The acoustic mode shape visualization can be made using either the pressure or the particle displacement fields. Figure 8 shows the contour plots of the pressure for the eighth mode (1602Hz) obtained analytically and experimentally. It should be noted that the poor results at the bottom right corner of the experimental mode shape of figure 8(b) is due to the fact that the corresponding location was inaccessible to the measurement microphone. In the three-dimensional case, the use of orthogonal plans slices allows the visualization of the entire volume.

The particle displacement mode shape can be computed from the pressure mode shape in the wavenumber domain using the spatial Discrete
representing the data by a two-dimensional regressive discrete Fourier series (RDFS), proposed by Arruda [10], can be used.

Figure 9 presents the analytical particle displacement field for the eighth mode. When using particle displacement mode shapes, there is no difficulty in visualizing modes in three-dimensions using a wireframe representation, as it is usually done in structural modal analysis.

7. Conclusions

The unit corrections which are necessary to compare analytical, numerical and experimental acoustic frequency response were exposed. Analytical and numerical methods predicting the acoustics of a rectangular cross-section shallow cavity were developed and the experimental modal analysis was performed with two different acoustic actuators. Comparison between analytical, numerical and experimental results showed good agreement. The experiment with the shaker-driven piston actuator presented high efficiency at low frequencies and smaller errors in the natural frequencies when compared to the analytical model. The loudspeaker actuator performed very well in a cavity of relatively high impedance. Its independence of the practical acoustic surrounding was proved. Its low efficiency at low frequencies (0 –
330 Hz) is due to the horn-drive loudspeaker characteristics. Two ways of visualizing acoustic modes (pressure and displacement fields) were shown, and a methodology to compute the particle displacement field from pressure measurements in the wavenumber domain using multi-dimensional Fourier transforms was proposed.

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