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A NOVEL DIFFERENTIAL CHAOS SHIFT KEYING MODULATION SCHEME

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In binary Differential Chaos Shift Keying (DCSK), the reference and information bearing chaotic wavelets are transmitted in two consecutive time slots. This TDMA approach provides two independent channels for the transmission of reference and information bearing wavelets but requires a delay component both in the modulator and demodulator circuits, furthermore, it halves the data attainable data rate. The wideband Radio Frequency (RF) delay lines at receiver are extremely difficult to implement with CMOS technology, therefore, the DCSK modulation cannot be exploited in many applications, such as ultra-wideband. To avoid the use of wideband RF delay lines at receiver, an alternative solution is proposed here where both the reference and information bearing wavelets are sent in the same time slot. The two wavelets are separated by Walsh codes instead of time delay. The new modulator and demodulator configurations are given, analytical expressions for the Bit Error Rate (BER) are derived and the derived BER expressions are verified by computer simulations over Additive White Gaussian Noise (AWGN) and multipath Rayleigh fading channels.

Keywords: Chaotic modulation; Code-Shifted DCSK (CS-DCSK); delay line; Bit Error Rate (BER); multipath Rayleigh fading channel.

1. Introduction

Chaos-based communication theory and, particularly, chaotic modulation schemes have been a hot research topic recently where, contrary to conventional modulation schemes, a wideband, non-periodic chaotic signal is used as carrier [Lau & Tse, 2003]. The wideband power spectral density, the excellent auto-correlation and cross-correlation properties of chaotic waveforms equip the chaos-based modulation scheme with a high robustness against multipath fading even in severe multipath environment. To satisfy the demands of different applications, many chaotic modulation schemes have been proposed, analyzed and optimized [Dedieu *et al.*, 1993; Kolumbán *et al.*, 1996; Kolumbán *et al.*, 1998; Kennedy *et al.*, 2000; Xia *et al.*, 2004; Ye *et al.*, 2005; Yao & Lawrance, 2006; Wang *et al.*, 2008], among which frequency-modulated differential chaos shift keying (FM-DCSK) modulation offers the best multipath performance. Binary DCSK and FM-DCSK belong to the class of transmitted-reference (TR) system [Rushforth, 1964] and they offer an alternative solution to spread-spectrum (SS) communication [Ye *et al.*, 2005]. Binary DCSK and FM-DCSK can be demodulated with a simple autocorrelation receiver (AcR) that can capture the entire signal energy without requiring the acquisition and synchronization of a spreading code and channel estimation [Kolumbán *et al.*, 1996].

In binary DCSK/FM-DCSK, one bit duration is divided into two equal time slots where a reference chaotic wavelet is transmitted in the first slot. Depending on the information bit to be transmitted, the information bearing wavelet, transmitted in the consecutive time slot, is either a repeated or inverted copy of the reference wavelet. The separation of reference and information bearing wavelets is done in the time domain and is implemented by a delay component at both the transmitter and receiver.

Recently, ultra-wideband (UWB) systems based on binary DCSK and FM-DCSK modulation schemes have been proposed [Kolumbán, 2005; Chong & Yong, 2008; Min *et al.*, 2010]. The UWB transceivers can be implemented by both digital and analog technology.

Due to the ultra-high sampling rate required, the all digital implementations of UWB receivers have an extremely high power consumption [Casu & Durisi, 2005]. The analog receiver configurations can remarkably reduce the power consumption, but the ultra-wideband RF delay lines required are extremely difficult to implement in CMOS [Bagga *et al.*, 2005; Stralen *et al.*, 2002]. For instance, a delay of 20 ns is realized with a 20 foot coaxial cable in [Stralen *et al.*, 2002]. Clearly, this is not an applicable solution in an integrated receiver. Hence, the development of an alternative architecture where the need for analog RF delay lines is minimized or avoided is essential.

The carrier is a chaotic wavelet in DCSK, while the chaotic signal is fed into an FM modulator and the FM modulator output is used as carrier in FM-DCSK. Otherwise, the two modulation schemes are identical, consequently, unless otherwise stated, only the DCSK modulation scheme is considered here. The rest of the paper is organized as follows: Starting from the multi-level DCSK modulation, Sec. 2 introduces the transceiver architecture for the CS-DCSK modulation scheme and defines its signals. Section 3 provides a detailed analysis of CS-DCSK BER performance and derives expressions in analytical form for the different channel conditions. Section 4 verifies the analytical BER expressions developed by computer simulations and compares the BER performances of the already known DCSK and new CS-DCSK modulation schemes. The conclusions are drawn in Sec. 5.

2. Principle of CS-DCSK Scheme

Starting from the M-ary DCSK modulation scheme and the block diagram of the generic DCSK transceiver, this section introduces the CS-DCSK transceiver architecture and derives the observation signal of the new modulation scheme.

2.1. *Model of the generic DCSK transceiver*

Exploiting the Walsh code, the binary FM-DCSK modulation scheme was generalized and the generic transceiver architecture for the M-ary FM-DCSK modulation scheme was elaborated in [Kis, 2005]. In a strict sense, the M-ary FM-DCSK does not belong to the class of TR systems, it is a generalization of the binary case exploiting the orthogonality of Walsh code sequences. The binary FM-DCSK which is defined by the second order Walsh code is a TR system.

Some ideas of M-ary FM-DCSK has been exploited to derive the CS-DCSK modulation. Only the DCSK case is considered here since a FM-DCSK transceiver can be derived from the DCSK case by exchanging the chaotic carrier wavelet with an FM chaotic one at the transmitter.

Let W_{2^n} define the Walsh code of order 2^n, $n = 0, 1, \ldots,$ and $W_{2^0} = W_1 = [+1]$. The Walsh code of order 2^n is constructed by Hadamard matrix

$$W_{2^n} = \begin{bmatrix} W_{2^{(n-1)}} & W_{2^{(n-1)}} \\ W_{2^{(n-1)}} & -W_{2^{(n-1)}} \end{bmatrix}. \tag{1}$$

Let w_m denote the mth row of W_{2^n}, $m = 1, 2, \ldots, 2^n$. Each row of Walsh code defined by Eq. (1) gives a Walsh code sequence. The order of Walsh code determines the size of alphabet.

Consider an alphabet set $\{0, 1, \ldots, M-1\}$ of symbols to be transmitted. Each symbol $b \in \{0, 1, \ldots, M-1\}$ is mapped into a Walsh code sequence. If the transmission of a single isolated symbol is considered then the radiated signal $s_b(t)$

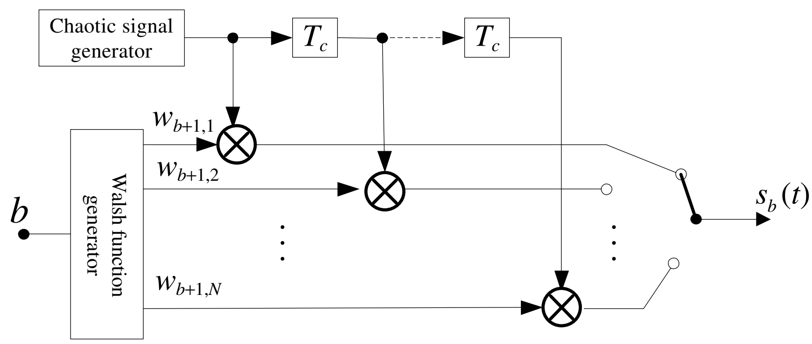

Fig. 1. Transmitter of M-ary DCSK based on Walsh code sequences.

is obtained as

$$s_b(t) = \sum_{k=0}^{N-1} w_{b+1,k+1} c(t - kT_c),$$
$$b = 0, 1, \ldots, M-1 \quad (2)$$

where $c(t)$ is the chaotic carrier wavelet. $T_s = NT_c$ denotes the symbol period and T_c is the duration of the chaotic carrier, i.e. the chip time. Note, in a binary modulation scheme $b \in \{0, 1\}$, the block diagram of M-ary DCSK modulator is derived from Eq. (2) and is depicted in Fig. 1.

Only the energy detector (ED) based receivers are feasible in chaotic communication. The ED-based detection algorithm for M-ary DCSK signals can be derived from the Generalized Maximum Likelihood (GML) decision rule [Kolumbán *et al.*, 2004; Lau *et al.*, 2003], the block diagram of an M-ary DCSK demodulator is shown in Fig. 2. The received signal $r_b(t)$ is fed into a delay component including $N-1$ elements, each having a delay of T_c. The output signal of each tap is multiplied by the corresponding element of the Walsh code sequence and the products are summarized. Since the different Walsh code sequences are orthogonal, the probability of receiving the lth symbol in an AWGN channel is characterized by the symbol energy $E_{l,b}$. The decision is done in favor of the symbol that is characterized by the largest symbol energy. The block diagram of the M-ary DCSK demodulator is shown in Fig. 2.

2.2. *CS-DCSK scheme*

The generic DCSK transceiver shown in Figs. 1 and 2 offers a multilevel modulation scheme that provides an optimal solution if the class of ED-based detectors is considered over an AWGN channel. Unfortunately, the implementation of M-ary DCSK receiver requires many RF delay lines that almost cannot be implemented in CMOS. The main feature of CS-DCSK modulation scheme proposed here is that the CS-DCSK receivers avoid the RF delay lines.

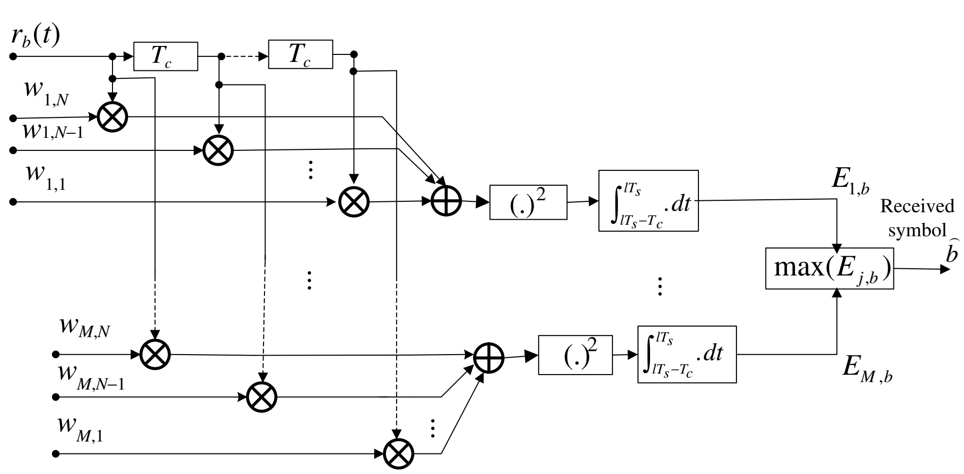

Fig. 2. Receiver of M-ary DCSK based on Walsh code sequences.

The CS-DCSK modulation is a binary modulation scheme where the reference and information bearing chaotic wavelets are transmitted in the same time slot but they are separated by the orthogonality of Walsh code sequences. If transmission of a single isolated symbol is considered then the transmitted CS-DCSK signal is obtained as

$$s_b(t) = \sum_{k=0}^{N-1} w_{R,k+1} c(t-kT_c) + a \sum_{k=0}^{N-1} w_{I,k+1} c(t-kT_c) \quad T_s = NT_c \tag{3}$$

where $a \in \{-1,+1\}$ is mapped from $b \in \{0,1\}$ which is the information bit to be transmitted. $w_{R,k}$ and $w_{I,k}$, respectively, denote the elements of two different Walsh code sequences used to distinguish the reference and information bearing wavelets and $c(t)$ is a chaotic wavelet with duration of T_c.

Equation (3) shows that both the reference and information bearing wavelets are transmitted in the same time slot. The orthogonality of two signals can be shown by substitution. Recall that

$$c(t-kT_c) = \begin{cases} c(t) & kT_c \le t < (k+1)T_c \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

So the inner product of the reference and information bearing wavelets can be written as,

$$\begin{aligned} \triangle &= \int_0^{T_s=NT_c} \sum_{k=0}^{N-1} w_{R,k+1} c(t-kT_c) a \\ &\quad \times \sum_{k=0}^{N-1} w_{I,k+1} c(t-kT_c) \mathrm{d}t \\ &= \int_0^{T_s=NT_c} a w_{R,1} w_{I,1} c^2(t) + a w_{R,2} w_{I,2} c^2(t-T_c) \\ &\quad + \cdots + a w_{R,N} w_{I,N} c^2[t-(N-1)T_c] \\ &= a(w_{R,1}w_{I,1} + w_{R,2}w_{I,2} + \cdots + w_{R,N}w_{I,N}) \frac{E_b}{2N} \\ &= a\mathbf{w_R}(\mathbf{w_I})^T \frac{E_b}{2N} \\ &= 0 \end{aligned} \tag{5}$$

where

$$E_b = 2\int_0^{T_s} c^2(t)\mathrm{d}t \quad \text{and} \quad \frac{E_b}{N} = 2\int_0^{T_c} c^2(t)\mathrm{d}t$$

$$\sum_{k=0}^{N-1} w_{I,k+1} w_{R,k+1} = \mathbf{w_I}(\mathbf{w_R})^T = 0$$

and the $(\cdot)^T$ is transpose of vector, and the fact that $c(t) = c(t-kT_c), \forall\, k$ has been exploited during the derivation. Note, the reference and information bearing wavelets are always orthogonal independently of the actual shape of the chaotic carrier,

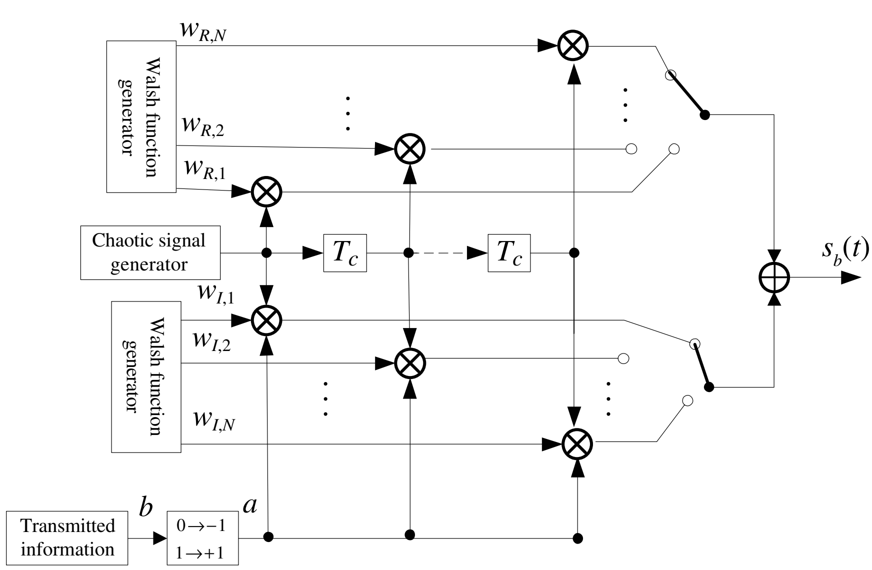

Fig. 3. Block diagram of the CS-DCSK transmitter.

the orthogonality of two wavelets is assured by the Walsh code sequences.

Equation (3) defines the CS-DCSK modulation scheme. The block diagram of CS-DCSK transmitter derived from Eq. (3) is shown in Fig. 3. The number of required delay lines is $N-1$ where the delay of each line is T_c. The CS-DCSK transmitter is very similar to the M-ary DCSK transmitter, however, note the important theoretical difference: in CS-DCSK there are a reference wavelet and an information bearing wavelet and both of them are transmitted in the same time slot. Hence, the CS-DCSK system is a binary modulation scheme. At least a second order Walsh code is required to implement a binary CS-DCSK modulation scheme. In this situation, the transmitter of CS-DCSK has the same delay requirement with that of traditional DCSK transmitter.

Figure 4 shows the block diagram of CS-DCSK receiver. Note the advantage of CS-DCSK receiver over the M-ary DCSK and traditional DCSK receivers, delay lines are not used in the CS-DCSK receiver.

The receive filter is a bandpass filter with a bandwidth $2B$. That bandwidth is large enough to pass the received signal $r(t)$ without any distortion. The received signal is corrupted by an additive Gaussian white noise (AWGN) $n(t)$ with zero mean and a variance of $N_0/2$. The receive filter output is

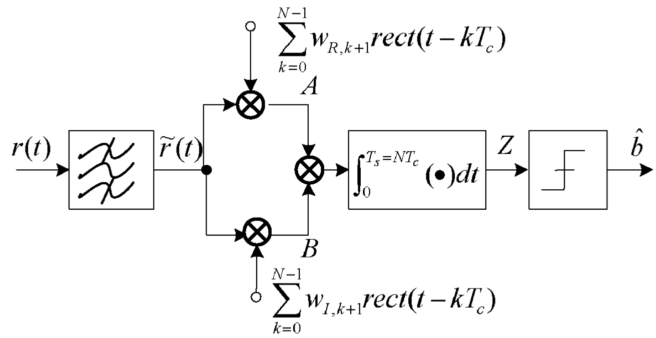

Fig. 4. Block diagram of CS-DCSK receiver.

obtained as

$$\tilde{r}(t) = \sum_{k=0}^{N-1} w_{R,k+1} c(t-kT_c) + a\sum_{k=0}^{N-1} w_{I,k+1} c(t-kT_c) + n(t). \tag{6}$$

To get the signals A and B, the output of the receive filter is multiplied by a rectangular function

$$\mathrm{rect}(t-kT_c) = \begin{cases} 1 & kT_c \le t < (k+1)T_c \\ 0 & \text{otherwise} \end{cases}$$

weighted by the two Walsh code sequences. The outputs of upper and lower mixers are obtained as

$$\begin{aligned} A &= s_b(t)\sum_{k=1}^{N-1} w_{R,k+1}\mathrm{rect}(t-kT_c) \\ &= \left[\sum_{k=0}^{N-1} w_{R,k+1}c(t-kT_c) + a\sum_{k=0}^{N-1} w_{I,k+1}c(t-kT_c)\right]\left[\sum_{j=0}^{N-1} w_{R,k+1}\mathrm{rect}(t-jT_c)\right] \\ &= w_{R,1}^2 c(t) + w_{R,2}^2 c(t-T_c) + w_{R,3}^2 c(t-2T_c) + \cdots + a[w_{R,1}w_{I,1}c(t) + w_{R,2}w_{I,2}c(t-T_c) \\ &\quad + w_{R,3}w_{I,3}c(t-2T_c)\cdots] \\ &= \sum_{k=0}^{N-1} c(t-kT_c) + a\sum_{k=0}^{N-1} w_{R,k+1}w_{I,k+1}c(t-kT_c) \end{aligned} \tag{7}$$

and

$$\begin{aligned} B &= s_b(t)\sum_{k=1}^{N-1} w_{I,k+1}\mathrm{rect}(t-kT_c) \\ &= \left[\sum_{k=0}^{N-1} w_{R,k+1}c(t-kT_c) + a\sum_{k=0}^{N-1} w_{I,k+1}c(t-kT_c)\right]\left[\sum_{j=0}^{N-1} w_{I,k+1}\mathrm{rect}(t-jT_c)\right] \end{aligned}$$

$$= w_{R,1}w_{I,1}c(t) + w_{R,2}w_{I,2}c(t-T_c) + w_{R,3}w_{I,3}c(t-2T_c) + \cdots$$
$$+ a[w_{I,1}^2 c(t) + w_{I,2}^2 c(t-T_c) + w_{I,3}^2 c(t-2T_c)\cdots]$$
$$= \sum_{k=0}^{N-1} w_{R,k+1}w_{I,k+1}c(t-kT_c) + a\sum_{k=0}^{N-1} c(t-kT_c). \quad (8)$$

The product of A and B is

$$AB = \left[\sum_{k=0}^{N-1} c(t-kT_c) + a\sum_{k=0}^{N-1} w_{R,k+1}w_{I,k+1}c(t-kT_c)\right]\left[\sum_{k=0}^{N-1} w_{R,k+1}w_{I,k+1}c(t-kT_c) + a\sum_{k=0}^{N-1} c(t-kT_c)\right]. \quad (9)$$

Since

$$c(t-iT_c)c(t-jT_c) = \begin{cases} c^2(t-iT_c) & i=j \\ 0 & \text{otherwise} \end{cases}$$

the product AB can be rearranged as

$$AB = \sum_{k=0}^{N-1} w_{R,k+1}w_{I,k+1}c^2(t-kT_c) + a\sum_{k=0}^{N-1} w_{R,k+1}^2 w_{I,k+1}^2 c^2(t-kT_c) + a\sum_{k=0}^{N-1} c^2(t-kT_c)$$
$$+ a^2\sum_{k=0}^{N-1} w_{R,k+1}w_{I,k+1}c^2(t-kT_c). \quad (10)$$

The observation signal is obtained as

$$Z = \int_0^{T_s=NT_c} AB dt = \left[\sum_{k=0}^{N-1} w_{R,k+1}w_{I,k+1} + a\sum_{k=0}^{N-1} w_{R,k+1}^2 w_{I,k+1}^2 + aN + a^2\sum_{k=0}^{N-1} w_{R,k+1}w_{I,k+1}\right]\frac{E_b}{2N} \quad (11)$$

where

$$E_b = 2\int_0^{T_s} c^2(t)dt \quad \text{and} \quad \frac{E_b}{N} = 2\int_0^{T_c} c^2(t)dt$$

Then

$$Z = \int_0^{T_s=NT_c} AB dt$$
$$= [0 + aN + aN + 0]\frac{E_b}{2N}$$
$$= aE_b, \quad a = \pm 1 \quad (12)$$

where $a \in \{-1, +1\}$ is mapped from $b \in \{0, 1\}$. The transmitted bits are estimated by the follow decision rule

$$\hat{b} = \begin{cases} 1 & \text{if } Z \geq 0 \\ 0 & \text{else } Z < 0. \end{cases} \quad (13)$$

3. BER Performance Analysis of CS-DCSK

This section derives analytical expressions for the noise performance of CS-DCSK over AWGN and multipath Rayleigh fading channels.

3.1. *Additive white Gaussian noise (AWGN)*

By Fig. 4, the observation variable is obtained as

$$Z = \sum_{k=0}^{N-1} \int_{kT_c}^{(k+1)T_c} [w_{R,k+1}\tilde{r}(t)][w_{I,k+1}\tilde{r}(t)]\mathrm{d}t$$
$$= \sum_{k=0}^{N-1} w_{R,k+1}w_{I,k+1}\int_{kT_c}^{(k+1)T_c} \tilde{r}^2(t)\mathrm{d}t$$

$$= \sum_{k=0}^{N-1} w_{R,k+1} w_{I,k+1} \int_{kT_c}^{(k+1)T_c} \times [(w_{R,k+1} + a w_{I,k+1}) c(t - kT_c) + n(t)]^2 \mathrm{d}t \quad (14)$$

The discrete chaotic signals are generated by the Logistic map $x(n+1) = 1 - 2x^2(n)$ in this work. Let f_s denote the sample rate. Then $\beta = T_c f_s$ and $\omega = T_s f_s = N\beta$ are the number of samples in a chip time and symbol duration, respectively. Parameter ω is referred to as Spreading Factor (SF) in the remaining part of the paper.

For simplicity, consider the transmission of a pure $a = +1$ sequence. Let c_j and η_j denote the samples of chaotic signal and channel noise, respectively. Then the observation variable is obtained from Eq. (14) in the discrete time domain as

$$Z = \sum_{k=0}^{N-1} w_{R,k+1} w_{I,k+1} \sum_{m=1}^{\beta} \times [(w_{R,k+1} + a w_{I,k+1}) c_{k\beta+m} + \eta_{k\beta+m}]^2 \quad (15)$$

where c_j is the discrete time equivalent of Eq. (4)

$$c_{k\beta+m} = \begin{cases} c_m & k\beta \le k\beta + m < (k+1)\beta \\ 0 & \text{otherwise.} \end{cases} \quad (16)$$

The decision variable is decomposed into three terms

$$Z = Z_{s\times s} + Z_{s\times n} + Z_{n\times n} \quad (17)$$

where

$$Z_{s\times s} = \sum_{k=0}^{N-1} w_{R,k+1} w_{I,k+1} \sum_{m=1}^{\beta} \times [(w_{R,k+1} + a w_{I,k+1}) c_{k\beta+m}]^2 \quad (18)$$

$$Z_{s\times n} = 2 \sum_{k=0}^{N-1} w_{R,k+1} w_{I,k+1} \sum_{m=1}^{\beta} \times [(w_{R,k+1} + a w_{I,k+1}) c_{k\beta+m} \eta_{k\beta+m}] \quad (19)$$

$$Z_{n\times n} = \sum_{k=0}^{N-1} w_{R,k+1} w_{I,k+1} \sum_{m=1}^{\beta} [\eta_{k\beta+m}]^2. \quad (20)$$

Expectation and variance of these variables are obtained as

$$\begin{aligned} E\{Z_{s\times s} \,|\, b=1\} &= E\{Z_{s\times s} \,|\, a=+1\} = 2N\beta\{c_j^2\} \\ E\{Z_{s\times n} \,|\, b=1\} &= E\{Z_{s\times n} \,|\, a=+1\} = 0 \\ E\{Z_{n\times n} \,|\, b=1\} &= E\{Z_{n\times n} \,|\, a=+1\} = 0 \\ \mathrm{var}\{Z_{s\times s} \,|\, b=1\} &= \mathrm{var}\{Z_{s\times s} \,|\, a=+1\} \\ &= 2N\beta\,\mathrm{var}\{c_j^2\} \\ \mathrm{var}\{Z_{s\times n} \,|\, b=1\} &= \mathrm{var}\{Z_{s\times n} \,|\, a=+1\} \\ &= 4N\beta N_0 E\{c_j^2\} \\ \mathrm{var}\{Z_{n\times n} \,|\, b=1\} &= \mathrm{var}\{Z_{n\times n} \,|\, a=+1\} = N\beta N_0^2 \end{aligned}$$

where $E\{\cdot\}$ and $\mathrm{var}\{\cdot\}$ represent the expectation and variance operators, respectively. Then we have

$$E\{Z \,|\, b=1\} = E\{Z \,|\, a=+1\} = 2N\beta E\{c_j^2\} \quad (21)$$

$$\mathrm{var}\{Z \,|\, b=1\} = \mathrm{var}\{Z \,|\, a=+1\} = 2N\beta\,\mathrm{var}\{c_j^2\} + 4N_0 N\beta E\{c_j^2\} + N\beta N_0^2. \quad (22)$$

The expectation and variance of observation variable for sending a pure "$a = -1$" sequence is derived in a similar manner

$$E\{Z \,|\, a=-1\} = -E\{Z \,|\, a=+1\} \quad (23)$$

$$\mathrm{var}\{Z \,|\, a=-1\} = \mathrm{var}\{Z \,|\, a=+1\}. \quad (24)$$

Assume that the transmitted bits are equiprobable and assume that the probability distribution of the observation variable can be approximated by a Gaussian one. Then the bit error rate of CS-DCSK modulation scheme is obtained as

$$\begin{aligned} \mathrm{BER} &= \frac{1}{2}\Pr(Z<0 \,|\, b=1) + \frac{1}{2}\Pr(Z\ge 0 \,|\, b=0) = \frac{1}{2}\Pr(Z<0 \,|\, a=+1) + \frac{1}{2}\Pr(Z\ge 0 \,|\, a=-1) \\ &= \frac{1}{2}\mathrm{erfc}\left(\frac{E\{Z \,|\, a=+1\}}{\sqrt{2\mathrm{var}\{Z \,|\, a=+1\}}}\right) = \frac{1}{2}\mathrm{erfc}\left(\frac{2N\beta E\{c_j^2\}}{\sqrt{2(2N\beta\,\mathrm{var}\{c_j^2\} + 4N_0 N\beta E\{c_j^2\} + N\beta N_0^2)}}\right) \end{aligned} \quad (25)$$

where erfc(·) is the complementary error function

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-\xi^2} d\xi. \quad (26)$$

For the Logistic map, we get

$$E\{c_j^2\} = \frac{1}{2} \quad \text{and} \quad \operatorname{var}\{c_j^2\} = \frac{1}{8}. \quad (27)$$

Substituting Eqs. (27) into Eq. (25), the expression for noise performance over AWGN channel is obtained

$$\begin{aligned} \text{BER} &= \frac{1}{2}\operatorname{erfc}\left(\left[\frac{1}{2N\beta} + \frac{4N_0}{E_b} + \frac{2N\beta N_0^2}{E_b^2}\right]^{-\frac{1}{2}}\right) \\ &= \frac{1}{2}\operatorname{erfc}\left(\left[\frac{1}{2N\beta} + \frac{4}{\frac{E_b}{N_0}} + \frac{2N\beta}{\left(\frac{E_b}{N_0}\right)^2}\right]^{-\frac{1}{2}}\right). \end{aligned} \quad (28)$$

3.2. *Rayleigh multipath channels*

Let the received signal be denoted by $r(t) = h(t) \otimes x(t) + n(t)$ and let the Channel Impulse Response (CIR) be defined by

$$h(t) = \sum_{l=0}^{L-1} \alpha_l \delta(t - \tau_l) \quad (29)$$

when a multipath channel is considered. In these equations "$\otimes$" denotes the convolution operator, L gives the number of propagation paths, α_l is the gain of the lth path, $\delta(t)$ denotes the Dirac delta function, and τ_l is the delay of the lth path.

Assume that the gains α_l of propagation paths are independent identical-distributed (i.i.d.) Rayleigh random variables. Given the noise performance for the AWGN channel, Xia *et al.* developed an approximation for the BER measured in a Rayleigh multipath channel [Xia *et al.*, 2004]. The conditional BER measured in a propagation path is

$$\begin{aligned} &\text{BER}(\alpha_0, \alpha_1, \ldots, \alpha_{L-1}) \\ &\quad \approx \frac{1}{2}\operatorname{erfc}\left(\left[\frac{1}{2N\beta} + \frac{4}{\gamma_b} + \frac{2N\beta}{\gamma_b^2}\right]^{-\frac{1}{2}}\right) \\ &\quad = \text{BER}(\gamma_b) \end{aligned} \quad (30)$$

where $\gamma_b = (E_b/N_0)(\alpha_0, \alpha_1, \ldots, \alpha_{L-1}) = \gamma_0 + \gamma_0 + \cdots + \gamma_{L-1}$, $\gamma_i = (E_b/N_0)\alpha_i^2, i = 0, 1, \ldots, L-1$. Denoting $\overline{\gamma}_i = E\{\gamma_i\} = (E_b/N_0)E\{\alpha_i^2\}$, when α_i is a Rayleigh random variable, the probability density function (PDF) of γ_i is an exponential distribution

$$f(x) = \frac{1}{\overline{\gamma}_k} \exp\left(\frac{-x}{\overline{\gamma}_k}\right). \quad (31)$$

If α_l is an i.i.d random variable with Raleigh distribution then the probability density function (pdf) of γ_b is obtained as

$$f(\gamma_b) = f(\gamma_0) \otimes f(\gamma_1) \cdots \otimes f(\gamma_{L-1}) \quad (32)$$

where $f(\gamma_k)$ is the pdf of instantaneous signal-to-noise ratio (SNR) measured in the ith path. The total BER measured in a multipath channel is obtained by averaging the conditional bit error rates

$$\text{BER} = \int_0^{\infty} \text{BER}(\gamma_b) f(\gamma_b) \mathrm{d}\gamma_b \quad (33)$$

Throughout this paper, two multipath channels with three propagation paths are considered. The statistical parameters of the three propagation paths are $[E\{\alpha_1^2\}, E\{\alpha_2^2\}, E\{\alpha_3^2\}] = [0.4, 0.4, 0.2]$ and $[E\{\alpha_1^2\}, E\{\alpha_2^2\}, E\{\alpha_3^2\}] = [0.6, 0.3, 0.1]$ respectively, and the delays of the three propagation paths are given by the delay vector $[\tau_0, \tau_1, \tau_2] = [0, 2T_s/\omega, 5T_s/\omega]$. Based on these two assumptions, the pdfs of SNRs are obtained as

$$f(\gamma_b) = \begin{cases} \left[\dfrac{\gamma_b}{\overline{\gamma}_1(\overline{\gamma}_1 - \overline{\gamma}_3)} - \dfrac{\overline{\gamma}_3}{(\overline{\gamma}_1 - \overline{\gamma}_3)^2}\right] e^{-\frac{\gamma_b}{\overline{\gamma}_1}} + \dfrac{\overline{\gamma}_3}{(\overline{\gamma}_1 - \overline{\gamma}_3)^2} e^{-\frac{\gamma_b}{\overline{\gamma}_1}} & E\{\alpha_1^2\} = E\{\alpha_2^2\} \neq E\{\alpha_3^2\} \\ \dfrac{1}{\overline{\gamma}_3(\overline{\gamma}_1 - \overline{\gamma}_2)} e^{-\frac{\gamma_b}{\overline{\gamma}_3}} \left[\dfrac{\overline{\gamma}_1\overline{\gamma}_3}{(\overline{\gamma}_1 - \overline{\gamma}_3)} e^{-\frac{\overline{\gamma}_1\overline{\gamma}_3}{(\overline{\gamma}_1 - \overline{\gamma}_3)\gamma_b}} - \dfrac{\overline{\gamma}_2\overline{\gamma}_3}{(\overline{\gamma}_2 - \overline{\gamma}_3)} e^{-\frac{\overline{\gamma}_2\overline{\gamma}_3}{(\overline{\gamma}_2 - \overline{\gamma}_3)\gamma_b}} + \dfrac{\overline{\gamma}_2\overline{\gamma}_3}{(\overline{\gamma}_2 - \overline{\gamma}_3)} - \dfrac{\overline{\gamma}_1\overline{\gamma}_3}{(\overline{\gamma}_1 - \overline{\gamma}_3)}\right] & \\ \qquad E\{\alpha_1^2\} \neq E\{\alpha_2^2\} \neq E\{\alpha_3^2\}. \end{cases} \quad (34)$$

Substituting Eqs. (30) and (34) into Eq. (33), the theoretical BER of three-path Rayleigh fading channel will be evaluated later numerically.

3.3. *Multiuser CS-DCSK communication system*

This paragraph extends the noise performance analysis of CS-DCSK modulation scheme discussed in Sec. 3.1 to multiuser application. To simplify the problem, the transmission of a single isolated symbol is considered.

The antenna receives the signal transmitted by N_u users

$$s_T(t) = \sum_{k=0}^{N-1} w_{R,k+1} c(t - kT_c) + \sum_{u=1}^{N_u} \left[a_u \sum_{k=0}^{N-1} w_{u,k+1} c(t - kT_c) \right], \quad T_s = NT_c \tag{35}$$

where N_u gives the number of users, a_u is the data symbol transmitted by the uth user, $T_s = NT_c$ is the symbol period, w_R and $w_u, u = 1, \ldots, N_u$ are orthogonal Walsh codes. The data rate is $1/T_s$ for each user. Since the receive filter passes the signals radiated by each user, the output of the mth user's receive filter is

$$\tilde{r}_m(t) = \sum_{k=0}^{N-1} w_{R,k+1} c(t - kT_c) + \sum_{u=1}^{N_u} \left[a_u \sum_{k=0}^{N-1} w_{u,k+1} c(t - kT_c) \right] + n(t). \tag{36}$$

The observation variable of mth user is obtained as

$$\begin{aligned} Z_m &= \sum_{k=0}^{N-1} \int_{kT_c}^{(k+1)T_c} (w_{R,k+1} \tilde{r}_m(t))(w_{m,k+1} \tilde{r}_m(t))\, dt \\ &= \sum_{k=0}^{N-1} \int_{kT_c}^{(k+1)T_c} (w_{R,k+1} \tilde{r}_m(t))(w_{m,k+1} \tilde{r}_m(t))\, dt \\ &= \sum_{k=0}^{N-1} w_{R,k+1} w_{m,k+1} \int_{kT_c}^{(k+1)T_c} \left[\left(w_{R,k+1} + \sum_{u=1}^{N_u} a_u w_{u,k+1} \right) c(t - kT_c) + n(t) \right]^2 dt. \end{aligned} \tag{37}$$

As done in Eq. (17) the observation variable is decomposed into three terms

$$\begin{aligned} Z_{s\times s} &= \sum_{k=0}^{N-1} w_{R,k+1} w_{m,k+1} \sum_{j=1}^{\beta} [(w_{R,k+1} + a_1 w_{1,k+1} + \cdots + a_m w_{m,k+1} + \cdots + a_{N_u} w_{N_u,k+1})]^2 c_{k\beta+j}^2 \\ &= 2 \sum_{k=0}^{N-1} w_{R,k+1} w_{m,k+1} a_m w_{R,k+1} w_{m,k+1} \sum_{j=1}^{\beta} c_{k\beta+j}^2 \\ &\quad + 2 \sum_{u=1}^{N_u-1} \sum_{l=u+1}^{N_u} a_u a_l \sum_{k=0}^{N-1} w_{R,k+1} w_{m,k+1} w_{u,k+1} w_{l,k+1} \sum_{j=1}^{\beta} c_{k\beta+j}^2 \\ &\quad + \sum_{k=0}^{N-1} w_{R,k+1} w_{m,k+1} w_{R,k+1}^2 \sum_{j=1}^{\beta} c_{k\beta+j}^2 + \sum_{k=0}^{N-1} w_{R,k+1} w_{m,k+1} \sum_{u=1}^{N_u} a_u^2 w_{u,k+1}^2 \sum_{j=1}^{\beta} c_{k\beta+j}^2 \\ &\quad + 2 \sum_{k=0}^{N-1} w_{R,k+1} w_{m,k+1} \sum_{u=1,u\neq m}^{N_u} a_u w_{R,k+1} w_{u,k+1} \sum_{j=1}^{\beta} c_{k\beta+j}^2 \end{aligned}$$

$$
\begin{aligned}
&= 2Na_m \sum_{j=1}^{\beta} c_j^2 + \underbrace{2 \sum_{u=1}^{N_u-1} \sum_{l=u+1}^{N_u} a_u a_l \sum_{k=0}^{N-1} w_{R,k+1} w_{m,k+1} w_{u,k+1} w_{l,k+1} \sum_{j=1}^{\beta} c_j^2}_{\text{MUI}} \\
&\quad + \underbrace{\sum_{k=0}^{N-1} w_{R,k+1} w_{m,k+1} \sum_{j=1}^{\beta} c_j^2}_{=0} + \underbrace{\sum_{u=1}^{N_u} \sum_{k=0}^{N-1} w_{R,k+1} w_{u,k+1} \sum_{j=1}^{\beta} c_j^2}_{=0} \\
&\quad + \underbrace{2 \sum_{u=1,u\neq m}^{N_u} \sum_{k=0}^{N-1} a_u w_{R,k+1}^2 w_{u,k+1} w_{m,k+1} \sum_{j=1}^{\beta} c_j^2}_{=0}
\end{aligned} \tag{38}
$$

$$
\begin{aligned}
Z_{s\times n} &= 2 \sum_{k=0}^{N-1} w_{R,k+1} w_{m,k+1} \\
&\quad \times \sum_{j=1}^{\beta} [w_{R,k+1} + a_1 w_{1,k+1} + a_2 w_{2,k+1} + \cdots + a_m w_{m,k+1} + \cdots + a_{N_u} w_{N_u,k+1}] c_{k\beta+j} \eta_{k\beta+j} \\
&= 2 \sum_{k=0}^{N-1} w_{R,k+1}^2 w_{m,k+1} \sum_{j=1}^{\beta} c_{k\beta+j} \eta_{k\beta+j} + 2 \sum_{u=1,u\neq m}^{N_u} a_u \sum_{k=0}^{N-1} w_{R,k+1} w_{m,k+1} w_{u,k+1} \sum_{j=1}^{\beta} c_{k\beta+j} \eta_{k\beta+j} \\
&\quad + 2a_m \sum_{k=0}^{N-1} w_{R,k+1} w_{m,k+1}^2 \sum_{j=1}^{\beta} c_{k\beta+j} \eta_{k\beta+j} \\
&= 2 \sum_{k=0}^{N-1} w_{m,k+1} \sum_{j=1}^{\beta} c_j \eta_{k\beta+j} + 2 \sum_{u=1,u\neq m}^{N_u} a_u \sum_{k=0}^{N-1} w_{R,k+1} w_{m,k+1} w_{u,k+1} \sum_{j=1}^{\beta} c_j \eta_{k\beta+j} \\
&\quad + 2a_m \sum_{k=0}^{N-1} w_{R,k+1} \sum_{j=1}^{\beta} c_j \eta_{k\beta+j}
\end{aligned} \tag{39}
$$

$$
Z_{n\times n} = \sum_{k=0}^{N-1} w_{R,k+1} w_{m,k+1} \sum_{j=1}^{\beta} [\eta_{k\beta+j}]^2. \tag{40}
$$

The effect of Multi-User Interference (MUI) appears in Eq. (38). Since the MUI term corrupts the noise performance of CS-DCSK radio system, it must be eliminated. Note, the MUI term equals to zero in Eq. (38) when

$$
\sum_{k=0}^{N-1} w_{R,k+1} w_{m,k+1} w_{i,k+1} w_{j,k+1} = 0, \qquad i, j, m = 1, 2, \ldots, N_u. \tag{41}
$$

We assumed at the beginning of this section that the different users are separated by different Walsh code sequences that are orthogonal. However, the Walsh code sequences are orthogonal only if they are synchronized in the multiuser network. Recall, the synchronization among the codes has to be established and maintained in every Code Division Multiple Access (CDMA) network.

If the transmitted bits are equiprobable and the Gaussian approximation can be applied then the

BER performance of a multiuser CS-DCSK system is obtained as

$$\begin{aligned}
\mathrm{BER}_{mu} &= \frac{1}{2}\Pr(Z_m < 0 \,|\, a_m = +1) + \frac{1}{2}\Pr(Z_m \geq 0 \,|\, a_m = -1) \\
&= \frac{1}{2}\mathrm{erfc}\left(\frac{E\{Z_m \,|\, a_m = +1\}}{\sqrt{2\mathrm{var}\{Z_m \,|\, a_m = +1\}}}\right) \\
&= \frac{1}{2}\mathrm{erfc}\left(\frac{2N\beta E\{c_j^2\}}{\sqrt{2(2N\beta\,\mathrm{var}\{c_j^2\} + 2(N_u+1)NN_0\beta E\{c_j^2\} + N\beta N_0^2)}}\right) \\
&= \frac{1}{2}\mathrm{erfc}\left(\left[2\left(\frac{\mathrm{var}\{c_j^2\}}{2N\beta E^2\{c_j^2\}} + \frac{(N_u+1)^2 N_0}{2(N_u+1)N\beta E^2\{c_j^2\}} + \frac{(N_u+1)^2 N\beta N_0^2}{4(N_u+1)^2 N^2\beta^2 E^2\{c_j^2\}}\right)\right]^{-1/2}\right) \\
&= \frac{1}{2}\mathrm{erfc}\left(\left[\frac{1}{2N\beta} + \frac{(N_u+1)^2}{\frac{N_u E_b}{N_0}} + \frac{(N_u+1)^2 N\beta}{2N_u^2\left(\frac{E_b}{N_0}\right)^2}\right]\right)
\end{aligned} \tag{42}$$

where $E_b = (N_u+1)N\beta E\{c_j^2\}/N_u$ is the received bit energy per user.

4. Results

The BER performances of CS-DCSK modulation scheme are evaluated by Monte-Carlo simulation and numerical calculation in this section for different channel conditions. Unless otherwise stated, only the second order Walsh code sequences will be used where $W = [+1, +1; +1, -1]$. Note, the second order Walsh code sequences make the CS-DCSK and traditional DCSK systems very similar. Remember, however, that in DCSK the two waveforms are transmitted in two consecutive time slots, while in CS-DCSK the two waveforms are transmitted in the same time slot.

4.1. *Noise performance over AWGN channel*

Noise performances of the CS-DCSK modulation scheme over AWGN channel are plotted in Fig. 5 for two spreading factors. Solid lines show the BER performance calculated from Eq. (28), while the values marked by circles, asterisks and diamonds give the results of computer simulations. This noise performance will be used later as benchmark when the effect of different channel conditions are evaluated. Note the very good agreement between the theoretical predictions and simulated data. A slight mismatch can be observed for SF = 40 since the Gaussian approximation is not valid for low spreading factors.

This effect can be observed in Fig. 6 where E_b/N_0 is fixed at 16 dB and the BER is plotted against the spreading factor. For large SF the theoretical predictions and simulated data are almost identical. Figure 6 shows that the noise performance depends on SF, the larger the spreading factor, the worse the noise performance.

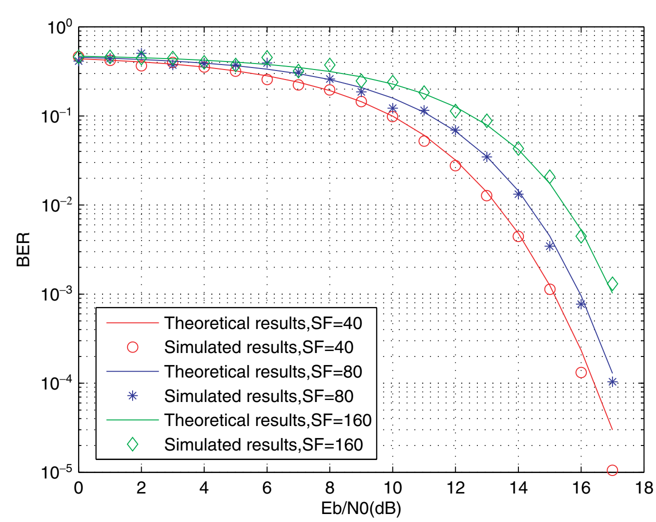

Fig. 5. BER performance of the CS-DCSK modulation scheme over AWGN channel. Solid lines show the BER performance calculated from the analytical expression, while the values marked by circles, asterisks and diamonds give the results of computer simulations.

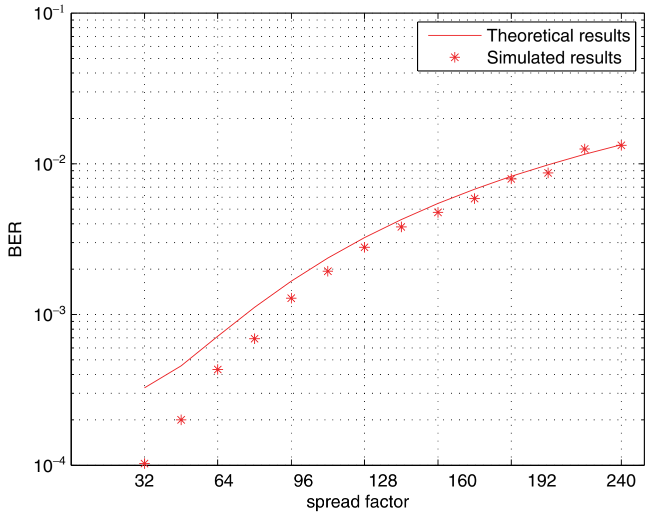

Fig. 6. Effect of spreading factor on BER performance of the CS-DCSK modulation scheme. E_b/N_0 has been fixed at 16 dB.

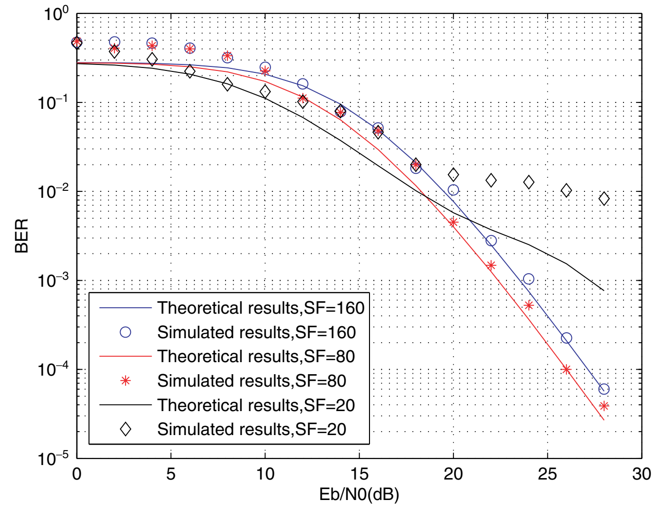

Fig. 8. BER performance of the CS-DCSK modulation scheme in a Rayleigh multipath propagation environment for channel model I. The parameter SF is the spreading factor.

Figure 7 shows that the order of Walsh code has practically no effect on the noise performance. This behavior is also reflected by Eq. (28), if the spreading factor $\omega = N\beta$ is kept constant then the noise performance remains unchanged.

4.2. *BER performance over multipath channels*

A tapped delay line model is used to describe the propagation conditions in a multipath channel. The impulse response of the model is given by Eq. (29) where the gains of propagation paths have a Rayleigh distribution. Two multipath channel profiles with three propagation paths in each are considered here. The statistical parameters of models I and II are $[E\{\alpha_1^2\}, E\{\alpha_2^2\}, E\{\alpha_3^2\}] = [0.4, 0.4, 0.2]$ and $[E\{\alpha_1^2\}, E\{\alpha_2^2\}, E\{\alpha_3^2\}] = [0.6, 0.3, 0.1]$. The delay vector is the same for both models: $[\tau_0, \tau_1, \tau_2] = [0, 2T_s/\omega, 5T_s/\omega]$.

Substituting Eqs. (30) and (34) into Eq. (33), the theoretical BER over Rayleigh multipath fading channel can be computed by a numerical integral. As shown in Figs. 8 and 9, the theoretical predictions and simulated data are almost identical for large spreading factors (SF $\geq$ 80).

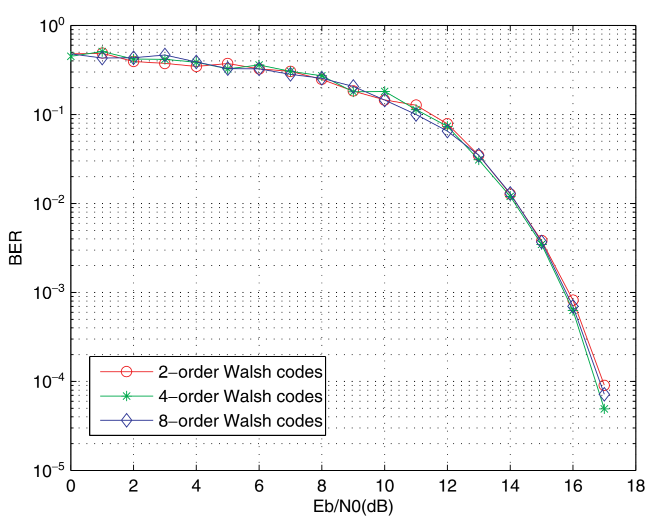

Fig. 7. Effect of the order of Walsh code on the BER performance when the spreading factor is kept constant ($\omega = N\beta = 80$).

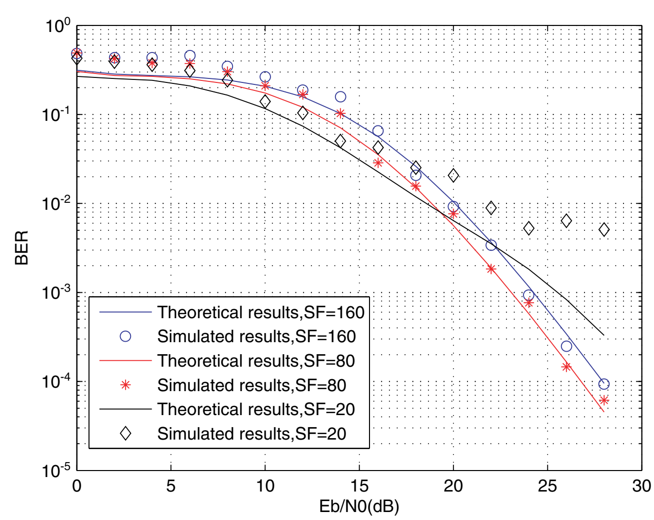

Fig. 9. BER performance of the CS-DCSK modulation scheme in a Rayleigh multipath propagation environment for channel model II.

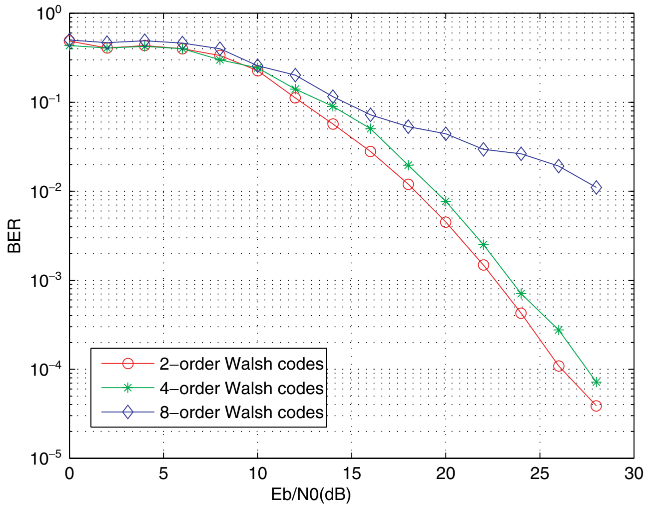

Fig. 10. BER performance of the CS-DCSK modulation scheme in a Rayleigh multipath propagation environment for channel model I. The spreading factor is 80, the parameter is the order of Walsh code.

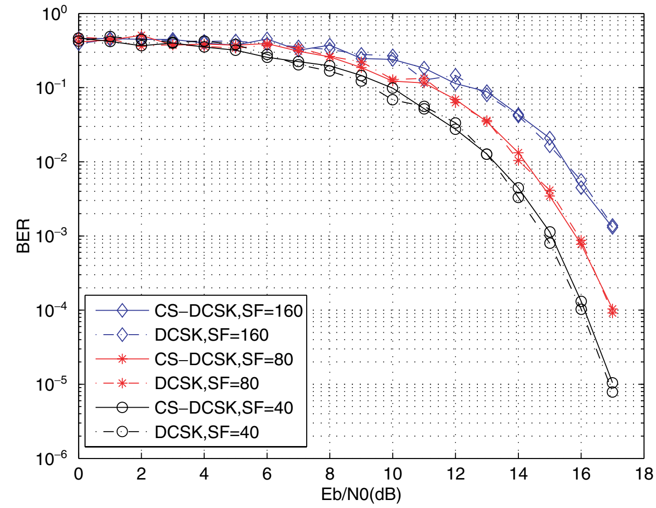

Fig. 11. Performance comparison of CS-DCSK and DCSK modulation schemes over AWGN channels for different spreading factors.

For SF = 20 the Gaussian approximation is not valid, that causes a considerable error for high values of E_b/N_0.

Figure 10 shows the effect of the order of Walsh code on the BER performance when the communication is established over a multipath Rayleigh channel. During the investigations, model I has been used and the spreading factor has been set to 80. Observe, in contrast with the AWGN channel, now the BER strongly depends on N, the larger the N, the worse the BER performance. This is because the larger N is, the smaller β is, for given spread-spectrum factor. For too small β results, the inter-symbol interference (ISI) cannot be neglected. Thus, the BER is deteriorated severely when N is 8 for spread-spectrum factor of 80. Here, maximum delay is $5T_s/\omega$ and β is $10T_s/\omega$. So the ISI cannot be neglected obviously.

4.3. *Performance comparison of CS-DCSK and DCSK modulation schemes*

The BER performances of CS-DCSK and DCSK modulation schemes are compared over an AWGN channel in Fig. 11 for different spreading factors. The figure shows that the noise performances of the two modulation schemes are practically identical.

The parameters of models I and II of the three-path Rayleigh multipath channel have been defined in Sec. 4.2. Those model parameters are used here to perform the comparison.

Figures 12 and 13 compare the BER performances of CS-DCSK and the DCSK modulation schemes over this three-path Rayleigh multipath channel. The former and latter figures give the results for models I and II, respectively. Observe, except for small spreading factors, the BER performances of the two modulation schemes are identical.

Finally, the performances of the two modulation schemes are compared over a two-path multipath channel where the gains of each path is 0.5,

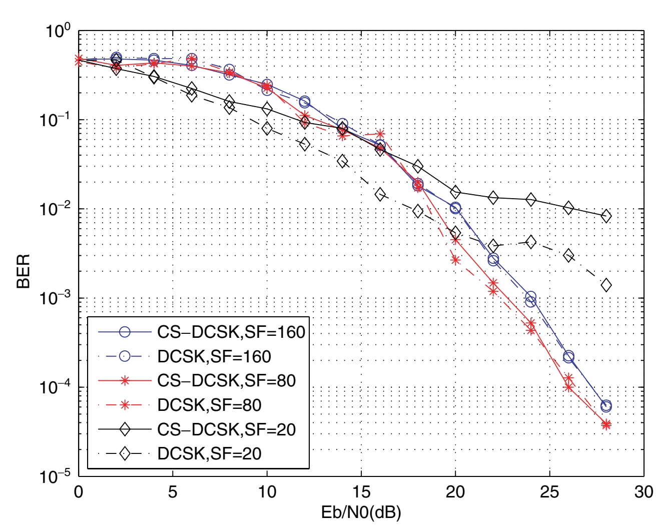

Fig. 12. Performance comparison of CS-DCSK and DCSK modulation schemes over Rayleigh multipath channel for different spreading factors. The channel is defined by model I.

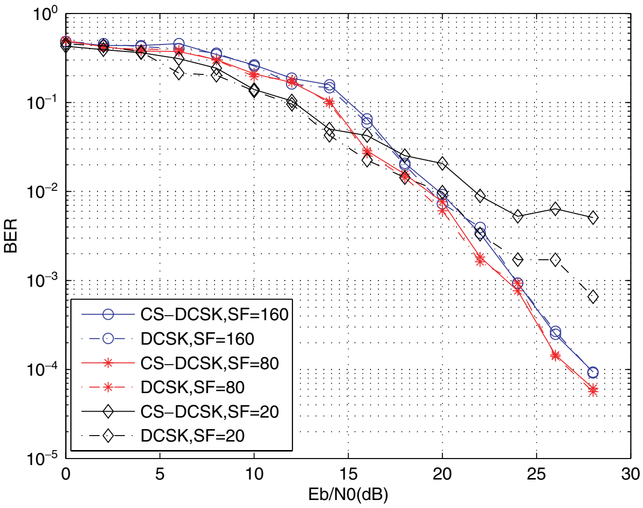

Fig. 13. Performance comparison of CS-DCSK and DCSK modulation schemes over Rayleigh multipath channel defined by model II.

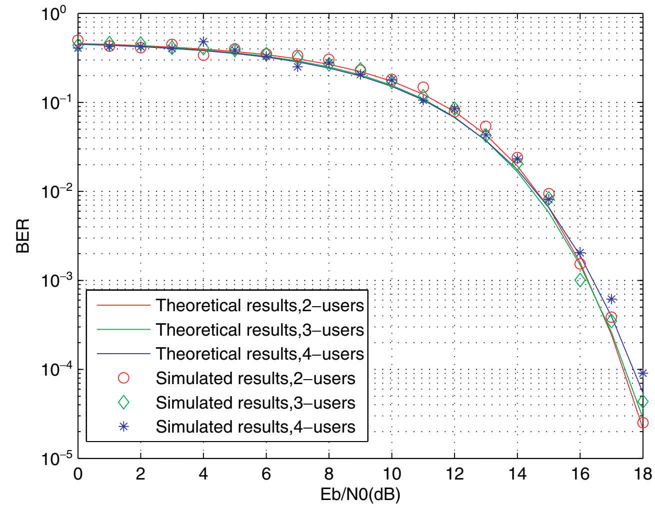

Fig. 15. BER performance of multiuser CS-DCSK over AWGN channel with different number of users for theoretical results (lines), and simulated results ("circle", "diamond", and "asterisk", respectively), spread-spectrum factor is 160.

and the excess delay of the second path, denoted by τ, varies from T_s/ω to $20T_s/\omega$.

The BER performances of CS-DCSK and DCSK modulation schemes over the two-path multipath channel are compared in Fig. 14 for $E_b/N_0 =$ 26 dB and SF $= 80$. As the excess delay is increased, the performance of CS-DCSK gets worse than that of DCSK, i.e. the CS-DCSK modulation is less robust against the excess delay. Fortunately, in most practical applications, since DCSK or CS-DCSK is an alterative spread spectrum scheme, the delays are much less than the spread factor.

4.4. *BER performance of multiuser CS-DCSK*

Section 3.3 proposed a new CS-DCSK modulation scheme equipped with multiuser capability where the different users are separated by Walsh codes. An analytical expression, see Eq. (42) has been developed to predict the BER performance of CS-DCSK in multiuser applications. During the derivation of Eq. (42) we assumed that the Walsh code sequences used by the different users are synchronized.

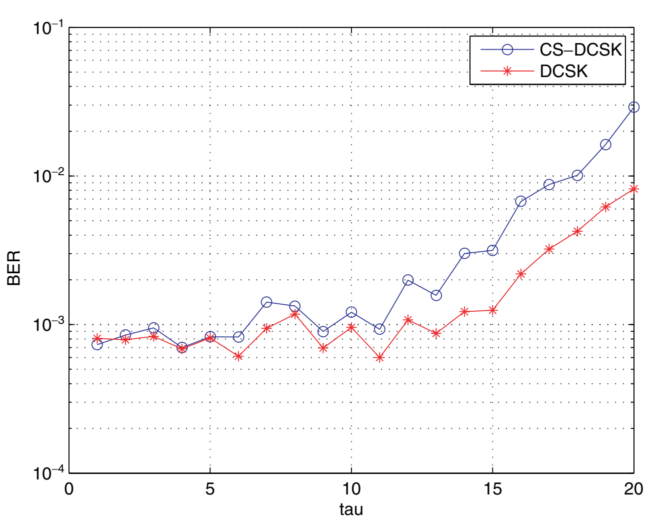

Fig. 14. BER performance comparison of CS-DCSK and DCSK modulation schemes over a two-path multipath channel for $E_b/N_0 = 26$ dB and SF $= 80$.

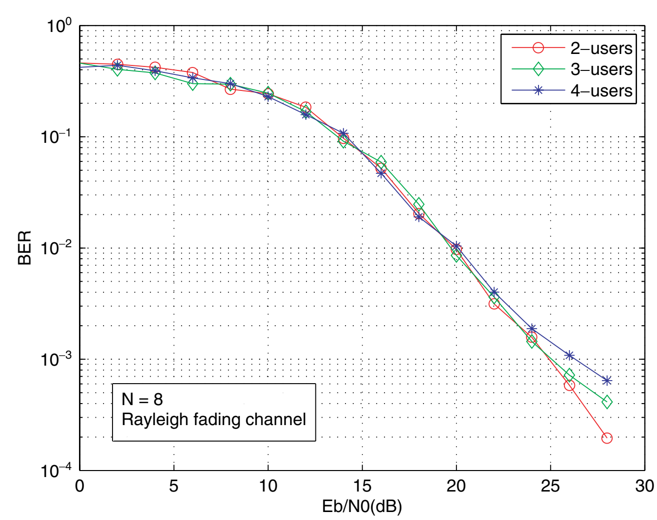

Fig. 16. BER performance of multiuser CS-DCSK over multipath Rayleigh fading channel (channel I), spread-spectrum factor is 160.

Figure 15 shows the multiuser performance of CS-DCSK over an AWGN channel. The spreading factor is 160 and an eighth-order Walsh code is used to separate the different user. In the binary case, this system can accommodate four users. As expected, the different users are separated by the orthogonality of Walsh code sequences and there is no deterioration in BER performance due to the multiuser application.

As shown in Fig. 16 the orthogonality of codes cannot be maintained in the Rayleigh multipath channel. The larger the number of accommodated users, the worse the BER performance of CS-DCSK. This is not a specific characteristic of CS-DCSK modulation scheme, the loss of orthogonality appears in every multiuser application.

5. Conclusions

The M-ary DCSK has become the best known modulation scheme in the field of chaos-based communication. Unfortunately it has a big disadvantage, delay lines are required both in the transmitter and receiver units. The problem becomes ever harder in ultra-wideband applications where the bandwidth of delay lines must be extremely wide. These delay lines cannot be implemented by CMOS technology.

A new variant of DCSK, called Code-Shifted DCSK is proposed here that allows to implement the CS-DCSK receivers without delay lines. This feature makes CS-DCSK extremely attractive especially when the system is implemented with CMOS technology.

In contrast to DCSK, the reference and information bearing waveforms are transmitted in the same time slot which approach also increases the data rate. The waveforms are separated by the Walsh code sequences. Exploiting the orthogonality of Walsh code sequences a CS-DCSK system offering a multiuser capability has also been developed.

Analytical expressions for the BER performance of CS-DCSK modulation scheme have been developed for AWGN channel, Rayleigh multipath channel and multiuser applications. The analytical expression have been verified by computer simulations.

The BER performance comparison of conventional DCSK and new CS-DCSK modulation schemes were performed for AWGN channel, Rayleigh multipath channel and multiuser applications. The results of comparison proved that the BER performance of CS-DCSK reaches or is very close to that of the DCSK. Since the implementation of a CS-DCSK receiver does not require delay lines, the CS-DCSK offers a much simpler and cheaper, furthermore CMOS-friendly implementation.

Acknowledgments

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