DIGITAL COMMUNICATION WITH CHAOTIC AND IMPULSE WAVELETS

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Abstract
Fourier analyzer concept that generalize the waveform communications to include chaotic carriers can be used to discuss and optimize detection problems. Using this new description it is possible to develop new detector configurations with improved performance for chaotic communications that may be used in ultra-wideband radio (UWB). Radio communications via channels already occupied by traditional telecommunication systems can be achieved by using UWB radio where extremely wideband wavelets are radiated in order to reduce the power spectral density (psd) of transmitted signal. Since the recovery of these UWB carriers is not feasible, noncoherent demodulation techniques have to be used. The paper shows the Fourier analyzer concept then evaluates and compares the noise performances of the feasible noncoherent UWB modulation schemes, namely, that of the noncoherent pulse polarity modulation and the transmitted reference system.

Keywords: subspace theory of detection, Fourier analyzer concept, chaotic communications, transmitted reference system, UWB radio

1. INTRODUCTION

Digital communications using wideband carriers have been becoming one of the most important applications of radio communications recently [1]. Typical applications are the wireless local area network (WLAN), ultra-wideband radio (UWB) and sensor network. It is not the thermal noise but the multipath propagation that limits the system performance in these, mostly indoor, applications; therefore, the transmitted signal must be a wideband signal.

The digital information to be transmitted is mapped into fixed, mostly sinusoidal waveforms in conventional communication systems. These systems are typically narrowband systems, but if it is needed the spectrum of them can be spread by a pseudo-noise sequence [2]. An alternative solution is the application of a wideband carrier, where the digital information to be transmitted is directly mapped into a wideband signal. The carrier can be a fixed waveform in impulse radio [1] or chaotic signal in chaotic UWB radio. The common property of these solutions is that the carrier is a wideband signal; consequently there is no need for an extra spectrum spreading sequence.

The theory of fixed waveform communication is well established. In conventional communication systems if the same symbol is transmitted, then the same waveform is radiated. In contrast to telecommunications using fixed carrier in case of chaotic communications the transmitted waveforms are continuously varying even if the same symbol is radiated repeteadly. This unique property has a very strong influence on the applicable detector configurations, makes difficulties to discuss the performance of these systems or to compare them to conventional ones. In order to overcome these difficulties the theory of waveform communications has to be extended to chaotic carriers.

A unified theory and model, the Fourier analyzer concept have to be established and developed, respectively, (i) that covers each kind of waveform communication systems, (ii) which may be used to determine the performance bounds of different waveform communication systems and (iii) that offers a systematic approach for the development of optimum detector configurations [3].

The *a priori* information available at the receiver is used in waveform communication to suppress channel noise and interference. As a rule of thumb we may say: the more the exploited *a priori* information, the better the system performance. The paper gives an exact measure for the *a priori* information. The measure proposed may be equally used in any kind of communications employing fixed or chaotic carriers.

In the first part of this paper the Fourier analyzer concept and the subspace theorem are introduced, then UWB radio is discussed based on the unified model.

2. GENERALIZATION OF WAVEFORM COMMUNICATIONS

In conventional waveform communications, each element of the signal set is a fixed waveform and it is represented as a linear combination of orthonormal basis functions which can be determined by the Gram-Schmidt orthogonalization procedure [2]. The most important difference between the conventional approach and the Fourier analyzer concept is that in the latter the basis functions are represented by harmonically related sinusoidal functions which have infinitely long duration.

In digital modulator the symbol m is mapped into a signal vector $\boldsymbol{s}_m=[s_{mn}]$. The elements $s_m(t)$ of signal set are generated as a linear combination of N real-valued orthonormal basis functions [2]

$$s_m(t)=\sum_{n=1}^{N} s_{mn} g_n(t) \qquad \begin{cases} 0 \le t < T \\ m = 1, 2, \ldots, M \\ n = 1, 2, \ldots, N \end{cases} \tag{1}$$

where $N \le M$. Each message is characterized by a distinct waveform (1). The signalling time period T is determined by the data rate. To avoid intersymbol interference (ISI) the basis functions must be zero outside the signalling time interval.

2.1. Signal model for detection

According to (1), the elements of signal set are represented by the basis functions that are known, or at least some of their characteristics are known at the receiver. This *a priori* knowledge is exploited to perform the detection and suppress channel noise and interference. Let a *received signal space* be defined in which each received signal such as transmitted signal, channel noise and interference may be expressed. Then the detection process is interpreted using the subspace theorem. Each basis function spans a subspace in the *signal space* and the detector maps the received signal into each subspace and generates an *observation vector* used later to perform the decision.

The general block diagram of a digital communication receiver is shown in Fig. 1. The transmitted signal $s_m(t)$ is corrupted by additive white Gaussian noise denoted by $n(t)$. The received signal is obtained as $r_m(t)=s_m(t)+n(t)$ and fed into a bandpass filter of RF bandwith $2B$ to select the desired channel. The detector observes the filtered signal $\tilde{s}_m(t)+\tilde{n}(t)$ over the bit duration T and generates an observation variable z_m.

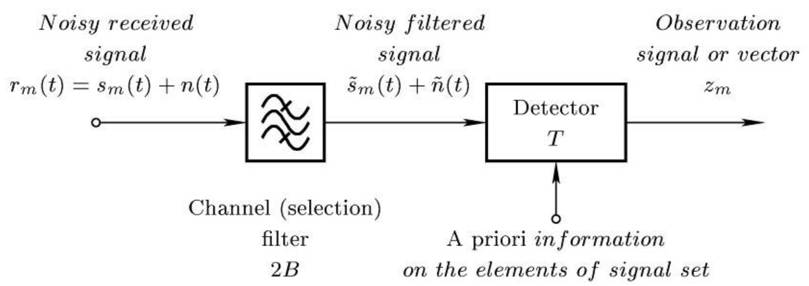

Figure 1. General block diagram of a digital communication receiver

The decision time intant, the bit duration T and the RF bandwidth $2B$ of transmitted signal $s_m(t)$ have to be known at the receiver to apply the Fourier analyzer concept.

2.2. Fourier analyzer concept

The Fourier transform that is widely used to represent an arbitrary waveform in the frequency domain cannot be used here because the dimension of its signal space goes to infinity. It has to be emphasized that the detector observes the channel filter output only over the bit duration T. Outside the observation period $\tilde{s}_m(t)$ may take any form. Consequently, from the detection point of view a periodic signal can be constructed of period T using

$$s_{T,m}(t) = \begin{cases} s_m(t) & \text{for } 0 \le t < T \\ s_m(t-CT) & \text{otherwise,} \end{cases} \tag{2}$$

where C is a nonzero integer. According to (2) the Fourier series representation can be used and a discrete spectrum is obtained. Because the signal is a bandpass signal the received signal space has a finite dimension. This signal space then has to be interpreted as a Hilbert space [4] spanned by the harmonically related cos(.) and sin(.) functions whereto the signal $s_{T,m}(t)$ has to be projected. In a Hilbert space an *inner product* and a *norm* have to be defined. The inner product of two arbitrary waveforms $s_1(t)$ and $s_2(t)$ is defined as

$$\langle s_1(t), s_2(t) \rangle = \int_0^T s_1(t) s_2(t) dt.$$

Signals with zero inner product are called orthogonal signals. The norm of a waveform $s(t)$, that equals the square root of energy carried by $s(t)$ is defined as

$$\|s(t)\| = \sqrt{\int_0^T s^2(t)dt}.$$

Demodulator input can be represented by a Fourier series over the bit duration as

$$s_m(t)\big|_{0\le t<T} = s_{T,m}(t) = \sum_{k=K_1}^{K_2}\left[a_k \cos\left(k\frac{2\pi}{T}t\right) + b_k \sin\left(k\frac{2\pi}{T}t\right)\right], \tag{3}$$

where a_k and b_k are weightening constants that determine the amplitude of the signal components while K_1 and K_2 are detemined by the signal dimension. Constants a_k and b_k can be obtained as

$$\begin{aligned} a_k &= \frac{2}{T}\int_0^T s_{T,m}(t)\cos\left(k\frac{2\pi}{T}t\right)dt = \frac{2}{T}\left\langle s_{T,m}(t), \cos\left(k\frac{2\pi}{T}t\right)\right\rangle \\ b_k &= \frac{2}{T}\int_0^T s_{T,m}(t)\sin\left(k\frac{2\pi}{T}t\right)dt = \frac{2}{T}\left\langle s_{T,m}(t), \sin\left(k\frac{2\pi}{T}t\right)\right\rangle. \end{aligned} \tag{4}$$

The signal dimension S_D as it has been shown in [3] is defined as $S_D = 2(K_1 - K_1 + 1) = 4BT$.

Chaotic and random signals may also appear at the detector input. The Fourier series representation introduced above remains valid for these signals, but the Fourier coeffcients defined by (4) will be random variables.

2.3. Properties of basis functions

Only a subspace of *received signal space* is required to represent the elements of signal set used to transmit the information. To minimize the effect of channel noise and interference on the observation variable, the detector observes only that subspace where the energy of transmitted waveform is greater than zero. The basis functions in (1) have been introduced to describe the elements of signal set with a minimum number of waveforms. The subspaces where the energy of transmitted signal is greater than zero are assigned by the basis functions. Each subspace belongs to a basis function. The union of these basis functions constitutes the *observation space*. Using the Fourier series representation the basis functions obtained in the *received signal space* can be written as

$$g_m(t)\big|_{0\le t<T} = g_{T,m}(t) = \sum_{k=K_1}^{K_2}\left[\alpha_k \cos\left(k\frac{2\pi}{T}t\right) + \beta_k \sin\left(k\frac{2\pi}{T}t\right)\right], \tag{5}$$

where α_k and β_k are from (4). The ortogonality of the basis functions makes them distinguishable in the *received signal space*.

2.4. Measure of *a priori* information

In waveform communications, either fixed or chaotic, two kinds of information are available at the receiver, the *must know* and *a priori* information. Without knowing the must know information, the detection may not be performed. The must know information includes the timing information expressed by the bit duration T here, and the spectral properties of transmitted signal characterized by the RF bandwidth $2B$ in the Fourier analyzer concept. The *a priori* information is exploited by the detection algorithm to suppress the effect of channel noise and interference. The detection may be performed even if only a very limited amount of a priori information is available at the receiver, but the less the a priori information, the worse the bit error rate (BER).

Equation (1) shows that the transmitter radiates signal energy only in the subspaces assigned by the basis functions. In the complementary subspace only noise and interference are received. The detector suppresses these unwanted signals in such a way that it observes only the subspaces assigned by the basis functions, i.e., where the transmitted signal energy is greater than zero. This technique is referred to as *subspace detection theorem*. If the Fourier components are exactly known at the receiver then the subspaces defined at the detector using the *a priori* information are identical with the subspaces assigned by each transmitted basis function and the maximum amount of noise and interference is suppressed. If the *a priori* information is not enough to define without any error the subspace of each basis function then the observation space differs from that one assigned at the transmitter. Due to the error, signal energy is lost and extra channel noise and interference is accommodated in the observation variable. Consequently, the system performance is corrupted.

3. ULTRA-WIDEBAND (UWB) RADIO

Using Fourier analyzer concept introduced in the previous part of this paper the UWB radio can be discussed exactly. Since the unified mathematical model is applicable to arbitrary waveforms, communication systems using chaotic or UWB signals as carrier become examinable.

The digital information to be transmitted is mapped to wideband wavelets of very short duration in UWB radio. The wavelets have a fixed waveform in impulse radio [1] and they are chaotic signals in chaotic UWB radio. Since the recovery of UWB wavelets is not feasible, noncoherent demodulation schemes have to be used: (i) pulse polarity modulation using one wavelet and template detection, and (ii) TR system using two wavelets and autocorrelation detection.

3.1. Pulse polarity modulation with template detection

The structure of UWB modulations using one wavelet is shown in Fig. 2, where g(t) denotes the wavelet having an arbitrary but fixed waveform. Because of its excellent spectral properties, bell-shaped gaussian impulses are used as $g(t)$ [1]. In pulse polarity modulation the information is carried by the sign of wavelet. The information $\hat{b}_m$ is recovered by correlating $\tilde{r}_m(t)$ with a template signal $p(t)$ as shown in Figs. 3 and 4.

The template signal is a windowing and weighting pulse

$$p(t) = \begin{cases} \dfrac{1}{\sqrt{\tau}} & \text{if } |t| < \dfrac{\tau}{2} \\ 0 & \text{otherwise,} \end{cases} \tag{6}$$

where τ is the observation time period.

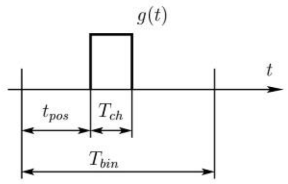

Fig. 2. Structure of UWB modulation using one wavelet.

The drawback of template detection as it shown by Fig. 5 is that the demodulator is very sensitive to the timing error. Any timing error reduces the separation of message points in the observation space and, consequently, results in a considerable performance degradation.

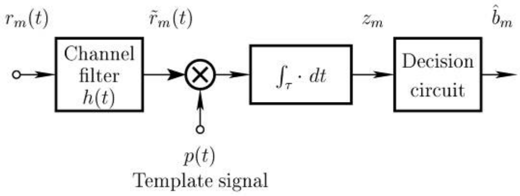

Fig. 3. Detection of pulse polarity modulation with a template signal.

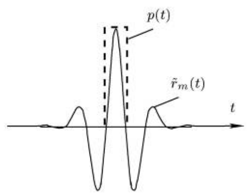

Fig. 4. Perfect alignment of received and template signal in a noise-free ideal channel.

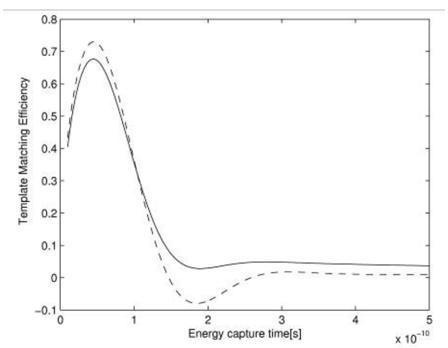

Fig. 5. Template matching efficiency as a function of the width of template signal. The RF bandwidths of Gaussian impulses are 2 GHz (dashed curve) and 2.5 GHz (solid curve).

The bit error rate of noncoherent pulse polarity modulation built with template detection is obtained from [3]

$$P_e = \frac{1}{2}\left(e_{tm}\sqrt{\frac{E_b}{N_0}} \right) \qquad (7)$$

where e_{tm} denotes the template matching efficiency.

3.2. Transmitted Reference (TR) system

In TR systems two wavelets, called chips, are used to transmit one bit information. The first chip serves as a reference, while the second one carries the information. The structure of modulated signal is shown in Fig. 6, where $g(t)$ denotes an arbitrary wavelet. The best noise performance is achieved by the antipodal modulation scheme, where the information bearing wavelet is equal to the delayed reference one for bit “1,” and to the inverted and delayed reference wavelet for bit “0.” The reference chip should be considered as a test signal used to measure the actual channel characteristics. This solution is very robust, it can be used even in a time-varying channel.

Due to the special structure of TR signal, the information bits may be recovered from the sign of correlation measured between the reference and information bearing chips as shown in Fig. 7.

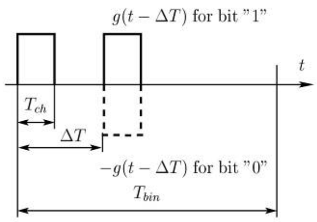

Fig. 6. Structure of modulation using two wavelets.

In contrast to template detection, in the autocorrelation receiver a noisy reference chip is correlated with a noisy information bearing one. Consequently, the cross correlation of two noisy sample functions corrupting the reference and information bearing chips appear in the observation variable that is responsible for the relatively bad noise performance.

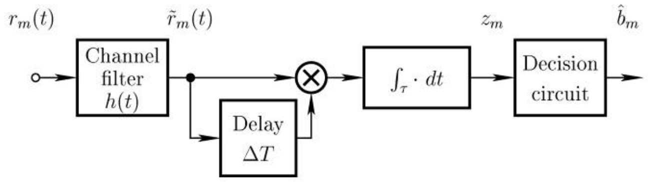

Fig. 7. Block diagram of TR autocorrelation receiver.

The bit error rate (BER) of a TR system is obtained as

$$P_e = \frac{1}{2^{2B\tau}} \exp\left(-\frac{E_b}{2N_0}\right) \sum_{i=0}^{2B\tau-1} \frac{\left(\frac{E_b}{2N_0}\right)^i}{i!} \sum_{j=0}^{2B\tau-1} \binom{j+2B\tau-1}{j-1} \tag{8}$$

where τ denotes the energy capture time of autocorrelation receiver and $2B$ is the RF bandwidth of channel filter [5].

3.3. Comparison of the noise performances

From (7) the noise performance of pulse polarity modulation implemented with template detection may be determined. Fig. 8 shows the noise performance for different channel bandwidths (*2B*) and template matching efficiency. According to (8) we can get Fig. 9, i.e., the noise performance of a TR system, built with an autocorrelation receiver.

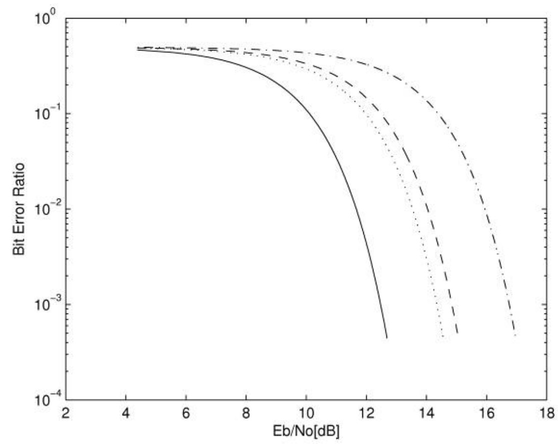

Fig. 8. Noise performance of pulse polarity modulation built with template detection. Solid curve: *2B* =2 GHz and e_{tm} = 0.69; dotted curve: *2B* =2 GHz and e_{tm} = 0.40; dashed curve: *2B* =500 MHz and e_{tm} = 0.35 and dash-dotted curve: *2B* =500 MHz and e_{tm} = 0.20.

Comparing the noise performances of the two systems, we find that in case of pulse polarity modulation at BER=10^{-3} with 0.1ns energy capture time the curve is very similar with that of TR system at $2B\tau$ = 17.

4. CONCLUSIONS

Exploiting the Fourier analyzer approach, a new model has been developed for waveform communications. The new model is valid for any kind of carriers, they may be either fixed, or chaotic waveforms.

The detector of a digital communication system observes the received waveform only over the signalling time period. Consequently, a periodic signal may be constructed which is identical to the received one over the signalling time period. The periodic signal is represented by a Fourier series, i.e., it has a discrete spectrum. The Fourier base of signal space is determined by the timing information which is always available at the receiver. The fundamental frequency of Fourier base equals the bit duration. The signal space constitutes a Hilbert space, its dimension, referred to as signal dimension, is determined by the product of bit duration and bandwidth of channel filter. The subspaces of each basis function are defined by the Fourier coefficients. Unfortunately, the Fourier coefficients are not always exactly known at the receiver. The accuracy with which the Fourier coefficients are known determines the type of receivers.

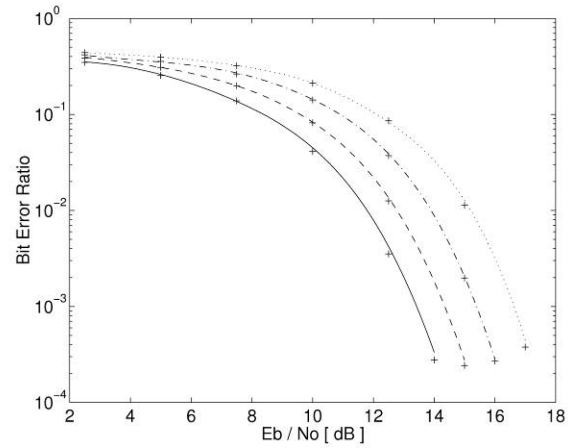

Fig. 9. Noise performance of a TR system built with an autocorrelation receiver. From left to right $2B\tau$ is 8.5, 17, 34 and 68.

Using the unified mathematical modell UWB radio become well examinable. Digital communications using UWB radio can be established by transmitted one wavelet received by a template detector or two wavelets received by an autocorrelation detector, respectively. The main advantage of template detection is that the template signal is a noise-free signal, its application results in a better noise performance if the width of template is perfectly matched to the UWB wavelet. However, the template detector is extremely sensitive to the mismatch including timing error. Even a small error in timing may prevent the communications. Because the UWB radio operates in the microwave frequency region, it seems to be very hard to fulfill the strict timing requirements.

FM-DCSK and the TR modulation scheme offers an alternative noncoherent UWB modulation scheme that may be used with either chaotic or deterministic wavelets. Like the noncoherent pulse polarity modulation, the noise performance of TR systems also depends on the energy capture time. But its sensitivity to the timing error is much less than that of the noncoherent pulse polarity modulation. The detection in TR systems can be achieved by a simple autocorrelation receiver. In this receiver the noisy reference chip is correlated with a noisy information bearing one, and due to the cross correlation of two noise sample functions the noise performance of TR system is relatively bad.

The noise performances of UWB impulse radio and chaotic FM-DCSK system have been compared. Figures 8 and 9 show that these systems offer a very similar noise performance.

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Curriculum Vitae (CV)

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