Angle of Arrival Estimation
Based on Interferometer Principle

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Abstract *– The angle of arrival estimation problem based on interferometer principle has great accuracy in the angle of arrival estimation. To increase the accuracy there are two main ways. In the first one we add more antennas to the configuration, hence we get more measurement data, but it means that the physical configuration has to be changed. In the second method we design numerical algorithms which gives better estimations. The prize is the complexity and the increased computation time. In this paper a method from the second set is shown and analysed. The algorithm is based on the cross correlation technique.*

I. INTRODUCTION

While radio direction finding for navigation purposes is losing in importance due to the availability of satellite navigation systems, the requirement for determining the location of emitter increases with the mobility of communication equipment. In the literature lots of situations can be found with respecting to direction finding. Nowadays one of the most focused applications are the mobile communication systems. To determine the direction of the mobile terminals it is possible to use space division multiplexing (SDM) using adaptive antenna arrays. This way helps to save the frequency spectrum as a final resource, and provide better coverage of the given area. In this kind of application the speed of the algorithm very important against the frequency agile signals of mobile communication systems.

In literature and introduction papers of devices there are several methods according to different requirements. In this work one of the widely used direction finding method, namely the phased interferometer technique is discussed. This solution has some advantages: very high resolution can achieve, and conformal arrays are possible. But the disadvantages are the relative high cost and the larger size.

In this case a four element antenna array configuration is used. The signals of the output of the antenna elements are measured synchronisedly and converted into digital domain after the corresponding down-converting. In the digital signal processing block the different estimation algorithms are realized with parallel computing, because of the strong computing time constraint.

This measurement configuration induces two separate statistical estimation algorithms. Firstly, the phase difference between two chosen antennas has to be estimated with very small error. Secondly, using the results of the delay estimations between antennas the estimated direction of the emitter is computed. As a matter of fact the first problem is a time delay estimation in the time-domain. In general without using any appropriate information about properties of the signal of emitter(s) it is a very hard problem with high accuracy requirement. Now, in the discussed situation there are radio frequency signals with different modulation methods, such as amplitude modulation (AM), frequency modulation (FM), phase shift keying (PSK), quadrature amplitude modulation (QAM), etc.

First of all this paper gives a brief description of the hardware configuration and the physical setup of antenna elements. Then the estimation problems are mentioned and analysed. In the last two sections a new method is published according to the assumed direction finding situation. In the end there is a brief conclusion section and some further research ideas are formulated.

II. PHYSICAL CONFIGURATION AND THEORY OF DIRECTION FINDING

The direction finder setup can be seen in Fig. 1. We use four antennas in the array whose input signals are converted in the first IF band by the down converter block. The maximal frequency component of this frequency band is such that we can convert signals into digital domain with high frequency A/D converters. The direction estimator component (bearing calculation) of the configuration denotes a digital signal processing part. In this block the computing is in digital domain using DSP and FPGA processors. This kind of setup suits the so-called software radio architecture, thus the analog

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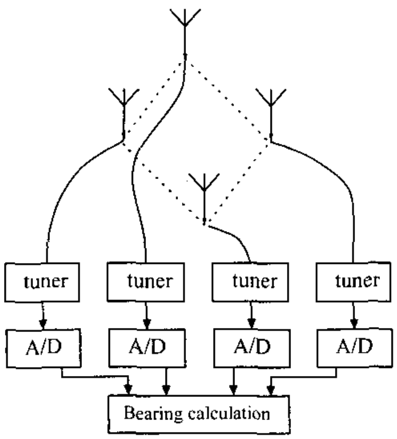

Fig. 1. Configuration of a four channel interferometer direction finder

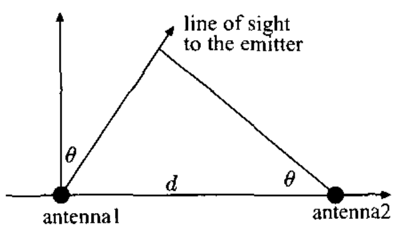

Fig. 2. Principles of 2-element interferometer

signals are sampled as soon as possible. Therefore we get high sampling rate and digital signals in pass-band.

The antenna configuration is such that the four antennas are at corners of a square. The minimal distance between two antennas determines the maximal frequency of the input signal. If λ denotes the wavelength of incoming signal and d denotes the distance of two antennas then the following inequality limits the highest frequency component:

$$d \leq \frac{\lambda}{2} = \frac{c}{2f} \tag{1}$$

Furthermore the physical realization of antennas causes that this setup can be used only in a typical frequency band. The filters in the down converter are specified to operate in this band, hence in digital domain we already known which interval of the time delay is valid.

The interferometer principle is the following. We originate the direction finding problem in time delay measurement. The Fig. 2 shows the basic geometry. The principle is that a plane wave arriving at an angle is received by one antenna earlier than the other due to the difference in path length.

If the distance of two antennas denotes d and the speed of light $c = 2.99792458 \times 10^8 \frac{\mathrm{m}}{\mathrm{s}}$ then the time delay between signals of antennas is

$$\tau = \frac{d \sin\theta}{c} \tag{2}$$

where θ is the angle of the arrival. So with this analytical relationship we can obtain the directional information from the spatial position of the lines or surfaces of equal phase. Hence to solve the angle of arrival (AOA) determination problem we need estimate time delays and from these results an angle. The details of these numerical algorithms are specified in the next sections.

III. DELAY ESTIMATION

In our treatment the estimation of time delay suffers from a problem. The time grid (or the frequency grid) , i.e. the sampling time does not give enough resolution for the desired accuracy. Therefore the algorithm which seems to be usable at the first sight has to be refine, like the publicated, specialised ones in the papers. In the literature lots of possible methods can be found but all of these are valid only in special cases. In these paper the presented algorithms are also such, i.e. application of these limited for direction finding (delay estimation) for communication signals.

In this paper we are assuming that the type of the emitter signal is known, so the modulation method and the type of baseband signal are a priori informations in delay estimation problem. Furthermore, we assume that the emitter signal is a narrow-band one. In practice this condition is current because typically the frequency of carrier signal is less than 3 GHz and the bit-rate of baseband signal is less then 0.1% of the carrier frequency. Hence the 5 MHz bandwidth is enough for our application. It causes that with high sampling rate in the IF band the measure of changing of the baseband signal with respecting one segment is little. In the case of analogue baseband signal it means that the modulated signal has narrow-band spectra and in the case of digital modulations the bit rate should have to be slow. Nowadays, the current technology allows us to apply such software radio structure which has central frequency 10.7 MHz sampled with frequency 40 MHz.

Continuous-wave modulation

Firstly, let us study the case of analogue baseband signals. We consider two main types, namely the amplitude modulation (AM) and the frequency modulation (FM). The latter one as we mentioned is a narrow-band FM signal and since the properties of narrow-band FM signals can be derived from AM signals, in this paper it is enough to study the AM ones [1]. For example, an AM signal with carrier frequency 10 MHz (in the IF band) and an audio baseband signal pass the assumptions above.

If we are sampling such signal with sampling frequency $f_s = 40$ MHz, then with conditions above in one segment there are lots of periods of carrier. And the amplitude of the carrier remains approximately constant because of the narrow band modulation.

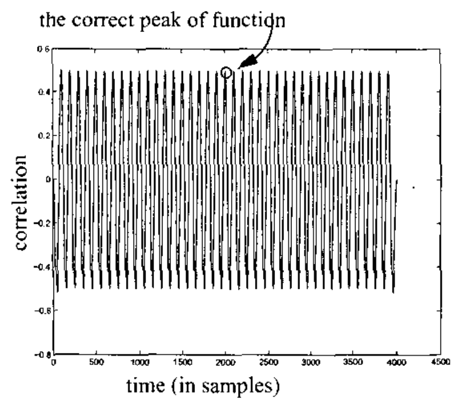

Fig. 3. Correlation function of analog modulated signals

Even so we cannot guarantee that the sampling is coherent, the result of FFT is approximately a line, because we get more periods. The problem is that the frequency grid cannot guarantee desired resolution. And the determination of the maximum of a sine is not a well-conditioned problem especially with presented noise. To increase this we have to resample (interpolate) the spectra. This means that low-pass digital filters have to be designed. A more efficient method is to use analytical model. Obviously, the application is limited to the corresponding modulation method. But, if we use analytical model only in the case of part of cross-correlation function, then the computing time and complexity of model will be reduced. The background of this methods is that the cross correlation function has a local maximum at the true value of the delay.

With using the fact, that the auto-covariance function of a sinus function is also a sine, we can approximate the neighbourhood of the corresponding, global maxima with a parabola. Since the parabola fitting problem is an efficient and quick numerical algorithm, it takes short time to determine the local maxima. To fit a cosine function on the given time grid is not such easy problem, especially to determine the frequency is not an easy and quick algorithm. This is the reason, why the parabola fitting methods gives more acceptable result with respecting to computing time. Of course, if we consider a better model for communication signals and formulate a non-linear minimisation method then we can get better estimation for the time delay. The main problem is that solution of non-linear minimisation problem expect some cases has no predictable computing time and is too complex.

In the Fig. 3 a typical correlation function can be seen. It is approximately a sine function and has lots of local maxima. The correct one can be selected by the known antenna distance d, because from Eq. (2) the inequality $d \geq \tau c$ gives an upper bound for τ. In Fig. 3 the good peak shows the maximum which is adequate to this condition (in the figure the half of the number of samples is equivalent with $\tau = 0$). After the

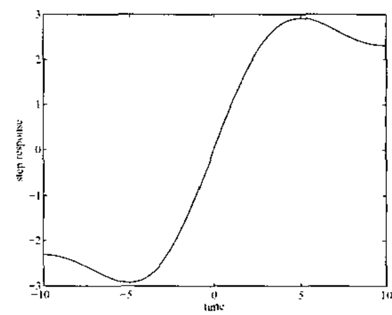

Fig. 4. Filtered pulse by raised cosine filter

localisation of correct maximum of the correlation function we have to call the parabola fitting routine to determine the more precise the maximum of the sine function.

Digital modulation

Secondly, we study the digital modulations. The message source emits pulse signals with given pulse rate. This signal is filtered by a Nyquist filter (in this case a raised cosine one) which is commonly used in communication system to avoid inter-symbol interference. And the result is up-converted to the desired frequency band by the mixer block. As we mentioned above we assuming that the bit rate is slow with respecting the carrier frequency. In our receiver we are processing segments of samples and we suppose that in one segment there is only fixed number (max. 10 pieces) of bit changing.

The impulse response $p(t)$ of a raised cosine filter is the following:

$$p(t) = (\mathrm{sinc}(2Wt)) \left(\frac{\cos(2\pi\alpha Wt)}{1 - 16\alpha^2 W^2 t^2} \right) \tag{3}$$

.where the parameter α is the roll-off factor. The frequency parameter f_1 and bandwidth W are related by

$$\alpha = 1 - \frac{f_1}{W} \tag{4}$$

Fig. 4 depicts the step response of raised cosine filter. As a matter of fact it is a filtered pulse.

As in the previous subsection in Fig. 5 the correlation function can be seen. Because in the case of digital modulation we cannot state that the modulation signal is quasi constant, the cross correlation function is not a sine function. But, like in the figure to find the correct local maximum of the function we have to use the parameters of the receiver.

Parabola fitting

In this subsection we study the approximation of cross correlation function. Because of its symmetrical property in the neighbourhood of the true delay value the correlation

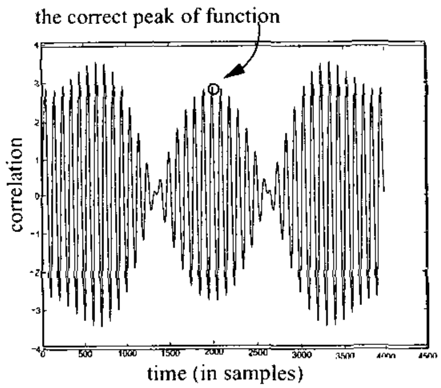

Fig. 5. Correlation function of digital modulated signals

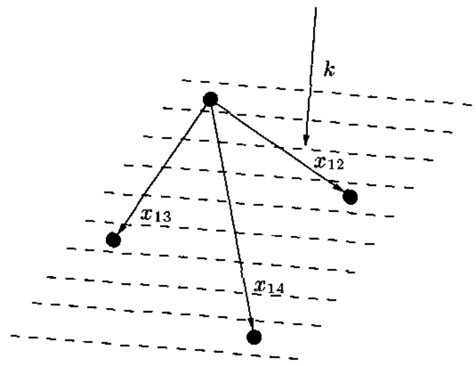

Fig. 6. Propagation vector

function can be approximated with a parabola. It means that with a few points (samples) we get an approximation and its analitycal maxima gives a good estimation. If we using the frequency domain identification method then statistically we get better result, but if the segment of the signal consists at least one bit changing then using the cross correlation gives better estimation.

We suppose that the neighbourhood of the τ value has M points. Then in the approximation problem we define the following norm that is the usually least squares norm.

$$\| a - b \| = \sum_{j=1}^{M} |a(x_j) - b(x_j)|^2 \tag{5}$$

where x_j are the sampling points in the corresponding neighbourhood and a, b are two functions from this interval to the real line. The solution (which is a projection) is exists and unique. In our case a is the cross correlation function and b is a second-order polynom. The background of this kind of approximation is Taylor serie. Furthermore, since the cross-correlation function is a linear combination of sinus, this approximation sounds well.

Numerically the projection finding leads to solution of the following linear equations.

$$\begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ \vdots & & \vdots \\ 1 & x_M & x_M^2 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} a(x_1) \\ a(x_2) \\ \vdots \\ a(x_M) \end{bmatrix} \tag{6}$$

with using that

$$b(t) = b_0 + b_1 t + b_2 t^2 \tag{7}$$

IV. ANGLE OF ARRIVAL ESTIMATION

After for antenna pairs the delay estimations are finished, the angle of arrival can be computed from the propagation vector.

Now, the propagation vector k of target signal can be defined as

$$k = \frac{1}{c} \frac{x_0 - x_{\mathrm{src}}}{\| x_0 - x_{\mathrm{src}} \|} \tag{8}$$

where x_0 denotes the centre of the array. $\| . \|$ denotes the L2-norm of a vector. Furthermore, the vector between the ith and mth antenna is denoted by

$$x_{im} = x_m - x_i \tag{9}$$

where x_i and x_m are two dimensional vectors representing the antenna locations in an orthogonal coordinate system.

Hence, the time delay t_{im} between the signals received by the ith and mth antenna can be calculated as

$$\tau_{im} = k^T x_{im} \tag{10}$$

Put it in matrix form, this equation for all antenna pairs can be represented by

$$V k = \hat{\tau} \tag{11}$$

with

$$V = \left[x_{i_1 m_1} \quad x_{i_2 m_2} \cdots x_{i_M m_M} \right]^T \tag{12}$$

$$\hat{\tau} = \left[\hat{\tau}_{i_1 m_1} \quad \hat{\tau}_{i_2 m_2} \cdots \hat{\tau}_{i_M m_M} \right]^T \tag{13}$$

Hence from k we can obtain the corresponding angle.

Algorithms

Now, we give a compact description of the algorithms. In every step the corresponding computational algorithms can be run parallel, so the total processing time can be reduced significantly.

- Do the D/A conversion and sampling for every antenna.
- Calculate the cross-correlation function for the proper antenna pairs.
- Find the correct local maximums.
- After the parabola fitting estimate delays for antenna pairs.
- Calculate propagation vector and AOA from the obtained delay values.

V. CONCLUSIONS

In this paper we gave a brief description about a direction finding algorithm with interferometer principle which can be used for high-precision measurements in a short time interval. We described the main problems which arrived in the case of analog and digital mobile communication systems. The published algorithm is robust which can be explained in theoretical way. This algorithm has the advantage that it is easy to compute in parallel way. The future works focus on the investigations of the behaviour of the algorithm under more different type of modulation formats and the sensitivity of the algorithm for the different errors come from the physical realisation of the hardware platform.

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