

Signal Processing in Distributed Systems with Limited Resources

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Abstract. Distributed wireless systems offer several advantages over their traditional wired and centralized counterparts, e.g., flexibility, easy installation, reconfiguration, and scalability. Although the resources in these distributed systems are often limited in order to achieve cost-effective operation, the concept of distributed system design is in the focus of intensive research even in those application fields that are extremely susceptible against the uncertainties caused by decentralized operation. Such applications are closed-loop signal processing systems where signals are fed back through a shared communication channel. There are three challenging problems of distributed closed-loop signal processing systems: bandwidth limit of the communication medium, synchronization of sensor nodes and data loss in communication. The paper introduces these problems and proposes some possible solutions.

Keywords: distributed systems, signal processing, wireless sensor network, data loss, bandwidth limit, synchronization, resonators

1 Introduction

Due to recent advances in communication technology, cheap and easy-to-use solutions are commercially available to build up complex distributed systems [1]. In these systems, autonomous nodes can communicate and cooperate with each other over a shared communication channel; usually over a wireless medium. Based on this technology, amazing concepts like wireless sensor networks (WSN) or Internet of Things (IoT) have already become reality [2]. Besides the first applications involving low sampling rate data acquisition (e.g., environmental monitoring), powerful systems enable the direct real-time interaction between the nodes [1]. Hence distributed signal processing can also be performed. Recently some control systems take the advantage of flexible and scalable communication [3].

However, the design of distributed signal processing systems is challenging, due to the inherent unreliability of networked communication. The problem is

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particularly difficult if a control system is to be designed, as the stability of a networked closed-loop system is not obvious [4]. Moreover, in several cases the resources of the nodes are also limited in order to ensure cost-effective and power-aware operation [2]. Most of the challenges stem from three basic roots [3]: bandwidth limit of communication medium [6], autonomous operation of sensor nodes [5] and data loss in communication [7]. Our paper focuses on these three of the main challenges which are illustrated through a practical example which is a wireless active noise control application. Beyond the analysis of the above problems, some possible solutions are also presented.

2 Test System

2.1 System Model

The architecture of a distributed signal processing system is depicted in Fig. 1. Beside the structure is flexible enough to represent a wide range of distributed systems, it contains all of the important components required for signal processing.

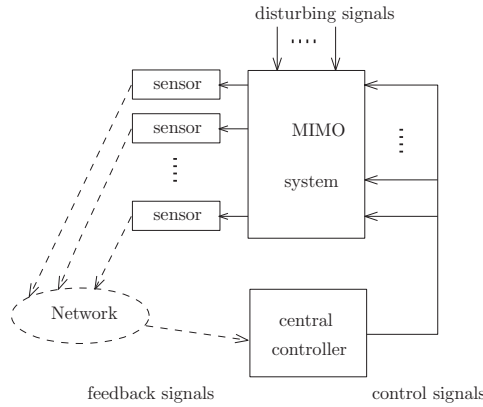


Fig. 1. Block diagram of the system model.

A sensor network is attached to a multiple-input multiple-output (MIMO) physical system, and the individual sensors transmit raw or preprocessed measurement data to a central processing unit over a shared communication network. The central unit aggregates the sensors' data, and generates control signals which are directly connected to the physical MIMO plant. The system model in Fig. 1 clearly shows that signal sensing and processing are performed in a distributed manner by the sensor network and a central controller.

The networked communication of sensors means a significant reduction in the physical complexity of the system, since the communication can be solved

over a wireless medium, which eliminates the need of cabling. A well-known communication standard for establishing wireless sensor network is ZigBee [2], which is suited for low-power local networks.

Perhaps the most critical part of the signal path is the feedback through the sensor network, since sensor nodes often contain only a simple microcontroller with limited resources in order to achieve low-power and cost-effective operation, and the network communication is also prone to inject different kinds of errors into the signal flow.

The system model is asymmetric in the sense that sensors transmit their measurement results over a multiple access network, however, control signals are directly connected to the plant. The reason of this asymmetry is that the interaction with a physical system often requires considerable amount of power, hence the deployment of power cables are often inevitable at the actuators, which reduces the significance of a shared communication medium.

2.2 Test Application

As an illustrative example, the problems and results will be presented through an active noise control (ANC) application. In ANC the phenomena of destructive interference is used to suppress low-frequency acoustic noise by radiating a so-called anti-noise [9]. Microphones are used to sense the noise to be suppressed, and loudspeakers are used to radiate the anti-noise. ANC requires a kind of adaptive signal processing algorithm. In this paper we investigate the so-called resonator-based adaptive controller (RAC), which is specialized to suppress periodic disturbances.

We do not deal with the ANC problem in detail, but we use it to demonstrate the problems that emerge in a real-time, closed-loop application. A question arises, why ANC is chosen as test application. An ANC system is easy to install and reconfigure. The plant is an acoustic system which is present everywhere. Sensors and actuators, i.e. microphones and loudspeakers are commercial products so they are easily available.

An ANC system is a MIMO system. It consists of several microphones and loudspeakers, so a real sensor network can be built using wireless microphones. An ANC system is scalable: the number of microphones and loudspeakers in the ANC applications vary in a wide range, from single input - single output (SISO) systems to bigger ones. Complicated algorithms can be tested with simple systems, but other, well-understood procedures can be checked out in a challenging MIMO environment.

Fig. 2 shows the schematic diagram and the linearized model of a wireless active noise control test system where the noise to be suppressed is sensed by a wireless sensor. Here the resonator-based ANC is presented by a single channel system (one input, one output), but readers are referred to [8] for further details about an extensive MIMO system which can be used as a general framework for the investigation of challenges in distributed real-time signal processing systems.

In the figure, $A(z)$ denotes the transfer function of the physical system to be controlled (including also the delay of the network), y_n and u_n denote the

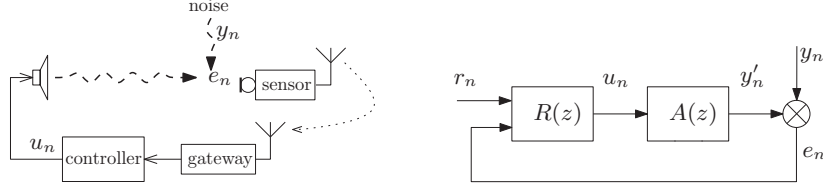


Fig. 2. Block diagram of a test system and its linear system model.

disturbing and the control signals, respectively, and $R(z)$ represents the control algorithm described by the following equations [10]:

$$u_n = \mathbf{c}^T \hat{\mathbf{x}}_n \quad (1)$$

$$\hat{\mathbf{x}}_{n+1} = \langle z_i \rangle \hat{\mathbf{x}}_n + \alpha \mathbf{W} \mathbf{z} e_n \quad (2)$$

$$\mathbf{W} = \langle \hat{A}^{-1}(z_i) \rangle, \quad (3)$$

where α a so-called convergence parameter, $\mathbf{c} = [1 \dots 1]^T$ is a vector of size $N \times 1$ containing ones, $\mathbf{z} = [z_0 \dots z_{N-1}]^T$ is a vector of size $N \times 1$ and $\langle z_i \rangle$ is a diagonal matrix of size $N \times N$ both of which containing the complex exponentials $z_i = e^{j2\pi f_i}$. Variables f_i are the discrete frequencies of harmonic components of the disturbing signal; f_i values are often termed as resonator frequencies.

The matrix $\mathbf{W}$ is a diagonal matrix which contains the inverse of the estimate of the transfer function of the plant at the resonator frequencies. The system is stable if the phase difference between $\hat{A}(z_i)$ and $A(z_i)$ is within 90° [10].

The signal r_n is the so-called reference signal which carries information about the disturbing signal, y_n . In the case of RAC it is used to determine the frequency of y_n .

3 Bandwidth Limit

3.1 Problem Statement

In real-time systems, a hard time limit is specified for the transmission time for all of the data collected by the sensors, so the bandwidth limit of the communication channel can be the bottleneck of the whole system. Taking a realistic example, if a ZigBee network with 250 kbps bandwidth is used, and an acoustic signal of a bandwidth of 1-2 kHz should be transmitted in real-time, only 3-4 sensors can be deployed in the network taking also into account the communication overhead.

The bandwidth of a communication link is highly determined by standards, costs and the power consumption, so the bandwidth constraints are often alleviated by using the computational capacity of the sensor nodes. Intelligent sensors are able to achieve data reduction by preprocessing the signal. Since sensor nodes often have limited computational resources, only a simple method can be used for data compression.

3.2 Proposed Method for Data Reduction

To achieve data reduction in a simple way, we propose the deployment of the so-called signed-error principle. It has already been developed to reduce the computational complexity of adaptive filters, however, its application in signal processing structures with dynamic feedback is quite new [12]. The principle is simple: the sensors transmit only the sign of the observed error signal, which means that significant reduction can be achieved in the amount of data to be transmitted.

The state equations of the sign-error algorithm can easily be derived from the original equations of RAC (see: (1)–(2)) by taking the sign of the error signal e_n in the state update rule:

$$u_n = \mathbf{c}^T \hat{\mathbf{x}}_n \quad (4)$$

$$\hat{\mathbf{x}}_{n+1} = \langle z_i \rangle \hat{\mathbf{x}}_n + \alpha \mathbf{W} \mathbf{z} \text{sign}(e_n) \quad (5)$$

The main advantage of the sign-error RAC algorithm is that it offers a very simple method for data compression (practically only the truncation of the error), hence the algorithm is easy to implement in systems with limited resources. The drawback of the algorithm is that the truncation of the error results generally in longer convergence time and higher steady-state error compared to the original RAC.

A measurement result of a practical application of the sign-error algorithm can be seen in Fig. 3. The measurement was performed in a test system shown in Fig. 2. An artificial periodic noise was radiated as a disturbing signal, and the remaining noise (i.e., the error signal e_n) was measured by a microphone.

The measurements show that the extent of noise reduction was almost the same (22 dB and 25 dB for the original and sign-error algorithms, respectively). The settling time of the sign-error structure was longer than that of the original algorithm, however, a data reduction of 60% was achieved in the practical implementation.

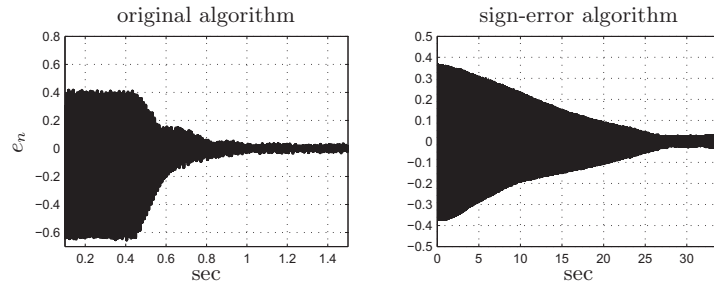


Fig. 3. Settling of a wireless ANC system controlled by the original and sign-error RAC.

4 Synchronization

4.1 Problem Statement

In traditional centralized signal processing systems, sampling and signal processing are performed on the same device. However, in distributed systems, where the sampling and the signal processing are performed on different hardware, the signal is distorted in a sense, which is to be compensated.

Perhaps the most unpleasant effect caused by the distributed signal sensing is the uncertain amount of delay in the data transmission. Since the constant delay can easily be compensated, system designers generally aim to use deterministic protocols for data transmission in order to ensure constant delay. However, the delay caused by the unsynchronized nodes has also serious effect.

Taking a realistic example, even 10 ppm difference in the oscillator frequencies results in 2.5 ms skew between the time references within 250 sec. An excess delay of 2.5 ms means 90° phase shift for a sine wave of the frequency of 100 Hz, which results in the instability of the RAC algorithm. This problem can only be solved by the synchronization of the network nodes.

4.2 Proposed Synchronization Algorithm

In order to ensure the synchronization of the sampling on the nodes, a PLL-like method was developed which is depicted in Fig. 4.

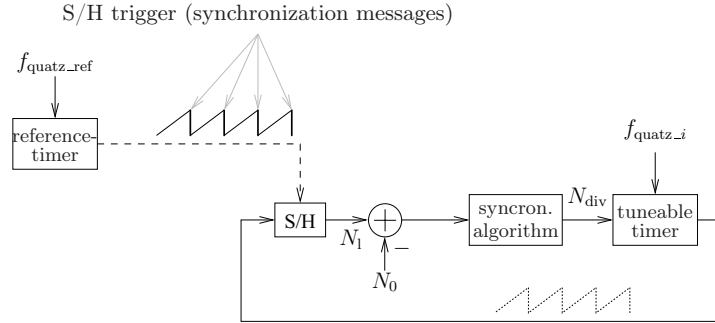


Fig. 4. Block diagram of the synchronization algorithm. Sawtooth signal with solid line: reference counter's value. Sawtooth signal with dotted line: the value of the counter of the timer to be synchronized.

The synchronization method requires a tuneable timer on the sensors, since synchronization is achieved by slowing down or speeding up the sampling. A tuneable timer is available on most of the microcontrollers. The timer is generally realized with a counter which operates with the clock frequency of the sensor

(f_{quartz}). After the counter has reached its programmed maximal value (N_{div}), it is cleared, and an interrupt is generated where the sampling of the observed signal is performed. The value N_{div} determines the sampling frequency: $f_s = f_{\text{quartz}}/N_{\text{div}}$. The time function of the counter's value is a sawtooth signal, at the falling edge of which the sampling is carried out.

The synchronization is performed in the following way. The algorithm requires a reference node which sends its synchronization messages over the radio channel at deterministic time instants with constant periodicity that is determined by its timer. At the reception time instants of these messages, the other nodes read the value of their own counter. In Fig. 4, N_1 denotes this value. Since the value of the sawtooth signal is proportional to its phase, this sampling and hold (S/H) operation is analogous to the phase detector function. Hence, with the tuning of the sampling frequency (changing N_{div}), the phase difference between the sawtooth signals on the reference and on the other nodes (i.e., N_1) can be kept constant.

Let N_0 denote the desired value of N_1 . The synchronization algorithm is as follows:

- if $N_1 < N_0 - \Delta N$, f_s should be increased (i.e., N_{div} should be decreased)
- if $N_1 > N_0 + \Delta N$, f_s should be decreased (i.e., N_{div} should be increased)
- if $N_0 + \Delta N \geq N_1 \geq N_0 - \Delta N$, f_s should be held on its nominal value

where ΔN is a tolerance parameter. ΔN is used because of the measurement uncertainty of the reception time of the messages (N_1). This is a very simple control algorithm, so it does not require much computational resources. It is important, since the algorithm could be implemented even on an eight bit microcontroller.

A measurement result of the synchronization algorithm can be seen in Fig. 5. The measurement was performed in the test system shown in Fig. 2. An artificial periodic noise was radiated as a disturbing signal, and the remaining noise (i.e., the error signal e_n) was measured by a microphone. The time skew (T_{drift}) between the sensor and the gateway was also measured.

As one can see in Fig. 5(a) and 5(b), after turning on the synchronization mechanism the delay became constant in the feedback loop, and the system remained stable after reaching the steady state. However, with inactive synchronization the delay in the feedback loop changed continuously, and as the delay had reached a critical level (shaded area), the system became unstable as shown in Fig. 5(c) and 5(d). The critical delay T_{crit} was calculated according to the stability condition of the RAC.

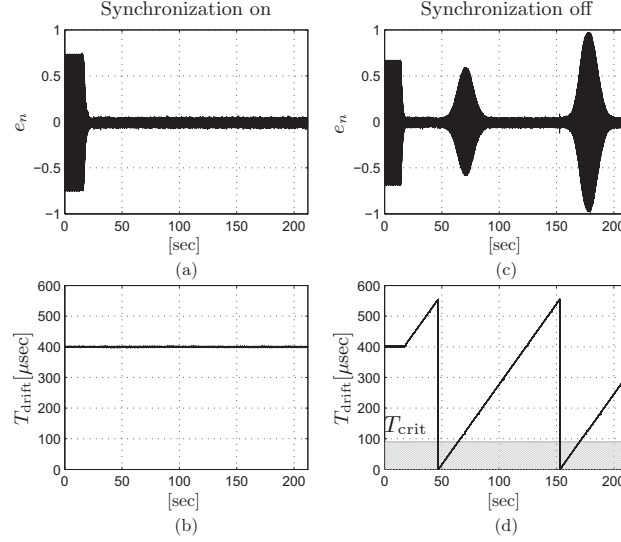


Fig. 5. Measurement results in an ANC system. (a) and (c): noise signal measured by the external microphone when synchronization is on and off, respectively. (b) and (d): delay (T_{drift}) between the notes when synchronization is on and off, respectively.

5 Data Loss

5.1 Problem Statement

Data loss cannot be avoided in real-time communication systems due to the time limit of data transmission and the inherently unreliable physical layer. Data loss can be especially dangerous in closed-loop systems, since if one or more samples of the feedback signals are lost, then the control loop becomes temporarily opened. The first straightforward question which emerges is whether the system remains stable depending on the data loss pattern [7].

In our model the stability of the closed-loop system can easily be ensured, since the plant itself is stable. However, the quality of the convergence is highly influenced by the data loss pattern. Our aim is to find definite conditions for the convergence of the adaptive algorithm. Convergence means that the state variables of the algorithm tend to their optimal values, where the optimal solution is the limit of the variables without data loss.

5.2 Proposed Test Methods

The main contribution of this paper relating to data loss is a set of conditions which enable us to predict whether the state variables of the resonator-based algorithms converge to their optimal values or not. The conditions can be evaluated using the pattern of data loss and the parameters of the algorithm. In order

to model the data loss, a so-called data availability indicator function, K_n , is introduced: $K_n = 1$ if the sample is processed, and $K_n = 0$ if the sample is lost at time index n .

Using this indicator function, the necessary condition of the convergence can be formulated as follows: *the state variables of the resonator-based adaptive controller don't converge to their optimal values if the set of vectors $\{c_n K_n\}$ don't span an N dimensional space.*

Indeed, the observability matrix of the system consisting of $\{c_n K_n\}$ vectors should have full rank. The detailed proof of this statement is found in [13]. The practically important case, when this condition is true, is when samples are missing every time from the same position(s) within the periods of the signal transmitted by the sensors. In other words, this is the case when the data loss pattern K_n is correlated with the signal.

Fortunately, one can find also such conditions under which the convergence can exactly be ensured. One of the practically most important conditions is that: *if the data loss ratio is less than a critical value, then the state variables of the resonator-based adaptive controller converge to their optimal values.*

The data loss ratio is precisely defined for different cases in [13], where the proof of the theorem is also given along with the critical upper bound on the data loss ratio which ensures the convergence. Loosely speaking, the data loss ratio is the ratio of the number of processed samples to the total number of samples.

Based on this basic sufficient condition, a set of different conditions can be derived for practical cases, e.g., random data loss (modeled by Bernoulli- or Markov-process) doesn't hinder the convergence.

As rule of thumb, the critical value of data loss ratio can be approximated by the reciprocal of the number of resonators [13]. For example, if the disturbing signal contains 5 harmonic components, i.e., there are $N = 2 \cdot 5 + 1 = 9$ resonators, and the data loss ratio is less than $1/9 = 11.1\%$, the convergence of the control algorithm can be strongly suspected even without exactly evaluating the conditions.

6 Conclusions

This paper presented three of the most important challenges which emerge during the design of distributed real-time signal processing systems with limited resources. These issues were presented through an application example which was an active noise control system.

In order to alleviate the limitations caused by finite bandwidth of communication channel, a simple data compression algorithm was presented, and some of its important properties were highlighted.

It was shown that the unsynchronized sensor nodes can cause even the instability of the whole system. A PLL-like synchronization algorithm was introduced which ensures the stability of the system with the synchronization of the sampling on the sensors.

The problem of data loss was handled by presenting a set of conditions which can be used to prove whether the state variables of the resonator-based adaptive controller converge to their optimal values. It means a potential danger for the proper convergence when the data loss pattern is repeated synchronously with the observed signal. However, if the data loss is a random process or the data loss probability is sufficiently low, the convergence can be ensured.

According to the authors' experience, the above three topics cover the signal processing problems arising in recent distributed systems. The research has to be continued in different directions. Signal processing researchers are motivated to adapt various algorithms for distributed systems. On the other hand, the development of distributed systems should be monitored, as by the spreading of new devices certain problems become marginal while others emerge.

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