A Common Structure for Recursive Discrete Transforms

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Abstract —**This paper presents a common framework for the recursive implementation of arbitrary discrete transformations. The transform coefficients to be applied are periodically time-varying and can be derived from the discrete basis functions of the transforms. The method is based on Hostetter's dead-beat observer approach to signal processing [1], [2], but instead of the ongoing calculation of the transform coefficients, explicit expressions are derived. The proposed structure can be efficiently used even for FIR and IIR filtering operations.**

I. Introduction

Recently, a new recursive method [1], [2] has been introduced especially suitable for running transformations and general multirate sampling situations. The algorithm is based on the state-variable formulation and the results of the observer theory. This approach is extremely attractive, but, except in some special cases [1], results in the ongoing calculation of the so-called observer gain. This paper derives explicit expressions for the observer gain; therefore, all the coefficients to be applied can be calculated in advance.

In Section II, the derivation of the main results is presented. The first one is the common observer structure suitable for arbitrary discrete transformation. The second one is the explicit expression of the observer gain, and finally the applicability of this structure to FIR and IIR filtering operations is introduced.

II. Derivation of the Common Structure

The key element of the observer-based approach to signal processing [1], [2] is a conceptual state-variable signal generating system model, where the state variables are the components to be calculated by the discrete transformation in hand. The state of a corresponding dead-beat observer will reach the required transform value in N steps, where N denotes the transform size. The block diagram of the conceptual model and the corresponding observer is given in Fig. 1. For every discrete transformation, to provide a dead-beat observer behavior, the $\{c_m(k)\}$, and the $\{g_m(k)\}$, $m=0, 1,\cdots, N-1$ values should be the kth components of the basis and reciprocal basis vectors of the transformation, respectively. To show this, let's denote

$$x(k)=[x_0(k),x_1(k),\cdots,x_{N-1}(k)]^T$$

$$\hat{x}(k)=[\hat{x}_0(k),\hat{x}_1(k),\cdots,\hat{x}_{N-1}(k)]^T$$

$$c(k)=[c_0(k),c_1(k),\cdots,c_{N-1}(k)]^T$$

$$g(k)=[g_0(k),g_1(k),\cdots,g_{N-1}(k)]^T$$

and, by expressing the error of the observer and the state of the

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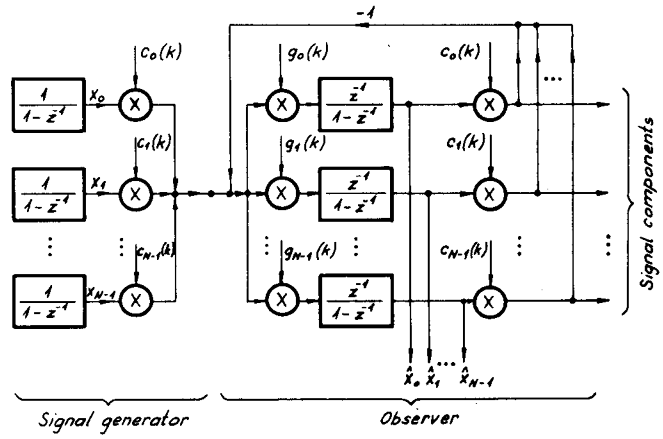

Fig. 1. The basic conceptual signal model and the corresponding observer.

observed system [2], we have

$$x(k+1)-\hat{x}(k+1)=(I-g(k)c^T(k))(x(k)-\hat{x}(k))$$
$$=\left[\prod_{i=0}^{k}(I-g(i)c^T(i))\right](x(0)-\hat{x}(0)). \quad (1)$$

The error reaches zero in N steps (or less) if

$$\prod_{i=0}^{N-1}(I-g(i)c^T(i))=0. \quad (2)$$

But this is true for an arbitrary basis/reciprocal basis system, since due to the orthogonality of the two bases (if $i\neq j$)

$$g(i)c^T(i)g(j)c^T(j)=0 \quad (3)$$

and

$$\sum_{i=0}^{N-1}g(i)c^T(i)=I \quad (4)$$

since every basis/reciprocal basis system can be expressed by the unit vector system $\{e(i)\}$, $i=0,1,\cdots,N-1$, using a nonsingular transformation F

$$c(i)=Fe(i) \quad g^T(i)=e^T(i)F^{-1} \quad (5)$$

and thus

$$\sum_{i=0}^{N-1}g(i)c^T(i)=\sum_{i=0}^{N-1}(F^{-1})^Te(i)e^T(i)F^T$$
$$=(F^{-1})^T\left(\sum_{i=0}^{N-1}e(i)e^T(i)\right)F^T=I. \quad (6)$$

After the first N samples, the calculation of the transformed values can be continued by periodically applying the two bases of the transformation.

Fig. 2 presents a less general, but rather useful conceptual model and the corresponding observer [4]. If $u(k)=[u_0(k), u_1(k),\cdots, u_{N-1}(k)]^T$, and $v(k)=[v_0(k), v_1(k),\cdots, v_{N-1}(k)]^T$ are the basis and the reciprocal basis, respectively,

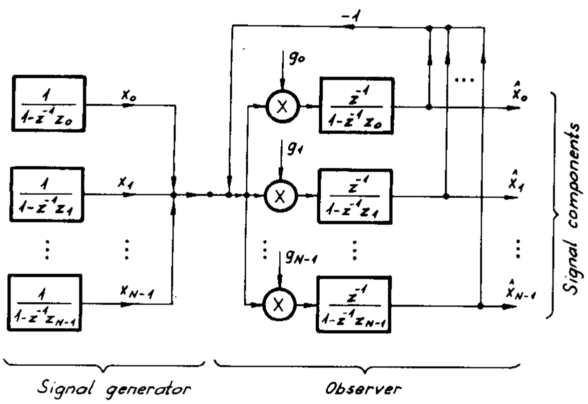

Fig. 2. Another conceptual signal model and the corresponding observer.

then in Fig. 2, assuming $u_m(k) \neq 0$

$$z_m(k) = \frac{u_m(k+1)}{u_m(k)}, \qquad m = 0,1,\cdots,N-1 \tag{7}$$

and

$$g_m(k) = v_m(k)u_m(k+1), \qquad m = 0,1,\cdots,N-1 \tag{8}$$

will result in a dead-beat observer. In this model, the time-varying first-order sections generate the basis of the transformation. If the condition $u_m(k) \neq 0$ cannot be fulfilled, higher order sections should be used. As an example, the Fourier transformation is obtained if in Fig. 1

$$\left.\begin{aligned} c_m(k) &= e^{j(2\pi/N)mk} \\ g_m(k) &= \frac{1}{N}e^{-j(2\pi/N)mk} \end{aligned}\right\} \qquad m = 0,1,\cdots,N-1 \tag{9}$$

while in Fig. 2

$$z_m = e^{j(2\pi/N)m} \quad g_m = \frac{1}{N}e^{j(2\pi/N)m} \qquad m = 0,1,\cdots,N-1 \tag{10}$$

As another example, the Walsh transformation is obtained if in Fig. 1

$$\left.\begin{aligned} c_m(k) &= wal(m,k) \\ g_m(k) &= \frac{1}{N}wal(m,k) \end{aligned}\right\} \qquad m = 0,1,\cdots,N-1 \tag{11}$$

while in Fig. 2

$$\left.\begin{aligned} z_m(k) &= \frac{wal(m,k+1)}{wal(m,k)} \\ g_m(k) &= \frac{1}{N}wal(m,k)\,wal(m,k+1) \end{aligned}\right\} \qquad m = 0,1,\cdots,N-1. \tag{12}$$

Both the recursive Fourier and Walsh transformations are of practical interest, and since (10) is independent of k, and multiplication by (11) can be reduced to additions and a single division by N, both can be implemented in a relatively simple way.

On the basis of the observers in Figs. 1 and 2, the realization of FIR and IIR filters is also possible. Since the observers are dead-beat, they have finite memory; thus, only the linear combination of the transformed output samples is to be generated additionally. This is an obvious generalization of the so-called "frequency sampling method" [3].

For an IIR filter, the observers cannot be dead-beat. If $\{p_n\}$, $n = 0,1,\cdots,N-1$, are the poles of the filter, we can apply, e.g., a Fig. 2-type time-invariant observer with

$$g_m = z_m \frac{\prod_{n=0}^{N-1}\left(1 - p_n z_m^{-1}\right)}{\prod_{\substack{n=0 \\ n \neq m}}^{N-1}\left(1 - z_n z_m^{-1}\right)} \tag{13}$$

(see [5]). IIR filter zeros are generated in the very same manner as for FIR filters. Expressions similar to (13) can be developed easily for every observer type of this paper.

It is very interesting to note that the introduced structures are closely related to the Lagrange (and Hermite) interpolation, and that this relation proves to be very helpful in the development of expressions like (13) [5].

III. Conclusions

In this paper, explicit expressions has been derived for the coefficients of observers which implement recursive discrete transformations. Since these observers, and even the related FIR and IIR filters, can be realized by the very same structure (Fig. 1), it can be considered a common base for every linear signal-processing operation.

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High-Speed Distributed-Arithmetic Realization of a Second-Order Normal-Form Digital Filter

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Abstract—**In a recent publication [1], an excellent tradeoff study was presented to show how one could design a normal-form second-order digital filter to meet prescribed performance criteria. In this note, an extremely efficient set of realizations is shown, one in which the speed and complexity can be effectively traded.**

A digital filter structure with optimum low roundoff noise, minimum coefficient inaccuracy errors, and absence of limit cycles is the normal form [1].

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