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Digital Filters Based on Recursive Walsh-Hadamard Transformation

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Abstract—In this paper the applicability of the recursive Walsh-Hadamard transformation [1] to FIR and IIR filtering is investigated. It is shown that using the recently introduced common structure for recursive transforms [2] the usual frequency-domain FIR filtering problem can be easily converted into a Walsh sequency-domain filtering problem. It is also shown that a simple modification of this structure results in a possible alternative for IIR filter implementations.

I. INTRODUCTION

It is well known from the literature that the Walsh-Hadamard transformation (WHT) [3], and especially its fast version, can be efficiently used for the calculation of the discrete Fourier transformation (DFT) [4], for implementing adaptive filters [5], and for DFT spectrum filter realizations [6]. In this paper, the recursive form of the WHT is investigated, and for its implementation the recently introduced common structure for recursive transformations [2] is suggested.

The derivation of this common structure is based on the state variable formulation and the results of the observer theory [1], while its applicability to FIR and IIR filtering operations comes from the generalization of the "frequency sampling method" [7]. The block diagram of the proposed structure is given in Fig. 1. For every discrete transformation to provide a dead-beat observer behavior, the $\{c_m(k)\}$ and $\{g_m(k)\}$, $m = 0, 1, \dots, N-1$, values should be the k th components of the basis and reciprocal basis vectors of the transformation, respectively [2].

In this paper we concentrate on the recursive Walsh-Hadamard transformation [1], which is obtained if in Fig. 1:

$$\left. \begin{aligned} c_m(k) &= \text{wal}(m, k) \\ g_m(k) &= \frac{1}{N} \text{wal}(m, k) \end{aligned} \right\}, \quad m = 0, 1, \dots, N-1 \quad (1)$$

and as usual, N is an integer power of 2. This transformation is of practical interest, since multiplication by (1) can be reduced to additions, and a single division by N , that is a simple shift operation.

In Section II the properties of the recursive WHT structure are examined. It is shown what is the relation between the recursive DFT and WHT, and what is the condition of "structural passivity" [8] for the recursive WHT. Section III presents a new method for implementing IIR filters. This method is based on a modified form of the recursive WHT structure. In Section IV some aspects of the hardware realization are discussed.

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II. PROPERTIES OF THE RECURSIVE WALSH-HADAMARD TRANSFORMER

Since the overall behavior of the transformer in Fig. 1 is time invariant, for the analysis we can replace the internally time-variant components, in every channel, by the transfer function $K_m(z)$, $m = 0, 1, \dots, N-1$, as it is shown in Fig. 2. $K_m(z)$ is a function of real coefficients, and can be decomposed as a linear combination of elementary transfer functions having one (complex) pole located on the unit circle [9]. These resonator-pole positions are the N th roots of the unity (z_n , $n = 0, 1, \dots, N-1$), and the "weight" of these elementary transfer functions comes from the Fourier series expansion of the Walsh-Hadamard functions applied. If the input signal of the transformer is band-limited in the usual sense, i.e., by one half of the sampling frequency, the effect of "modulating" by the discrete Walsh functions is equivalent to the effect of a limited number of discrete harmonical components. Thus

$$K_m(z) = \sum_{n=0}^{N-1} v_{mn} H_n(z) \quad (2)$$

where $\{v_{mn}\}$, $m, n = 0, 1, \dots, N-1$, are the appropriate weighting coefficients, and

$$H_n(z) = \frac{1}{N} \frac{z_n z^{-1}}{1 - z_n z^{-1}}, \quad n = 0, 1, \dots, N-1 \quad (3)$$

is the elementary transfer function. It is interesting to note that for the recursive DFT $K_m(z) = H_m(z)$, $m = 0, 1, \dots, N-1$ [2].

Concerning the stability of the recursive structure (see Fig. 2) the transfer function from the input to point P should be investigated. The stability of this feedback loop in the linear sense comes from the fact that all the poles of the transfer function

$$H_P(z) = \frac{\sum_{n=0}^{N-1} K_n(z)}{1 + \sum_{n=0}^{N-1} K_n(z)} \quad (4)$$

are at the origin of the z -plane unless coefficient quantization errors occur. Note, however, that for the recursive Walsh-Hadamard transformer the only source of coefficient quantization error is the division by N , and if we apply "magnitude-truncation" strategy, it is easy to show, that both the frequency domain criteria given in [10], and the structural boundedness property detailed in [8] can be fulfilled. This latter property is simply due to the fact that $H_P(z)$ in (4) implements an all-pass transfer function, the magnitude of which can not exceed the unity if "magnitude-truncation" is applied. Thus in the Walsh-Hadamard transformer implemented as in Fig. 1, the zero-input limit cycles can be suppressed (see also [11]).

The implementation of FIR filters using this transformer is rather straightforward, since only the appropriate linear combination of the transformer outputs should be generated. These

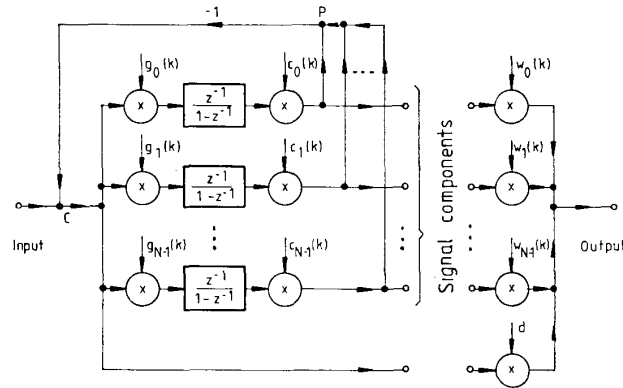


Fig. 1. The suggested common structure.

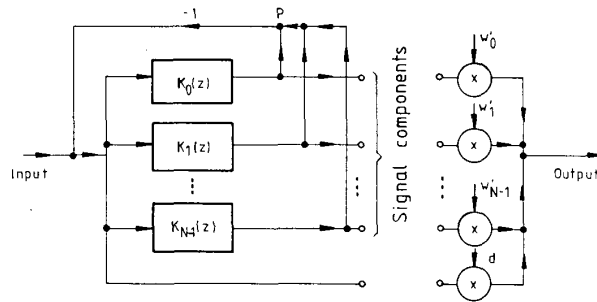


Fig. 2. "Time-invariant" Walsh-Hadamard transformer.

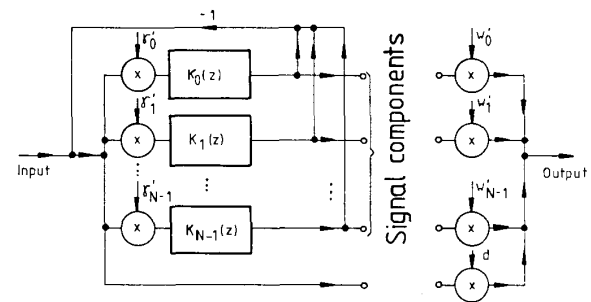


Fig. 3. Modified Walsh-Hadamard transformer.

"taps" do not influence the stability of the transformer structure. Thus the transfer function for an FIR filter has the form of

$$H(z) = \frac{d' + \sum_{n=0}^{N-1} w'_n K_n(z)}{1 + \sum_{n=0}^{N-1} K_n(z)} \quad (5)$$

where the d' and w'_n , $n=0,1,\dots,N-1$, are the weighting coefficients. The calculation of these coefficients is relatively simple. By introducing $\{v_{mn}\} = V$, $\mathbf{w} = [w_0, w_1, \dots, w_{N-1}]^T$, $\mathbf{w}' = [w'_0, w'_1, \dots, w'_{N-1}]^T$, where

$$w'_m = H(z_m), \quad m=0,1,\dots,N-1 \quad (6)$$

as for the "frequency sampling method" [7], we have

$$\mathbf{w}' = V^{-T} \mathbf{w} \quad (7)$$

(here V^{-T} stands for "transpose of the inverse of V "), and finally,

$$d' = H(0). \quad (8)$$

The calculation of V is straightforward

$$V = \{v_{mn}\} = \mathbf{W}\mathbf{F}^{-1} \quad (9)$$

where \mathbf{W} is the Walsh-Hadamard, while \mathbf{F} is the discrete Fourier transformation matrix.

III. THE IMPLEMENTATION OF IIR FILTERS

If we introduce additional weighting coefficients into the transformer structure like in Fig. 3, the implementation of IIR filters is also possible. For the recursive Fourier transformation these coefficients are given by

$$\gamma_m = \prod_{n=0}^{N-1} (1 - p_n z_n^{-1}), \quad m=0,1,\dots,N-1 \quad (10)$$

where $\{p_n\}$, $n=0,1,\dots,N-1$, are the poles of the filter (see [2]). The number γ_m is usually complex, but if the poles are real numbers, or occur in complex-conjugate pairs, the complex conjugate of γ_m will also appear.

Let us denote $\boldsymbol{\gamma} = [\gamma_0, \gamma_1, \dots, \gamma_{N-1}]^T$, $\boldsymbol{\gamma}' = [\gamma'_0, \gamma'_1, \dots, \gamma'_{N-1}]^T$, and apply the structure of Fig. 3, then we have for the IIR filter

$$\boldsymbol{\gamma}' = V^{-T} \boldsymbol{\gamma}. \quad (11)$$

The tap coefficients can be derived from (7) if w_m and w'_m are replaced by $w_m \gamma_m$ and $w'_m \gamma'_m$, respectively, ($m=0,1,\dots,N-1$). It is interesting to note that if the transfer function to be implemented is of real coefficients, then both the w'_m and γ'_m values ($m=0,1,\dots,N-1$) will be real numbers.

It is worth mentioning, however, that the introduction of the γ'_m parameters destroys the orthogonality of the transformer structure, and thus the general criteria of nonlinear stability and low sensitivity [8] are not fulfilled. Still for its computational simplicity in certain cases even this Walsh-Hadamard transformer can be suitable for IIR filter realizations.

IV. IMPLEMENTATIONAL ASPECTS

The hardware implementation of the recursive Walsh-Hadamard transformer is relatively simple, since it requires only additions and a single shift operation to perform division by N . The Walsh function samples can be stored in a memory, or can be generated by a simple digital circuit. To implement FIR and IIR filters the multiplications by d' , w'_m , and γ'_m , $m = 0, 1, \dots, N-1$, obviously can not be avoided, however, this structure is still canonic with respect to the number of (nontrivial) multiplications.

The recursive Walsh-Hadamard transformer has been implemented by the authors on a microprogrammable signal processor based on bit-sliced elements [12]. It was found that for this hardware, and for every input sample, the number of microprogram steps to perform Walsh-Hadamard transformation is

$$S(N) = 6N + \log_2 N + 13, \quad (13)$$

where N equals the transform size. The number $S(N)$ includes the steps required for the Walsh function generation as well.

V. CONCLUSIONS

In this paper the applicability of the recursive Walsh-Hadamard transformer has been investigated. It turned out that using the common structure proposed, both FIR and IIR filters can be efficiently implemented, and that for the transformer itself, and for FIR filters, the structural boundedness of the common feedback loop can be easily guaranteed. This property implies the ability of suppressing zero-input limit cycles. The generalization of these results for arbitrary discrete transformation is straightforward.

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The Infeasibility of a Zero-Current Switching Class E Amplifier

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Abstract—It has been shown that a class E amplifier may be configured to provide nearly 100-percent efficiency and zero-current or zero-voltage switching. A particular configuration, which supplies zero-current-switching when the transistor output capacitance is zero, is considered for the case when the transistor output capacitance is greater than zero. Equations for the switch voltage and current, the dc input voltage and current, the output voltage and current and powers are determined for the amplifier. The equations show that with this configuration, a zero-current-switching class E amplifier can not be physically realized.

I. INTRODUCTION

Class E switching-mode amplifiers provide efficiencies approaching 100 percent by reducing the power dissipated in the active device switch during one of the switch transitions. Conceptually this may be accomplished by reducing the switch voltage or current to zero during the off-to-on or on-to-off transition, respectively.

For a zero-current-switching class E amplifier with minimum power dissipation the three following conditions are needed:

- zero switch current at the on-to-off transition;
- zero slope for the current at the on-to-off transition;
- zero switch current at the off-to-on transition.

Class E amplifiers can be configured to provide zero-current-switching when the transistor has zero output capacitance [1]-[4]. At high frequencies the internal capacitance of the transistor is no longer negligible. It is the purpose of this letter to show the infeasibility of a zero-current-switching amplifier with a nonzero transistor output capacitance.

II. CIRCUIT DESCRIPTION

The basic class E amplifier (Fig. 1) consists of an active device operated as a switch at the input frequency ω and a load network which includes a capacitor shunting the active device and a series *RLC* output circuit. Circuit operation is determined by the switch when on and by the transient response of the load network when the switch is off. The analytical description of the circuit begins with the following assumptions.

- 1) The load current is sinusoidal at the output frequency. This is approached with a large Q of the output circuit.
- 2) The active device acts as an ideal switch: thus any nonzero shunt capacitor voltage present at switch turn-on is discharged by a current impulse. The switch is bidirectional, i.e., it allows positive and negative current when on and withstands positive and negative voltage when off.
- 3) The shunt capacitance includes device capacitance and is independent of device voltage.
- 4) All equivalent circuit elements are ideal.

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