STRUCTURALLY PASSIVE RESONATOR-BASED DIGITAL FILTERS

Gábor Péceli

Department of Measurement and Instrument Engineering
Technical University of Budapest, H-1521 Budapest, Hungary

ABSTRACT

In this paper a new implementation of FIR and IIR filter transfer functions is presented that is structurally passive and, hence, has very low pass-band sensitivity. This implementation is based on a recently introduced common structure for recursive discrete transforms [1]. This structure consists of digital resonators in a feedback loop. It is shown that by locating the resonator poles properly, the passivity of the feedback loop can be easily guaranteed.

INTRODUCTION

Recently a common structure for recursive discrete transforms has been suggested [1], which seems to be suitable to form a (possibly VLSI implemented) common base for every linear filtering-like signal processing operation. The derivation of this structure and its parameters is based on the state-variable formulation, and the results of the observer theory [2], while its applicability to FIR and IIR filtering operations comes from the generalization of the "frequency-sampling" method [3]. The block diagram of the structure is given in Fig.1. This structure is based on digital resonators embedded into a feedback loop. Due to this feedback, the properties of this structure substantially differ from that of the well-known Lagrange or "frequency-sampling" structures [3,4].

In the next section, the possible resonator positioning startegies are investigated, and a simple condition is derived, which guarantees the passivity of the feedback loop. It is also shown that by applying appropriate quantization strategy, even this structure is able to suppress zero-input limit cycles.

In the last section some aspects of digital filter implementations based on this new structure are considered.

DERIVATION OF THE STRUCTURALLY PASSIVE FORM

In this paper we will concentrate on the time-invariant version of the common transformer structure (see Fig.1), and for simplicity we do not consider the case of multiple resonator poles, thus we have in every channel of the structure as an internal transfer function

$$H_m(z)=\frac{g_m z^{-1}}{1-z_m z^{-1}},\quad m=0,1,\dots,N-1, \qquad (1)$$

which is usually a function of complex coefficients having a pole on the unit circle. The global transfer function for every channel has the form of

$$T_m(z)=\frac{H_m(z)}{1+\sum_{n=0}^{N-1} H_n(z)},\quad m=0,1,\dots,N-1, \qquad (2)$$

while the overall transfer function of an FIR or IIR filter based on this structure can be written in the following form

$$H(z)=d+\sum_{m=0}^{N-1}(w_m-d)T_m(z)=\frac{d+\sum_{m=0}^{N-1} w_m H_m(z)}{1+\sum_{n=0}^{N-1} H_n(z)}. \qquad (3)$$

In this paper we will suppose that the poles and zeros of H(z) are real numbers or occur in complex conjugate pairs, and as usual, even d is a real number.

It is a very interesting property of this structure that at the resonator pole positions

$$H(z_m)=w_m,\quad m=0,1,\dots,N-1, \qquad (4)$$

from which we can deduce that the sensitivity of the transfer function at

CH2458-8/88/0000-1421$1.00

$z=z_m$ is zero with respect to any $H_n(z)$, w_n and z_n $(n=0,1,\dots,N-1)$ except the $n=m$ case. This property is due to the infinite loop gain at these frequencies providing complete independency of the coefficients within the feedback loop. The detailed coefficient sensitivity formulas are presented elsewhere [5].

It is very important to note that this common feedback implements a perfect pole-zero cancellation mechanism. The poles in (1), which are not necessarily located on the unit circle in a general case, will be transformed by the feedback loop into zeros, which automatically, and perfectly cancel their "generators" in expression (3). This is the reason why the application of ideal resonators does not cause implementational problems, since every resonator pole is cancelled by a zero generated by the overall common feedback from the very same resonator pole, and this is true even if implementational errors occur.

The design of the filter parameters for this structure is rather straightforward. If the resonator positions are known, the weighting coefficients are given by (4), while if (p_n), $n=0,1,\dots,N-1$, are the poles of the filter

$$g_m = z_m \frac{\prod_{n=0}^{N-1} (1 - p_n z_m^{-1})}{\prod_{n=0, n\neq m}^{N-1} (1 - z_n z_m^{-1})} \tag{5}$$

see [4]. The resonator poles, at the price of some redundancy, can be located arbitrarily. Excellent pass-band behavior can be achieved if the resonator poles are distributed in the pass-band. For elliptic filters, to provide good stop-band behavior, and spare weighting coefficients, the resonator poles can be located to the transmission zero positions of the filter transfer function. A very useful pole positioning strategy can be derived, if we investigate the transfer function from the input to point P (see Fig.1)

$$H_p(z) = \frac{\sum_{n=0}^{N-1} H_n(z)}{1 + \sum_{n=0}^{N-1} H_n(z)} \tag{6}$$

It is easy to show that the magnitude of this transfer function is less or equal to the unity (i.e. it is passive in this sense), if

$$\mathrm{Re} \sum_{n=0}^{N-1} H_n(z) \geq -\frac{1}{2} . \tag{7}$$

Condition (7) can be fulfilled at every frequency, if

$$g_m = z_m r_m , \quad m=0,1,\dots,N-1, \tag{8}$$

where r_m is a real constant, and

$$\sum_{m=0}^{N-1} \mathrm{Re}\left[\frac{g_m}{z_m} \right] = \sum_{m=0}^{N-1} r_m \leq 1 . \tag{9}$$

In Fig.2 we present two simple second-order sections, which "structurally" force the fulfilment of condition (8). It is important to note that these second-order sections implement

$$G_m(z) = w_m H_m(z) + w_m^* H_m^*(z) , \tag{10}$$

where asterisk denotes a complex conjugate.

If in (9) equality holds, $H_p(z)$ implements an all-pass filter, and if so, we know even its zeros, since they are in mirror image relationship with the poles of $H(z)$. This property is the key to the determination of those resonator pole positions which will provide the above properties. These positions coincide with the zeros of $1-H_p(z)$, since the input of the resonators (see point C in Fig.1) is the difference of the filter input and the output at point P. We will have two sets of resonator poles, since the filter poles do not specify the sign of $H_p(z)$.

If the number of resonator poles equals the number of the filter poles, $H_p(z)$ can not be an all-pass transfer function, because it is forced to have at least one zero at the origin, otherwise the loop would be delay-free. The resonator pole positions, however, can be determined rather similarly by finding the zeros of $1-H_A(z)$, where $H_A(z)$ is an all-pass function having the same poles as the filter has. These resonator poles meet condition (8), and one of the two resonator pole sets will insure also the fulfilment of condition (9).

If conditions (8) and (9) are fulfilled, the transfer function $H_p(z)$ is a bounded real (BR), or if it is all-pass, lossless bounded real (LBR) transfer function [6,7]. As it is shown in the literature (e.g. [6,7,8]), the BR (or LBR) property promises low sensitivity in the pass-band, and if appropriate quantization strategy is applied, even the zero-input limit cycles can be avoided. If the

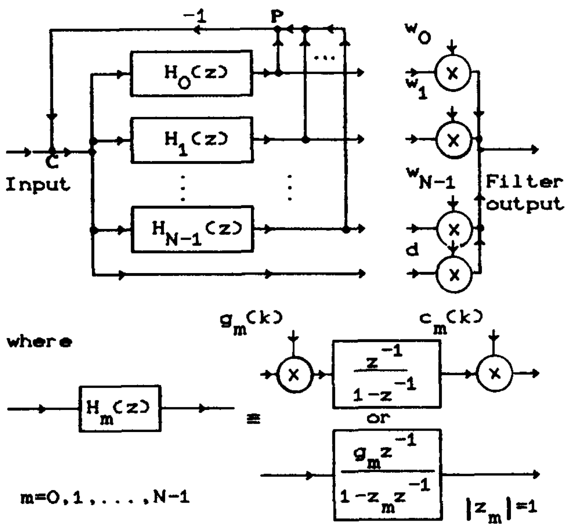

Figure 1. The suggested common structure

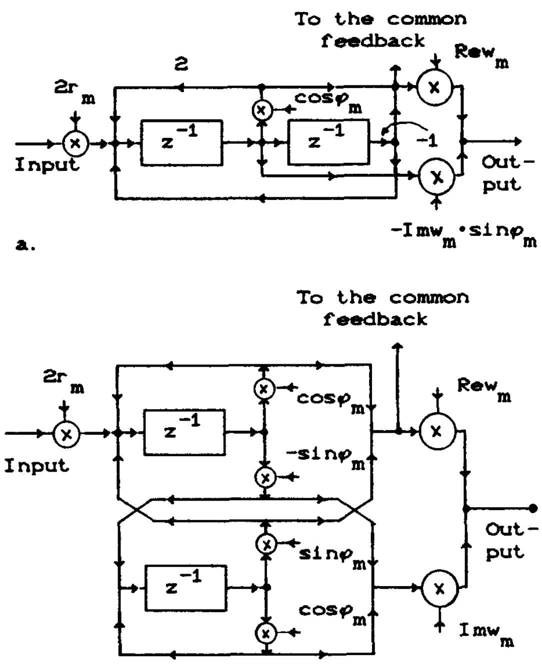

Figure 2. Second-order resonator sructures $(z_m = e^{j\varphi_m})$

coeficients (r_m), $m=0,1,\ldots,N-1$, are positive, and fulfil condition (9), and the magnitude of all the other (nontrivial) coefficients is less or equal to the unity, both stability (in the linear sense), and the boundedness property are guaranteed. The zero-input limit cycles will be suppressed, if the (unavoidable) quantizers are placed to the "entrance" of the delay elements, and "magnitude truncation" strategy is applied (see Fig.3).

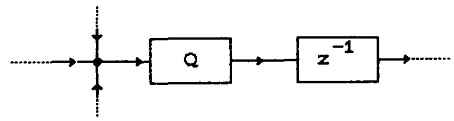

Figure 3. The proper position of the quantizer

As it is clear from (3), to implement an arbitrary FIR or IIR filter, only appropriate taps should be added to the structure described by $H_p(z)$ (see Fig.1). Since these taps do not influence the behavior of the feedback loop, the nice properties mentioned above, will not be destroyed.

For an FIR filter the poles of $H_p(z)$ are placed to the origin. If we force the fulfilment of equality in condition (9), we will get the recursive Fourier transformation structure presented in [1], that gives back the "frequency-sampling" method for FIR filter design.

Concentrating on the "time-invariant" version of the common structure for recursive discrete transforms (see Fig.1), which can be considered to be a one input, N output digital filter, it is easy to show, that the condition of the structural boudedness property is the fulfilment of (8), and

$$\sum_{m=0}^{N-1} r_m = N \sum_{m=0}^{N-1} r_m^2 = 1 \,, \qquad (11)$$

which means, that $r_m = \frac{1}{N}$, $m=0,1,\ldots,N-1$, and thus the discrete Fourier transformation is the only transformation that can be implemented recursively in a structurally bounded manner using the "time-invariant" version of the proposed structure.

UTILIZATION OF THE NEW STRUCTURE

The proposed structure can be applied in a wide variety of forms. One form is the direct utilization of the common structure (see Fig.1) having as internal building blocks the second-order sections suggested in Fig.2.

An alternative utilization can be the implementation of those filters, which are based on parallel connection of all-pass filters [8,9].

Obviously simple first- and second-order building blocks can also be implemented, and if the structure suggested in Fig.2.a is applied, the second-order section obtained will be canonic concerning the nontrivial multiplications. The structure suggested in Fig.2.b is essentially the well-known coupled-form [10], that pormises lower roundoff noise at the price of additional multiplications.

CONCLUSIONS

In this paper a simple condition of structural passivity has been derived for digital filters consisting of digital resonators in a common feedback loop. Due to this structural passivity the structure obtained has very low pass-band sensitivity, and, by applying appropriate quantization strategy, suppresses zero-input limit cycles.

REFERENCES

[1] G.Peceli,"A Common Structure for Recursive Discrete Transforms", IEEE Transactions on Circuits and Systems, Vol.CAS-33, No.10, pp.1035-1036, Oct. .1986.

[2] G.H.Hostetter, "Recursive Discrete Fourier Transformation", IEEE Transactions on Acoustics, Speech and Signal Processing, Vol.ASSP-28, No.2 pp. 183-190, Apr.1980.

[3] L.R.Rabiner and B.Gold, Theory and Application of Digital Signal Processing, Englewood Cliffs, NJ: Prentice-Hall, 1975, p.47.

[4] G.Peceli, "Investigation of recursive signal processing algorithms", Engineering Sciences Thesis,Budapest, 1985.(In Hungarian).

[5] G.Peceli, "Sensitivity Properties of Resonator-Based Digital Filters", submitted for publication.

[6] P.P.Vaidyanathan,S.K.Mitra,"Passivity Properties of Low-Sensitivity Digital Filter Structures", IEEE Transactions on Circuits and Systems, Vol.CAS-32. No.3, pp.217-224, March 1985.

[7] P.P.Vaidyanathan, "The Discrete-Time Bounded-Real Lemma in Digital Filtering", IEEE Transactions on Circuits & Systems, Vol.CAS-32,No.9, pp.918-924, Sept. 1985.

[8] P.P.Vaidyanathan, "A New Approach to the Realization of Low-Sensitivity Digital Filters" IEEE Transactions on Acoustics, Speech, and Signal Processing,Vol.ASSP-34,No.2, pp.350-361, April 1986.

[9] L.Gazsi, "Explicit Formulas for Lattice Wave Digital Filters", IEEE Transactions on Circuits and Systems, Vol.CAS-32, No.1, pp.68-88, Jan.1985.

[10] J.Szczupak,S.K.Mitra, "On Digital Filter Structures with Low Coefficient Sensitivities", Proc. of the IEEE, Vol.66, No.9, pp. 1082-1083, Sept. 1978.