Iterative Fuzzy Model Inversion

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Abstract

Nowadays model based techniques play very important role in solving measurement and control problems. Recently for representing nonlinear systems fuzzy models became very popular. For evaluating measurement data and for controller design also the inverse models are of considerable interest. In this paper a technique to perform fuzzy model inversion is introduced. The method is based on solving a nonlinear equation derived from the multiple-input single-output (MISO) forward fuzzy model simple by interchanging the role of the output and one of the inputs. The utilization of the inverse model can be either a direct compensation of some measurement nonlinearities or a controller mechanism for nonlinear plants. For discrete-time inputs the proposed technique provides good performance if the iterative inversion is fast enough compared to system variations, i.e. the iteration is convergent within the sampling period applied. The proposed method can be considered also as a simple nonlinear state observer, which reconstructs the selected input of the forward fuzzy model from its output using an appropriate strategy and a copy of the fuzzy model itself. It is also shown that using this observer concept completely inverted models can also be derived.

1. Introduction

Model based schemes play an important role among the measurement and control strategies applied to dynamic plants. The basically linear approaches to fault diagnosis [1], optimal state estimation [2] and controller design [3] are well understood and successfully combined with adaptive techniques (see. e.g. [4]) to provide optimum performance. Nonlinear techniques, however, are far from this maturity or still are not well understood. There is a wide variety of possible models to be applied based on both classical methods [5] and recent advances in handling [6] information, but up till now practically no systematic method was available which could be offered to solve a larger family of nonlinear control problems. The efforts on the field of fuzzy modeling and control, however, seem to result in a real breakthrough also in this respect. With the advent of adaptive fuzzy controllers very many control problems could be efficiently solved and the model based approach to fuzzy controller design became a reality [7]. Using model based techniques in measurement and control also the inverse models play a definite role [4].

Recently different techniques have been published ([7]-[10]) for inverting certain fuzzy models, however, exact inverse models can be derived only with direct limitations on the fuzzy models applied. In this paper an alternative approach is investigated which is based on the quite general concept of state observation widely used in measurement and filtering applications. The key element of this concept is to force a model of a physical system to "copy" the behavior of the system to be observed (see Fig. 1.). This scheme is the so called observer structure which is a common structural representation for the majority of iterative data and/or signal processing algorithms. From Fig. 1. it is obvious that here the inversion of dynamic system models is considered. Traditionally the observer is a device to measure the states of dynamic systems having state variable representation. These states, however, can be regarded as unknown inputs, and therefore their "copy" within the observer as the result of model inversion. Here the fuzzy models appear as static nonlinear Multiple Input Single Output (MISO) mappings from the state variables to the system output, i.e. represent the output equation. A copy of this output equation is also present within the observer, and in this case observer dynamics is simply due to the iterative nature of the algorithm. At this point it is important to note that the inversion is not unique if more than one input is considered, i.e. from one observer input value more than one output is to be calculated. In this paper only unique inversions are investigated, therefore the calculation of one controllable input is regarded based on the desired output and uncontrollable input values.

The paper is organized as follows. The possible role of inverse fuzzy models and the main features of the explicit

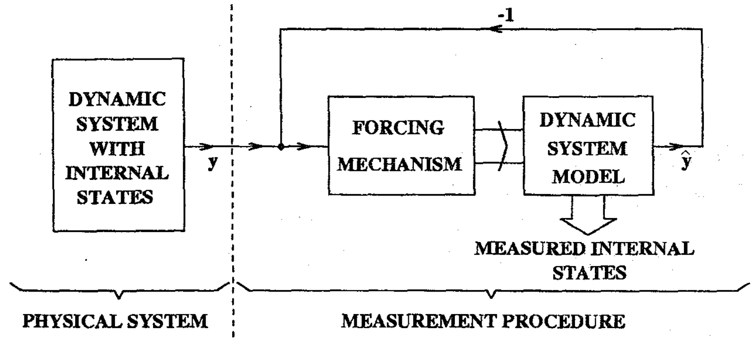

Figure 1. The observer concept

inversion methods are described in Section 2. Section 3 presents the observer based iterative inversion technique. A simple example illustrating the proposed method is given in Section 4, while Section 5 provides the conclusions.

2. Inverse fuzzy models in measurement and control

Nowadays solving measurement and control problems involves model-integrated computing. This integration means that the available knowledge finds a proper form of representation and becomes an active component of the computer program to be executed during the operation of the measuring and control devices. Since fuzzy models represent a very challenging alternative to transform typically linguistic a priori knowledge into computing facilities therefore it worth reconsidering all the already available model based techniques whether a fuzzy model can contribute to better performance or not.

The role of the inverse models in measurements is obvious: observations are mappings from the measured quantity. This mapping is performed by a measuring channel the inverse model of which is inherent in the data/signal processing phase of the measurement. In control applications inverse plant models are to be applied as controllers in feedforward (open-loop) systems, as well as in various alternative control schemes. Additionally there are very successful control structures incorporating both forward and inverse plant models (see e.g. [4]). In measurement and control applications the forward and inverse models based on fuzzy techniques are typically MISO systems (see Fig. 2.) representing static nonlinear mappings. Typical Single Input Single Output (SISO) dynamic system models are composed of two delay lines and a MISO fuzzy model as in Fig. 3. The first delay line is a memory for a limited number of last input samples while the second one contains the last

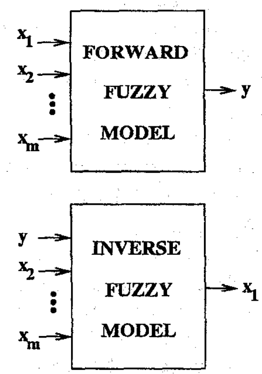

Figure 2. Forward and inverse model

segment of the outputs. In the scheme of Fig. 3. of last input $\hat{y}(n+1)$ is an estimated system output for time instant *n+1* calculated from input and estimated output samples available at time instant *n*. It is important to note that there is an inherent delay in such and similar systems since the model evaluations take time. The (partial) inverse of such a dynamic system is also a SISO scheme (see Fig. 4.) which is in correspondence with Fig. 3. except the role of the input and output is transposed. The input *r(n)* is a sample of the reference signal and a predicted value of the input will be the model output. It is obvious from these figures that if the forward fuzzy model is available only the inverse (nonlinear static) mapping must be derived. There are different alternatives to perform such a derivation. One alternative is to invert the fuzzy model using the classical regression technique based on input-output data. To solve this regression problem iterative algorithms can also be considered. The result of such a procedure is an approximation of the inverse, and the accuracy of this approximation depends on the efficiency of the model fitting applied. In this case,

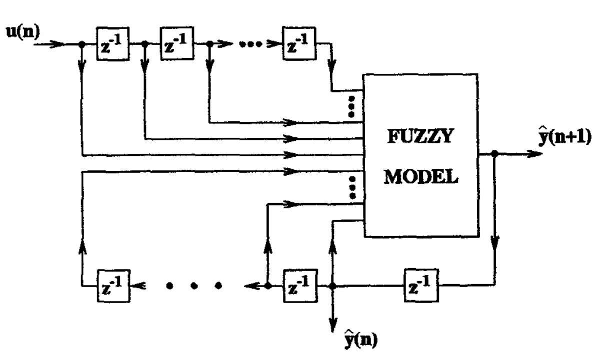

Figure 3. Dynamic system model

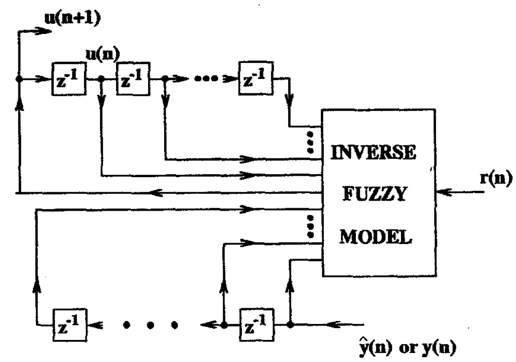

Figure 4. Inverse model

however, the inverse is not necessarily a fuzzy model in its original sense.

Recently very interesting methods have been reported for exactly inverting certain type of fuzzy models [7]. For the case of the standard Mamdani fuzzy model with singletons in the rule consequents exact inverse can be derived. Obviously the general conditions of invertibility must be met: the forward fuzzy model should be strictly monotone with respect to the input which is considered to be replaced by the output. If the forward model implements a noninvertible function it must be decomposed into invertible parts and should be inverted separately. There are, however, several other prerequisites of the exact inversion concerning both antecedent and consequent sets, rule base, implication and T-norm, as well as the defuzzification method. The construction of the inverse proved to be relatively simple at the price of strong limitations. There are other promising approaches ([8], [9]) where the inversion is solved on linguistic level, i.e. rule base inversion is performed. These techniques can accommodate various fuzzy concept but must be combined with fuzzy rule base reduction algorithms if the number of the input sets is relatively high.

3. Observer based iterative inversion

The general concept of the observer is represented by Fig. 1. The physical system produces output y and we suppose that its behavior can be described by a dynamic system model e.g. like the structure given on Fig. 3. This system description becomes the inherent part of the measurement procedure and is forced to behave similarly to the physical system. A more detailed description of this idea for static nonlinear fuzzy model is given by Fig. 5. Here a three input one output fuzzy model is considered. Input x_1 is considered unknown and therefore to be observed via comparing the output of the physical system and that of the model. If the correction (forcing) mechanism is appropriate the observer will converge to the required state and produce the estimate of the unknown input. The strength of this approach is that this iterative evaluation is easy to implement e.g. using standard digital signal processors and the complete system of Fig. 5. can be embedded into a real-time environment, since the necessary number of iterations to get the inverse can be performed within one sampling time slot of the measurement or control application. For the correction several techniques can be proposed based on the vast literature of numerical methods (see e.g. [11]) since the proposed iterative solution is nothing else than the numerical solution of a single variable nonlinear equation. The iteration is based on the following general formula

$$\hat{x}(n+1) = \hat{x}(n) + correction[y - \hat{y}, \hat{x}(n), f(), \mu] \qquad (1)$$

where $f(\ )$. stands for nonlinear function to be inverted and μ for the step size. In the case of Newton iteration (1) has the form of

$$\hat{x}(n+1) = \hat{x}(n) + \mu \frac{(y - \hat{y})}{f'(\hat{x}(n))} \qquad (2)$$

where $f'()$ denotes the derivative of the $f(\ )$. This latter must be evaluated locally using simple numerical technique:

$$f'(\hat{x}(n)) \cong \frac{f(\hat{x}(n)+\Delta) - f(\hat{x}(n)-\Delta)}{2\Delta} \qquad (3)$$

The computational complexity of this iterative procedure depends mainly on the complexity of the forward fuzzy model itself. It is important to note, however, that after the first convergence if the input of this observer changes relatively smoothly then relatively few iterations will be required to achieve an acceptable inverted value.

In principle the proposed method can be generalized even for two or more (forward model) inputs (see Fig. 5.).

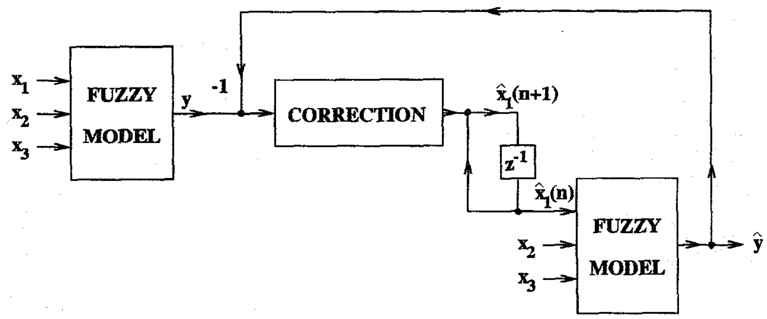

Figure 5. Block diagram of the iterative inversion scheme for the three input one output example

For this generalization, however, it must make clear that to calculate two or more inputs at least the same number of ("measured") outputs are required. With static nonlinear MISO models this is not possible. If the fuzzy model on Fig. 5. is replaced by a dynamic, state variable model, where the "inputs" will correspond to the state variables, following the state transitions new outputs can be "measured" and as many values can be collected as required to the iterative solution of the multi input problem. This idea obviously requires the "observability" [2] of the states and the application of state variable models instead of the schemes of Fig. 3. and Fig. 4. The complexity of the iterative technique to be applied in such cases is under investigation.

4. An illustrative example

In this Section, to illustrate the proposed iterative inversion method, a simple example with three input one output forward fuzzy model is presented. The purpose of this model is to describe the frequency, sound intensity and age dependency of the human hearing system [12]. The output of the model is the sound intensity felt by the person, i.e. the subjective intensity. The inversion of this fuzzy model might be interesting if the input sound intensity is to be estimated from known subjective intensity, frequency and age information.

The inputs are represented by Gaussian shape membership functions: five-five sets for the frequency and the input intensity, respectively (see Fig. 6.), and another three to represent the age (see Fig. 7.). The characterization of the output is solved by five triangular shape sets (see Fig. 8.). The rule base of the model consists of 61 rules which is a linguistic equivalent of the widely known, measurement-based plots describing the human hearing system.

Fig. 9 shows the obtained surface as a function of frequency and sound intensity at age of 60 years. The inverse was calculated by the proposed iterative method with $\mu=1$ and termination at $|y-\hat{y}| < 10^{-6}$ or 200 iterations. For the purpose of comparison the inverse is reinverted. The error surface is given in Fig. 10. The number of iterations can be kept at a relatively low level as it is illustrated by Fig. 11.

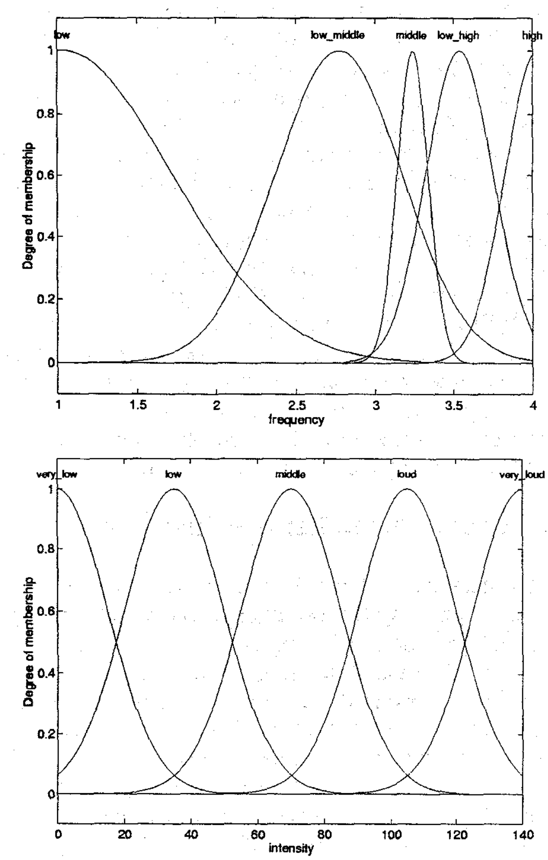

Figure 6. Input fuzzy sets for the frequency and input intensity

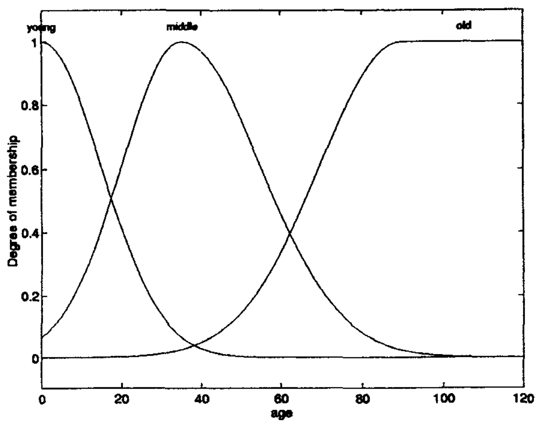

Figure 7. Input fuzzy sets for the age

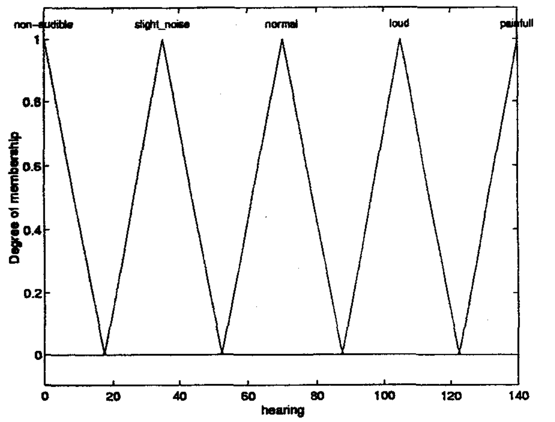

Figure 8. Output fuzzy sets

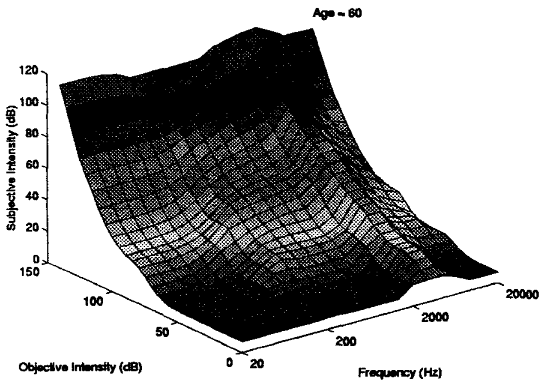

Figure 9. The obtained surface as a function of frequency and sound intensity at age of 60 years

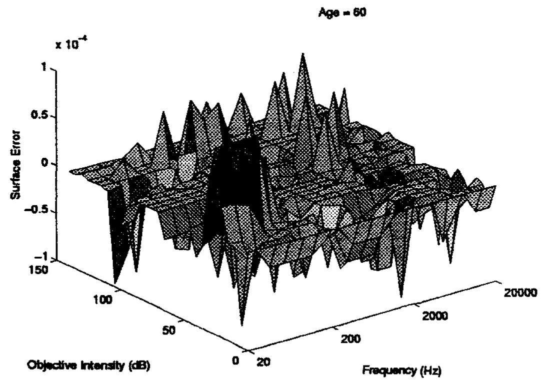

Figure 10. The error surface

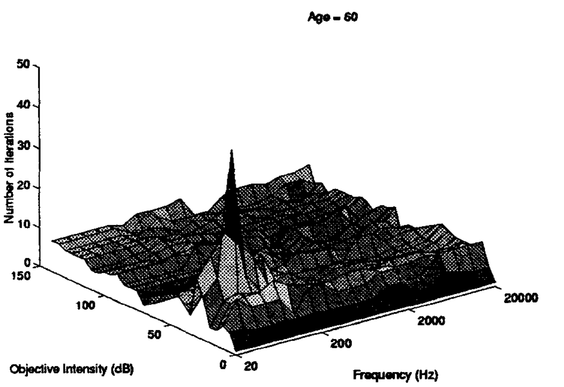

Figure 11. The number of iterations

5. Conclusions

In this paper an "on-line" iterative technique has been proposed to solve the inversion of fuzzy models for measurement and control applications. The derivation of this iterative technique is related to the state observer concept which proved to be very successful in the interpretation of the different techniques applied on this field. Additionally a step toward completely inverted models can also initiated by introducing state variable dynamic models combined with fuzzy logic based components.

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