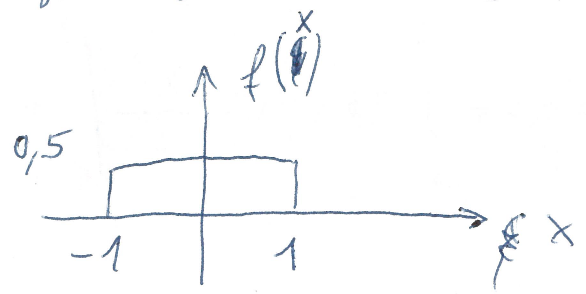
# $\boxed{\text{3. gyakorlat}}$

3.1. $\xi$ egyenletes eloszlású $[-1, 1]$-en




$\mu = ?$

$$\cancel{\mu = \mathbb{E}\{\xi\}} \quad \mu = \mathbb{E}\{x\} = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_{-1}^{1} \tfrac{1}{2} x\,dx = \left[\frac{x^2}{4}\right]_{-1}^{1} = 0$$

$$\sigma^2 \underset{\text{Steiner-tétel}}{=} \mathbb{E}\{x^2\} - \mathbb{E}^2\{x\} \underset{\text{most}}{=} \mathbb{E}\{x^2\}$$

$$\sigma^2 = \mathbb{E}\{x^2\} = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_{-1}^{1} \tfrac{1}{2} x^2\,dx = \left[\frac{x^3}{6}\right]_{-1}^{+1} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

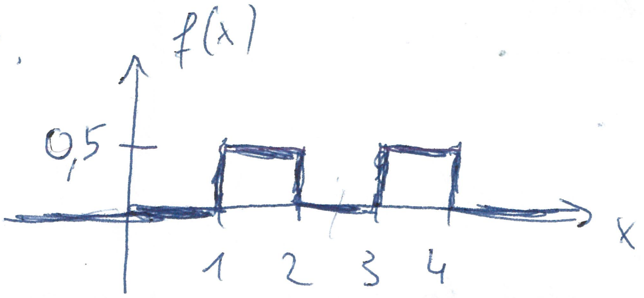
$$\sigma = \sqrt{\frac{1}{3}} = 0{,}5774$$

Általánosan: $f(x) = \begin{cases} \dfrac{1}{b-a} & \text{ha } a < x \leq b \\ 0 & \text{egyébként} \end{cases}$

$\Downarrow$

$$\boxed{\mathbb{E}\{x\} = \frac{a+b}{2}} \qquad \boxed{\sigma^2 = \frac{(b-a)^2}{12}}$$

3.2.




$$\mu = \int_{-\infty}^{\infty} x f(x)\,dx = \int_1^2 0{,}5x\,dx + \int_3^4 0{,}5x\,dx = \left.\frac{x^2}{4}\right|_1^2 + \left.\frac{x^2}{4}\right|_3^4 =$$

$$= \frac{4-1}{4} + \frac{16-9}{4} = \frac{10}{4} = \boxed{2{,}5}$$

$$\mathbb{E}\{x^2\} = \int_{-\infty}^{\infty} x^2 f x\,dx = \int_1^2 0{,}5x^2\,dx + \int_3^4 0{,}5x^2\,dx =$$

$$= 0{,}5 \cdot \left.\frac{x^3}{3}\right|_1^2 + 0{,}5 \left.\frac{x^3}{3}\right|_3^4 = \frac{8-1}{6} + \frac{64-27}{6} =$$

$$= \frac{44}{6} = 7{,}3333$$

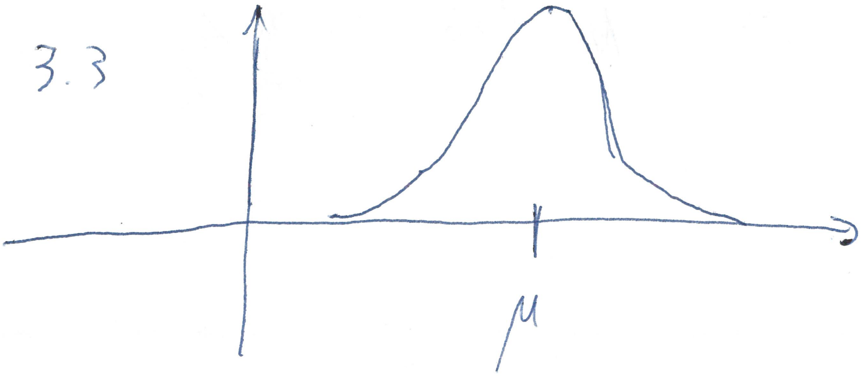
$$\sigma^2 = \mathbb{E}\{x^2\} - \mathbb{E}^2\{x\} = 7{,}33 - 6{,}25 = 1{,}083$$

$$\boxed{\sigma = 1{,}04}$$

b) Teljes szélesség 3 → 90% $d = 2{,}8$ ; 2-nek 90%-a 1,8 ; ehhez jön még a [2;3] intervallum

Elhelyezkedés tetszőleges az [1;4] intervallum belsejében.

3.3



$$f(x) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{1}{2} \cdot \frac{(x-\mu)^2}{\sigma^2}}$$

$\mu - \sigma \,;\ \mu + \sigma \rightarrow 68{,}26\%$

$\mu - 2\sigma ,\ \mu + 2\sigma \rightarrow 95{,}44\%$

$\mu - 3\sigma ;\ \mu + 3\sigma \rightarrow 99{,}73\%$

$\mu - 4\sigma ;\ \mu + 4\sigma \rightarrow 99{,}994\%$

$\mu - 6\sigma ,\ \mu + 6\sigma \rightarrow 99{,}9999998\%$

intervallum hossza

$$[1;2] \rightarrow 99{,}7\% \Rightarrow \sigma = \frac{2-1}{2 \cdot 3} = \frac{1}{6} = \boxed{0{,}1667}$$

3.10. $[0,5]$ egyenletes, 48 minta → összegezzük

$$\mu_1 = \frac{5-0}{2} = 2{,}5 \qquad \sigma_1^2 = \frac{(5-0)^2}{12} = 2{,}083$$

Független mintákat összegzünk ⇒ ~~Szorások~~ varianciák összeadódnak
(a várható értékek is persze...)

$$\mu_2 = N \cdot 2{,}5 = 48 \cdot 2{,}5 = 120 \qquad \sigma_2^2 = N \cdot \frac{5^2}{12} = 100$$

$$\sigma_2 = 10$$

$$\frac{x_i - 120}{\sigma_2} \longrightarrow z_i = \frac{x_i - \mu_2}{\sigma_2}$$

minden egyes ~~[illegible]~~ !! generált mintára

3.24. $a$ ; $-a$ 50-50% $N = 256$ → standard normális

$a = 2$

$$\mathbb{E}\{X\} = 0$$

$$\sigma_1^2 = a^2 = 4$$

N mintát összegezve

$$\mu_2 = 0 \qquad \sigma_2^2 = N \cdot a^2 = 1024 \qquad \sigma = 32$$

$$z_i = \frac{x_i - \mu_2}{\sigma_2} = \frac{x_i}{\sigma_2} = \frac{x_i}{32}$$

3.11. normális zaj

13,6720 9,4190 $21,\overset{3489}{\cancel{3840}}$ 9,7298 14,6773 18,5959

90%-os konfidenciaintervallum

$b = \overset{0,95}{\cancel{0,05}}$ szimmetrikus eloszlás

$$\hat{\mu} = \frac{1}{N} \cdot \sum_{i=1}^{N} x_i = 14,5738$$

$$s = \sqrt{\frac{1}{N-1} \cdot \sum_{i=1}^{N} (x_i - \hat{\mu})^2} = 4,7527$$

$\uparrow$

Tapasztalati szórás $\rightarrow$ Student eloszlás $N-1$ szabadságfokkal

$t_{5;\,0,05}$

szabadságfok konfidenciaszint: $\frac{1-p}{2}$

$$P\left[\hat{\mu} - \frac{s}{\sqrt{N}} \cdot \underbrace{t_{5;0,05}}_{2,015} < \mu < \hat{\mu} + \frac{s}{\sqrt{N}} \cdot t_{5;0,05}\right] = 90\%$$

$$P\left[10,6642 < \mu < 18,4835\right] = 90\%$$