Budapest University of Technology and Economics
Faculty of Electrical Engineering and Informatics
Department of Measurement and Information Systems

Integer number representation

Tamás Raikovich

BUTE DMIS

Contents

1. **Unsigned integer numbers**
2. **Signed integer numbers**
   - Signed magnitude
   - Offset binary
   - One’s complement
   - Two’s complement

Unsigned integer numbers

- **Representing unsigned integer numbers is simple in binary number system**
- **The decimal value is the weighted sum of the binary digits (d_i=0,1), the weights (w_i) are the values of the binary places (2^i): $\sum_{i=0}^{N-1} w_i \cdot d_i = \sum_{i=0}^{N-1} 2^i \cdot d_i$**

Bit position	N-1 (MSb)	N-2	……	3	2	1	0 (LSb)
Weight (w_i)	2^{N-1}	2^{N-2}	……	2^3=8	2^2=4	2^1=2	2^0=1

- **Decimal to binary conversion → Textbook Chapter 1**
- **Range when N bits are used: 0…(2^N-1)**

Signed integer numbers

- **In case of signed integer number representation, the sign information (+ or -) has to be encoded into the binary code**
- **There are several methods for this:**
  - Signed magnitude coding
  - Offset binary coding
  - One’s complement (1’s complement) coding
  - Two’s complement (2’s complement) coding

Signed integer numbers

(Signed magnitude coding)

- **When N-bit signed magnitude coding is used:**
  - The most significant bit (MSb) determines the sign (0: positive, 1: negative)
  - The lower bits represent the absolute value

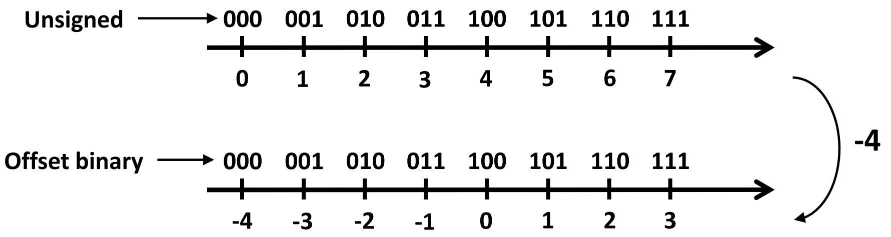
N-1 (MSb)	N-2	……	4	3	2	1	0 (LSb)
Sign bit	absolute value of the number						

- **Range: -(2^{N-1}-1)…+(2^{N-1}-1)**
- **Zero has two representations**
  - +0: ***00…00000*** and -0: ***10…00000***

Signed integer numbers

(Offset binary coding)

- **The N-bit offset binary codes can be get by subtracting half of the range (2^{N-1}) from the decimal values of the N-bit unsigned binary numbers**



- **The MSb is the sign bit: 1 → positive, 0 → negative**
- **Range: -(2^{N-1})…+(2^{N-1}-1)**
- **Zero has only one representation: *10…000000***

Signed integer numbers

(One’s complement coding)

- **When N-bit one’s complement coding is used, the –X is represented with the one’s complement of X**
  - $X_{(2)}$ + (1’s complement of $X)_{(2)}$ = $(2^N\text{-}1)_{(2)}$ = $\underbrace{1\ldots111}_{N\,\text{"1"}\,bits}$
  - Therefore: (1’s complement of $X)_{(2)}$ = $\overline{X_{(2)}}$
- **The MSb is the sign bit: 0 → positive, 1 → negative**
- **To get the decimal value, the weights are**
  - $w_{N-1} = -2^{N-1}+1$ (weight of the MSb) and $w_i = 2^i$ ($i < N-1$)
- **Range: $-(2^{N-1}-1)\ldots+(2^{N-1}-1)$**
- **Zero has two representations**
  - +0: ***00…00000*** and -0: ***11…11111***

Signed integer numbers

(Two’s complement coding)

- **When N-bit two’s complement coding is used, the –X is represented with the two’s complement of X**
  - $X_{(2)}$ + (2’s complement of $X)_{(2)}$ = $2^N{}_{(2)}$ = $\underbrace{10\ldots0000}_{\text{"1"}\,and\,N\,\text{"0"}}$
  - Therefore: (2’s complement of $X)_{(2)}$ = $\overline{X_{(2)}}+1$
- **The MSb is the sign bit: 0 → positive, 1 → negative**
- **To get the decimal value, the weights are**
  - $w_{N-1} = -2^{N-1}$ (weight of the MSb) and $w_i = 2^i$ ($i < N-1$)
- **Range: $-(2^{N-1})\ldots+(2^{N-1}-1)$**
- **Zero has only one representation: *00…000000***
- **Two’s complement ↔ offset binary conversion**
  - Inverting the sign bit (MSb)

Signed integer numbers

(Example)

Converting the +41 and the -41 decimal values to 8-bit binary numbers in different coding systems

Coding system	41	-41
signed magnitude	00101001	10101001
offset binary	10101001	01010111
one’s complement	00101001	11010110
two’s complement	00101001	11010111