Robust Broadband Periodic Excitation Design

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***Abstract*—This paper considers a rather practical problem arising when inexperienced users misuse the (otherwise well-designed) periodic broadband excitation signals during the measurement or signal processing phase of an identification process. Using a fractional period of the excitation signal instead of full periods may affect not only the precision because of the well-known leakage effect, but may cause a serious loss of information on the measured system as well. The power of the excitation signal in certain frequency bands may be much lower (30–40 dB) than it would be expected, and thus the measurement in a noisy environment may give poor result. A solution is presented here to make periodic broadband (multisine) excitation signals more robust against such misuse. The suggested solution is analyzed, and the theoretical results are verified by practical examples.**

***Index Terms*—Excitation, identification, parameter estimation, periodic functions, robustness, spectral analysis.**

I. INTRODUCTION

IN system identification processes the device under test is excited by an appropriate excitation signal, and the response of the system is measured. Given the excitation and the respective system response the desired system parameters can be derived by different digital signal processing methods.

The excitation signal sometimes comes from the nature, but most commonly is artificially generated. In the latter case, the designer of the excitation signal can select the appropriate signal type and can often set a lot of signal parameters to achieve optimal result. In practice, the following types of general-purpose excitation signals are widely used: random noise, pseudorandom binary sequences, swept sine (periodic chirp), and multisine.

Since the deterministic signals usually have superior properties compared to the random noise [1], they are commonly used, and also effective algorithms ([2], [3]), and handy design tools are available to design and generate such signals [4]. The main advantage of such signals is that there is no leakage effect (precise FRF-measurements can be made), the desired band of excitation usually can be selected (especially by multisine), and good crest factor values can be achieved to provide good signal-to-noise ratio during the measurement. However, the good properties hold only if the designed signal is properly used. Inexperienced users may not adequately use the carefully designed signals, namely they may tend to use fractional periods instead of full ones. (Typical examples are when the measurement process would be too long and is terminated, or the recorded data is divided into two parts by the user for identification and validation purposes.) The result may be quite unexpected: certain frequencies in the band of interest are excited very poorly (even 30–40 dB loss can happen) thus resulting in bad signal-to-noise ratio during the measurement process. (Note that while the leakage effect can be taken into account in the signal processing algorithms, the bad SNR due to loss of information can not be compensated.) This effect can happen not only in the swept sine, but even in the multisine case. While the swept sine signals do not allow helping this problem, the multisine excitation signals can be designed to avoid this phenomenon.

Section II will formulate the problem and examples will be given on the effect of misused excitation signals. Also a simple and easy-to-use solution is suggested to design multisine signals which are robust: when not a full period is used the proposed excitation signal has only a small power loss in the band of interest, and thus satisfactory results can be obtained even in case of misuse. In Section III the suggested solution is analyzed, and the theoretical results are verified through practical examples. Based on the results Section IV provides guidelines to design robust multisine excitation signals.

II. EFFECT OF FRACTIONAL PERIODS ON THE POWER SPECTRUM

Periodic excitation signals can be—and usually are—designed to excite only a frequency band of interest and they have no significant power outside that band. This helps to keep excitation power as low as possible and to avoid unnecessary nonlinear effects. But it is even more important that the excitation really be present in those frequencies (or nearby) where the system is to be measured and modeled. If certain bands are not excited sufficiently then the amount of information gathered to model those bands may be low, and the result of the identification process would be poor.

However, misused excitation signals may result in quite different spectra than it would be necessary: the band of interest may be partly unexcited. Fig. 1 illustrates the phenomenon using popular excitation signals: swept signal, Schroeder multisine, and random phase multisine [1], [5].

In the case of the swept signal the frequency is swept between $f_{\min}$ and $f_{\max}$ in time with period T, where $f_{\min}$ and $f_{\max}$ are the lowest and highest frequencies to excite

$$x(t) = 2A\sin\left(\left(\frac{\pi(f_{\max} - f_{\min})}{T}t + 2\pi f_{\min}\right)t\right). \quad (1)$$

A multisine signal is a sum of harmonically related sinusoids

$$x(t) = \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \phi_k) \quad (2)$$

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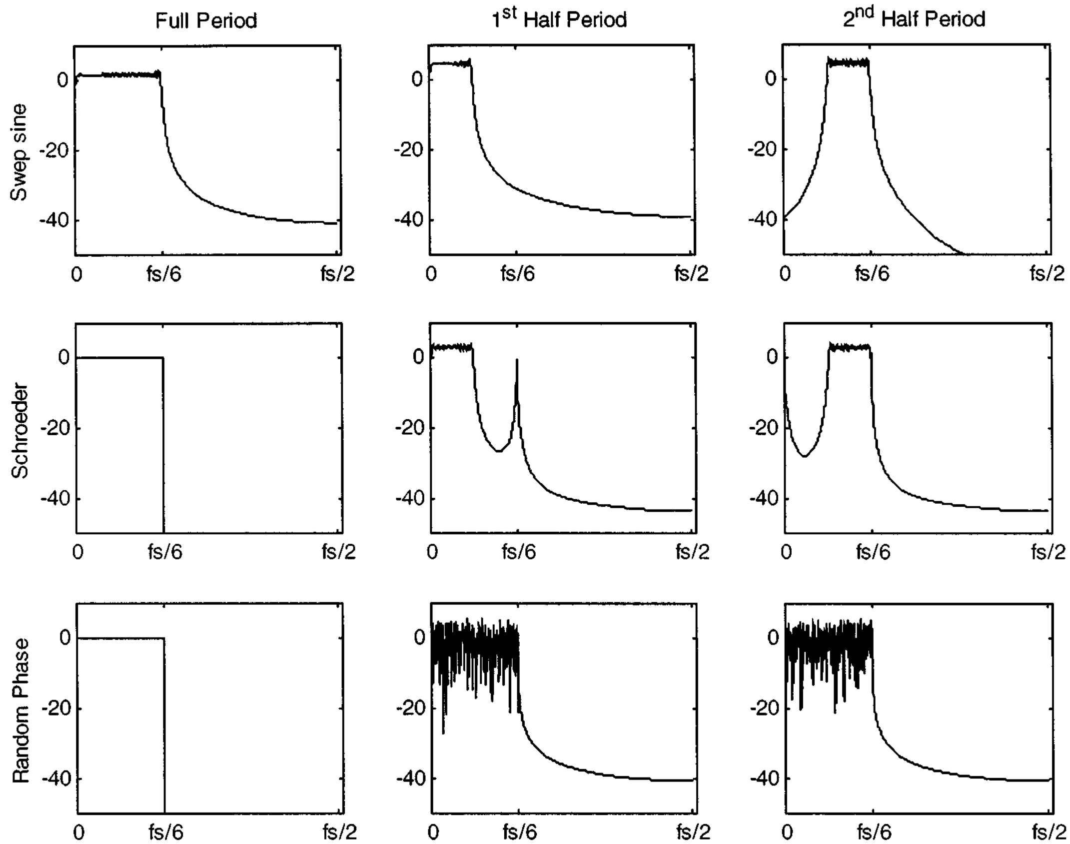

Fig. 1. Examples of correctly and incorrectly used excitation signals. First column: Spectra of correctly used signals; second and third column: spectra of truncated (half) periods. First row: Swept sine; second row: Schroeder multisine; third row: random phase multisine. The power spectra are shown in decibels.

where $f_k = l_k * f_0, l_k$ positive integer, so that $f_{\min} \leq f_k \leq f_{\max}$. The phases are often chosen so that the crest factor be small, like in the Schroeder multisine, where the phases are calculated by

$$\phi_k = -\frac{k(k-1)}{N}\pi. \quad (3)$$

All the signals in Fig. 1 were designed to excite the frequency band from 0 to $f_s/6$, where f_s is the sampling frequency. The spectra were calculated by DFT with a rectangular window, as it is common in identification processes. Using full periods the required results are obtained, but when only half of the signal length was used, the resulted spectra seriously differ from the required: in case of swept sine and Schroeder multisine a large part of the band of interest is not excited adequately. In the random phase case no wide unexcited bands can be seen. Note that in this respect, the behavior of a multisine signal can strongly resemble to the swept sine (Schroeder case), or it can completely be different (random phase case), depending on the phase values.

Unfortunately, this kind of misuse is very common since a lot of users do not have—and, of course, do not need to have—solid theoretical background in the field of digital signal processing. Instead the excitation signal should be robust against misuse. In the next section it will be proven that the random phase multisine is really robust against "fractional-period misuse." First a framework will be defined to analyze the statistical properties of the spectral power loss, and then the behavior of the random-phase multisine will be examined.

III. Statistical Properties of the Power Loss

In an identification process it is crucial, that during the measurement the frequency band of interest be well excited to gain enough information to determine the system's properties. To identify an arbitrary unknown system it would be necessary to excite all the frequency lines in the band of interest. However, in the case of practical systems with "not too rapidly changing" transfer functions it is not crucial if some lines are poorly excited when the surrounded lines have enough power. The data gained from the well-excited lines is enough to describe the system's properties in the close neighborhood. This means that it is usually satisfactory if the total power is sufficient in all *subbands* of the band of interest, where a *subband* contains more than one line. In practice the size of the subband depends on the system to be identified, but usually means a smaller fraction of the band of interest, e.g., one-tenth of it, and thus can contain some tens or even hundreds of lines.

In this framework the global quantity of "smallest excitation power" in the band of interest can be defined as the minimum of the average powers in all subbands

$$S = \min_j(P_j), \quad j = 1 \ldots N_s \quad (4)$$

$$P_j = \frac{1}{K}\sum_{i=1}^{K} |A_{i,j}|^2 \quad (5)$$

where K is the number of frequency lines in a subband, N_s is the number of subbands in the band of interest, and $A_{i,j}$ is the ith spectrum line in the jth subband.

It is clear that if S is sufficiently large even if only a fraction of the full period is used, then the excitation signal is robust against the "fractional period" misuse. To determine the stochastic behavior of S, the following general-purpose excitation signal will be considered:

- length of the full period is N;
- spectrum is flat in the band of interest between $f_{\min}$ and $f_{\max}$, and $f_{\min} < f_{\max} < f_s/2$, where f_s is the sampling frequency;
- excited frequencies are $f_k = f_s/N * k$, for all possible integer k, so that $f_{\min} \leq f_k \leq f_{\max}$;
- phase of the sinusoid components is random and equally distributed between $-\pi$ and $+\pi$.

The power spectrum of the above excitation signal computed by taking the absolute square of an N-point DFT has equal values $(|A_{i,j}|^2 = A^2)$ for all lines in the band of interest between $f_{\min}$ and $f_{\max}$, and zeros otherwise, and the phase of the DFT is random. When the signal sequence is truncated to $M < N$ points, then the power spectrum of the computed M-point DFT will also be random-like with expected value of A^2 in the band of interest. Thus the real and imaginary parts of the M-point DFT in the band of interest can be modeled by independent random variables with normal distribution $N(0, A^2/2)$.

According to (5), P_j (in the band of interest) is the sum of $2K$ squared, normally distributed random variables, so P_j is also a random variable, and its distribution is chi-square with $2K$ degrees of freedom. If the cumulative distribution function (c.d.f.) of a chi-square distribution with f degrees of freedom is denoted by $F_f(x)$, then the c.d.f. of P_j will be

$$P(P_j < x) = F_{2K}\left(\frac{2Kx}{A^2}\right) \tag{6}$$

where K is the number of lines in the subband.

If (after the M-point DFT) the band of interest is divided into N_s subbands with K lines in each, then the c.d.f. of S, e.g., the c.d.f. of the smallest P_j value in the band of interest can be expressed using order statistics [6]

$$P(S < x) = 1 - \left(1 - F_{2K}\left(\frac{2Kx}{A^2}\right)\right)^{N_s}. \tag{7}$$

It is clear that, apart from the normalizing coefficient A^2, the distribution depends only on the number of lines in the subbands (K) and the number of subbands (N_s). Based on (7), Table I contains the most probable power loss values as well as the upper limits for the maximum power loss with confidence level of 99%, for different K and N_s values.

From data shown in Table I, it can be seen that for a constant number of subbands (constant N_s) the power drop decreases as the number of lines (K) in the subbands increases. For a constant K the power drop increases as N_s increases. As it is intuitively expected, for a constant number of frequency lines in the band of interest (i.e., for constant $K \cdot N_s$) the power drop increases as the number of subbands increases.

TABLE I
MOST PROBABLE POWER DROP/MAXIMUM POWER DROP (WITH 99% CONFIDENCE LEVEL) VALUES IN DECIBELS FOR DIFFERENT N_s AND K VALUES

		Values of K			
		10	30	100	300
Values of N_s	5	1.9 / 4.9	1.0 / 2.6	0.5 / 1.4	0.3 / 0.8
	10	2.5 / 5.3	1.3 / 2.8	0.7 / 1.5	0.4 / 0.9
	20	3.0 / 5.7	1.6 / 3.0	0.8 / 1.6	0.5 / 0.9
	30	3.3 / 6.0	1.7 / 3.1	0.9 / 1.6	0.5 / 0.9
	40	3.5 / 6.1	1.8 / 3.2	1.0 / 1.7	0.6 / 1.0

In a practical case, where the number of subbands is not too high, and in each subband there is a reasonably high number of frequency lines, the expected power drop of the misused random-phase multisine is around $1\ldots3$ dB, which is much less than the power drop in the case of the Schroeder or swept-sine cases (compare with Fig. 1).

To validate the theoretical results, 1000 excitation signals were designed with the same power spectrum, each with random phase. The band of interest was similar to that of Fig. 1, the number of points in the full period was $N = 2048$, and the signal before calculating the DFT was truncated to $M = 512$. For test purposes, five subbands were selected in the band of interest with ten consecutive lines in each $(N_s = 5, K = 10)$. The smallest average subband power (S) was calculated for each excitation signal according to (4). The experimental distribution is shown in Fig. 2(a), with the theoretical distribution function calculated from (7), and scaled according to the number of experiments. The match clearly validates the theoretical results.

In Fig. 2(b) and (c), similar plots can be seen for signals designed starting from random-phase multisines using the crest factor optimizer algorithms in [2] and [3]. The results verify the conjecture that the algorithms preserve the random-like behavior of the phase components, and thus the theoretical results for random phase multisine signals can be applied to determine the behavior of the modified signals as well.

Based on the results, the next section will provide guidelines of robust multisine excitation signal design.

IV. ROBUST MULTISINE SIGNAL DESIGN

If the designed multisine excitation signal will not surely be adequately used, then the designer or the automatic designer algorithm should take care of the robustness against fractional-period use. The free design parameters are the phase values. The sophisticated Schroeder multisine—which provides satisfactory low crest factor without any additional optimization, and thus is very popular—is one of the worst solutions from this point of view. Even the use of Schroeder multisine as an initial signal for crest-factor minimization algorithms ([2], [3]) is a bad choice, since the result of the minimization still behaves almost as badly as the initial signal, according to experiments.

A better choice is to start from a random phase multisine and then apply a crest factor minimization algorithm. According to experiments the known crest-factor minimization algorithms do

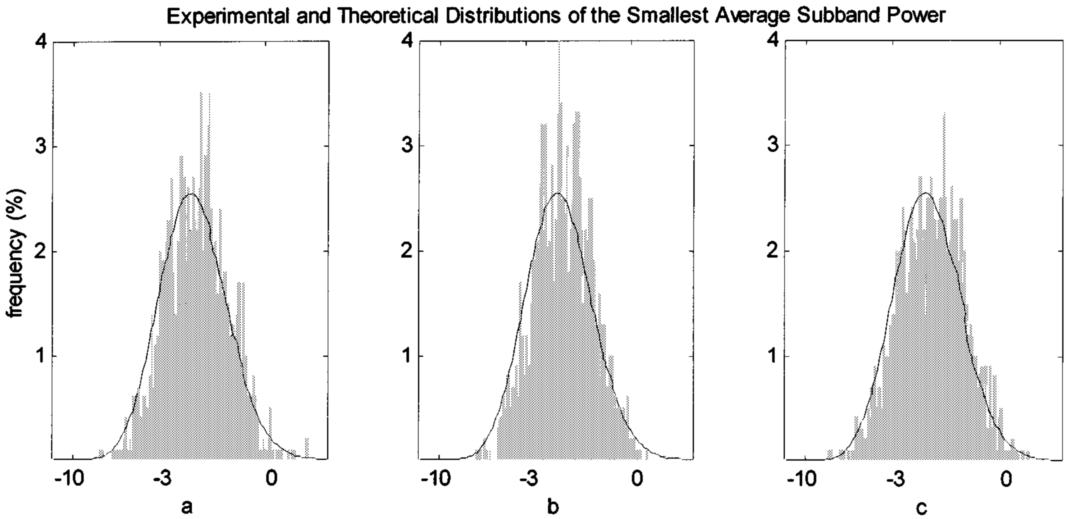

Fig. 2. Experimental (bars) and theoretical (solid line) distributions of the smallest average subband power values (horizontal axis in decibels, vertical axis in percentage) for a random-phase multisine excitation signal ($N_s = 5,\ K = 10, 1000$ experiments) (a) without crest factor minimization, (b) with crest factor minimization algorithm [2], and (c) with crest factor minimization algorithm [3].

TABLE II
TYPICAL CREST FACTOR VALUES OF MULTISINE EXCITATION SIGNALS

Schroeder	1.66
Schroeder with Algorithm [2]	1.49
Schroeder with Algorithm [3]	1.42
Random phase	3.0 … **3.5** … 4.8
Random phase with Algorithm [2]	1.53 … **1.63** … 1.76
Random phase with Algorithm [3]	1.37 … **1.39** … 1.41

not significantly change the random-like behavior of the spectrum, thus the gained theoretical results hold even if the initial random-phase signal is modified by the algorithms (see the histograms in Fig. 2).

Table II contains typical crest factor values of the Schroeder multisine, random-phase multisine, and the output of crest-factor minimization algorithms in [2] and [3] starting from Schroeder and random initial phases. It is clear, that the optimization methods give very good results when the initial state is a random phase multisine, and moreover, the gained signal is robust against misuse.

The only drawback of the random phase multisine signals is that an additional iterative optimization process is required to reach the desired low crest factor values. With the algorithms described in [2] and [3] the amount of time necessary to optimize a multisine signal's crest factor depends mainly on the desired number of excited frequency lines. Table III shows typical running times for the optimization algorithms with typical settings, on a Pentium-class PC, in MATLAB environment. From the results in Tables II and III it is clear that Algorithm [3] gives better crest factor values and is faster than Algorithm [2] when the number of excited lines is low. For very high number of frequency lines only Algorithm [2] can be used.

Note that the results shown in Table III are for typical settings, where the number of iterations is set high to achieve low crest factor values. If the number of iterations (and thus the running time) is decreased by a factor of five then the resulted crest factor values are typically 5–10% higher.

TABLE III
TYPICAL RUNNING TIME VALUES OF CREST-FACTOR MINIMIZATION ALGORITHMS FOR DIFFERENT TYPES OF EXCITATION SIGNALS

Number of excited lines	Running time of	
	Algorithm [2]	Algorithm [3]
10	3 sec	2 sec
30	4 sec	3 sec
100	9 sec	4 sec
300	45 sec	30 sec
1000	2 min	50 min
3000	10 min	-
10000	60 min	-

V. CONCLUSION

In this paper, a solution was suggested to avoid problems arising from the misuse of periodic broadband excitation signals. Examples illustrated that the power-loss of the spectrum in the band of interest can be serious when a fractional period of the popular Schroeder multisine signals is used. The power-loss causes insufficient excitation in the band of interest and thus the result of the identification process may be of lower quality.

It was proven that the random-phase multisine signals have superior properties; they are much less sensitive to the improper use. Even if some frequency lines are poorly excited, the excitation power is evenly distributed in the band of interest and no larger regions remain without excitation.

The theoretical results were verified by experiments. It was shown that the theoretical distribution function of the power loss and the measured distribution gained from 1000 random experiments match. An additional utilization of the theoretical results can be the prediction of the power loss in the band of interest when the excitation signal is misused.

The weak crest factor of the random-phase multisine can be effectively decreased by appropriate optimization methods. Two optimization algorithms known in the literature were tested. In the paper a combined method was proposed (random-phase multisine with optimization), which provides "good quality" multisine with low crest factor values, which is also robust against misuse.

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