Analyzing A/D and D/A converters

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1 Signals

1.1 Periodic signals

A signal is said to be periodic with $T$ period of time if $x(t + T) = x(t)$ for all values of $t$. One of the most important periodic signals is the sine wave:

$$x(t) = A \cdot \cos(2\pi ft + \phi),$$

where $A$, $f$, $\phi$ are the amplitude, frequency and initial phase, respectively. The complex form of the sine wave is:

$$A \cdot \cos(2\pi ft + \phi) = C_1 e^{j2\pi ft} + C_{-1} e^{-j2\pi ft},$$

where

$$C_1 = \frac{A}{2} e^{j\phi} \text{ and } C_{-1} = \overline{C_1}.$$  

Above equations show that the sine wave can be expressed as the sum of two complex exponential functions.

Periodic functions can be decomposed into the sum of infinite complex exponential functions using the Fourier series:

$$x(t) = \sum_{k=-\infty}^{\infty} C_k \cdot e^{j2\pi kft}.$$  

If $x(t)$ is a real function then $C_{-k} = \overline{C_k}$ holds true which guarantees that the sum is real. Based on the form of the sinewave, for a given $k$ value:

$$C_k e^{j2\pi ft} + C_{-k} e^{-j2\pi kft} = A \cdot \cos(2\pi kft + \phi).$$

This means that for real, periodic signals the Fourier series express the signal as the sum of sinewaves. The frequencies of the sines are the integer multiples of the base frequency $f$.

For example the Fourier series of the symmetric square wave:
\[ x(t) = \frac{4}{\pi} \left( \sin(2\pi ft) + \frac{1}{3} \sin(3 \times 2\pi ft) + \frac{1}{5} \sin(5 \times 2\pi ft) + \frac{1}{7} \sin(7 \times 2\pi ft) + \cdots \right) \]

Here the frequencies of the harmonics are the odd integer multiples of \( f \). If the squarewave is not symmetric (its duty cycle differs from 50\%) than its Fourier series will contain also even integer multiples of \( f \).

The Fourier series of the triangle wave is:

\[ x(t) = \frac{8}{\pi^2} \left( \sin(\omega 2\pi ft) - \frac{1}{3^2} \sin(3 \times 2\pi ft) + \frac{1}{5^2} \sin(5 \times 2\pi ft) - \frac{1}{7^2} \sin(7 \times 2\pi ft) + \cdots \right). \]

The following rule helps to determine the nature of the amplitude spectrums of periodic signals: if the \( n \)th derivative of an \( x(t) \) periodic function is the series of \( \delta(t) \) Dirac-functions, then its amplitude spectrum is bounded by

\[ \frac{1}{f^n}. \]

For example visit the Fourier series of the square wave and triangle wave functions.

Note: The generalization of the Fourier series for nonperiodic signals is the continuous Forirer transform but it has almost the same meaning.

### 1.2 Sampling

During the sampling the value of a continuous \( x(t) \) function is measured every \( T_s \) seconds. Next figure [1] shows the sampling of a signal started at \( t_0 \):

[Diagram of sampling process]

The reciprocal of \( T_s \) is the so-called sampling frequency \( (f_s) \). After the sampling the values of the function are known only in the \( t_1, t_2, ..., t_i \) sampling points.

#### 1.2.1 Discrete Fourier transform

Since the sampled signal is discrete its continuous Fourier transform cannot be determined. The spectrum can be calculated in discrete points using the discrete Fourier transform:

\[ X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{\frac{j2\pi kn}{N}}, \]
where $X(k)$ is the $k$th spectral value ($k = 0, ..., N - 1$), $x[n]$ is the $n$th sample of the sampled signal and $N$ is the total number of samples in the record. The expression shows that the spectrum is discrete, the resolution of the DFT ($\Delta f$, the difference between two adjacent points in Hz) is:

$$\Delta f = \frac{f_s}{N}.$$ 

For example, if $N = 1000$ samples are measured with $f_s = 10$ kHz frequency:

$$\Delta f = \frac{10 \text{ kHz}}{1000} = 10 \text{ Hz}.$$ 

The frequency of the $k$th spectral component is:

$$f_k = k \cdot \Delta f.$$ 

This means that the result of the DFT shows the components of the signal in the $[0, f_s - \Delta f]$ interval. For real signals $X(k) = \bar{X}(N - k)$ (complex conjugate symmetry).

### 1.2.2 Spectrum of sampled signals

When a signal is sampled with $f_s$ frequency its spectrum changes. Let be $X(f)$ the spectrum of the original signal and $X_m(f)$ the spectrum of the sampled signal. The relation between them is:

$$X_m(f) = \sum_{i=-\infty}^{\infty} X(f - i \cdot f_s).$$

Above equation shows that the spectrum of the sampled signal becomes periodic with $f_s$. Next figure [1] shows the effect of sampling on the spectrum:

If the sampling frequency is chosen to be too small then the spectrums are overlapping (this is called also aliasing). Due to the overlap the signal will be distorted because components of the original and repetitive spectrums cannot be separated. The Nyquist-Shannon sampling theorem helps to choose a proper sampling frequency:

If the $x(t)$ signal does not contain any spectral components higher than $B$ Hz, then no information is lost if it is sampled with $f_s > 2B$ frequency.

Note that the no loss of information also requires infinite number of samples. For finite number of samples $f_s > 2B$ guarantees only that there will be no overlap in the frequency domain the result of the DFT will not be distorted.)
1\textsuperscript{st} example: a sinewave of $f = 50$ Hz is to be measured. Considering the Nyquist-Shannon theorem $f_s > 100$ Hz sampling frequency is required.

2\textsuperscript{nd} example: a square wave of $f = 50$ Hz is to be measured. The Fourier series of the square wave contains infinite elements, thus $B = \infty$. This means that the square wave cannot be sampled without distortions due to overlaps in the frequency domain.

The second example showed a case when the fulfillment of the Nyquist-Shannon condition is not possible. Let $f_{in}$ be the frequency of the sampled signal and $f_s$ be the sampling frequency. In the upper case:

- Signal components from the $[0, \frac{f_s}{2}]$ domain appear correctly,
- Components from the $[\frac{f_s}{2}, f_s]$ domain overlap into the $[0, \frac{f_s}{2}]$ domain, the frequencies of such components seem to be $f_s - f_{in}$.
- Components from the $[f_s, \frac{3f_s}{2}]$ domain overlap into the $[0, \frac{f_s}{2}]$ domain, the frequencies of such components seem to be $2f_s - f_{in}$.
- And so on...

So if a sinewave with constantly increasing frequency between 0 Hz $f_s$ and is measured with an oscilloscope, on the display first we will see a sine with increasing frequency ($f_{in} < \frac{f_s}{2}$), then the frequency will seem to decrease ($\frac{f_s}{2} < f_{in} < f_s$).

To avoid the above case the signal is filtered with a so-called anti-aliasing filter before the sampling. The anti-aliasing filter is a low-pass filter with $\frac{f_s}{2}$ cut-off frequency. Real filters are not ideal which means that a transition band can be found between the passband and stopband where the rejection is not infinite.

Example: Square wave of 50 Hz frequency is filtered with an ideal lowpass filter of $f_c = 175$ Hz. The characteristics of the filter is the following:

$$F(f) = \begin{cases} 1, & |f| \leq f_c \\ 0, & |f| > f_c \end{cases}$$
The components of the square Fourier series of the square wave will pass the filter if their frequency is not greater than the cut-off frequency:

\[ |k \cdot f| < f_c. \]

The frequencies of the components are \( k \cdot 50 \text{ Hz} \) where \( k = 0, \pm 1, \pm 2 \pm \ldots \). The \( |k \cdot f| < f_c \) equations holds true for \( k = -3, -2, -1, 0, 1, 2, 3 \). The original, nonfiltered squarewave and its spectrum can be seen in the next two figures:

The characteristics of the filter is the following:

Next figure shows the filtered squarewave:
1.2.3 Sampling of periodic signals

Coherent sampling means that an integer number of periods are measured of the periodic signal. The finite number of samples can be modeled as the signal is multiplied by a rectangular window in the time domain. Assuming that the number of samples is \( N \):

\[
w(k) = \begin{cases} 
1, & 0 \leq k < N \\
0, & \text{otherwise}
\end{cases}
\]

The DFT of such window is the following:

\[
\frac{\sin(k\pi)}{\sin\left(\frac{k\pi}{N}\right)}
\]

This value of the function is the following:

\[
W(k) = \begin{cases} 
N, & k = 0, \pm N, \pm 2N, \pm \cdots \\
0, & k = \pm 1, \pm 2, \pm 3, \ldots \quad \text{otherwise}
\end{cases}
\]

Multiplication in the time domain means convolution in the frequency domain: the DFT of the input signal has to be convolved with the above function. For example, the DFT of the sampled sinewave is:

\[
\frac{\sin((k-D)\pi)}{\sin\left(\frac{(k-D)\pi}{N}\right)}
\]

where \( D \) is the sine’s frequency (the number of measured periods of the signal). If the sampling is coherent than \( D \) is an integer so \( k - D \) is an integer also and the spectrum of the sine will not be distorted. However, for noncoherent sampling \( k - D \) will not be integer and the samples of the \( W(k) \) will appear in the spectrum:
The appearance of the samples of $W(k)$ in the DFT is called leakage. When a sinewave is not sampled coherently, the DFT will show its amplitude wrongly because it will seem to be smaller, this is called picket fence effect.

The effects of leakage and picket fence can be decreased with the usage special time domain windows, for example the Blackman-Harris window:

1.2.4 Undersampling:
Undersampling means that a signal is sampled with a frequency which does not fulfill the Nyquist-Shannon condition. Sometimes, when the spectrum of the signal is known to be very narrow, aliasing does not cause distortions in the frequency domain:

A special case of undersampling is the sinewave sampled with $f_s = f_x$ frequency (where $f_x$ is the frequency of the sine). In this case, the values of the samples in the measured signal seem to be constant:
The amplitude of the measured DC signal depends on the phase of the samples of the sine.
2 A/D and D/A conversion

2.1 A/D conversion

A/D conversion means that a continuous quantity is converted to a digital number that represents its amplitude. An important step of the A/D conversion is quantization:

In the figure $x(t)$ is the continuous signal to be converted and $x_q(t)$ is the quantized signal. The $x(t)$ signal can have any value while the value of the $x_q(t)$ signal can be only the integer multiple of $q$, the quantum (also denoted by $LSB$ which means least significant bit). The unit of the $x(t)$ and $x_q(t)$ signals are the same (generally Volt). A/D conversion means that after quantization $x_q(t)$ is sampled and then digital numbers are assigned to the sample values. At the end of the conversion $x(t)$ is converted into an array of digital numbers.

The A/D converters have two important properties:

- The number of bits ($b$),
- The voltage range in which the conversion is done. $FS$ (Full Scale) is the maximum allowed input voltage. Unipolar converters work in the $[0, FS]$ domain while bipolar converters work in the $[-FS, FS]$ domain. $FSR$ denotes the full scale range, $FSR = FS$ for unipolar and $FSR = 2FS$ for bipolar converters.

If $b$ and $FSR$ is known than the nominal value of the quantum can be determined as:

$$q = \frac{FSR}{2^b}.$$

Next figure [1] shows the characteristics of a quantizer:

In the figure we can see that $y_i$ is the digital number which will be assigned to the values of $x$ between $x_i$ and $x_{i+1}$. So $x_i$ and $x_{i+1}$ bounds a voltage domain which belongs to the $y_i$ digital code.
Every digital code has a \( q \) wide voltage domain, these are called code bins. The bounds between two adjacent code bins are the so called transition levels of the A/D. In the figure \( x_i \) and \( x_{i+1} \) are transition levels and \( x_{i+1} + q = x_i \) for every \( i = 1 \ldots 2^b \). An A/D of \( b \) bits has \( 2^b \) code bins and digital output codes, and \( 2^b - 1 \) transition levels. The curve connecting the transition levels is called the characteristics of the A/D converter.

### 2.1.1 Quantization noise

During the quantization of the \( x(t) \) signal information is lost because the original signal cannot be restored based on \( x_q(t) \). The idea of the quantization noise is that \( x_q(t) \) can be considered as the “noisy” \( x(t) \), \( x_q(t) = x(t) + e(t) \) where \( e(t) \) is the quantization noise. The statistical properties of \( e(t) \) are the following:

- it is uniformly distributed in the \( [-\frac{q}{2}, \frac{q}{2}] \) interval,
- its expected value is \( \mu = 0 \),
- its standard deviation is \( \sigma = \frac{q}{\sqrt{12}} \).

This means that quantization can be modeled as the addition of independent, uniformly distributed noise to the signal. Below figure [1] shows the noise model of quantization:

![Noise Model of Quantization](image)

### 2.1.2 Analyzing A/D converters

Previous section presented ideal A/D converters where every code bin has the same width \( q \). This also means that the voltage values of the transition levels are on a straight line. This section presents the properties of nonideal A/D converters. The A/D converters are characterized by the deviations of their characteristics from the ideal, linear characteristics.

#### Characteristics of the ideal A/D

The characteristics of an A/D converter is the curve connecting its transition levels. In the case of an ideal A/D the level of \( x_q(t) \) for a given \( D \) digital code is:

\[
x_q(t) = D \cdot q.
\]

The width of the code bin has to be \( q \) and it is symmetric around \( x_q(t) \), so the transition levels bounding the code bin belonging to \( D \) are:

\[
T_{bottom} = D \cdot q - \frac{q}{2} \quad \text{and} \quad T_{top} = D \cdot q + \frac{q}{2}.
\]

This means that the transition levels can be calculated using the following formula:

\[
T_k = k \cdot q + \frac{q}{2}, \quad k = 0, 1, \ldots, 2^b - 1.
\]

Above formula is the expression of a straight line which has the following two parameters:

offset: \( C = \frac{q}{2} \), steepness: \( q \).
Characteristics of a nonideal A/D:

Actually, A/D converters are not ideal which means that the connecting curve of the transition levels is not a straight line. For real A/D-s, a line is fit to the highest and lowest transition levels, or using least squares method. This line is called the reference line of the A/D converter. Characterization of the A/D converter means two steps:

- analyzing the difference between the reference line and the characteristics of an ideal A/D,
- analyzing the difference between the reference line and the real characteristics.

The deviation of the reference line from the characteristics of an ideal A/D is described by the offset error and gain error. The ideal characteristics is calculated using the nominal full scale range and the number of bits:

\[ q_{id} = \frac{FSR}{2^b}. \]

Using the expression of the transition levels of an ideal A/D converter:

\[ T_k = k \cdot q_{id} + \frac{q_{id}}{2}. \]

Let be \( T_k = k \cdot m + C \) the equation of the reference line of the A/D. Then the offset error (\( e_{os} \)) and gain error (\( e_g \)) are the following:

\[ e_{os} = C - \frac{q_{id}}{2}, \quad e_g = m - q_{id}. \]

Note: the steepness of the reference line is the reference value of the LSB of the A/D converter.

Next figure shows the ideal characteristics (blue) and the reference line (red) of a 10 bit A/D working in the \([0 V, 5 V]\) nominal domain:

The differences between the connecting curve of the transition levels and the reference line are called nonlinearities (since the connecting curve is nonlinear). There are two types of nonlinearities: integral and differential.
The integral nonlinearity shows the difference between the position of each transition level and the corresponding point of the reference line. Below figure shows the real characteristics and the reference line fit to the first and last transition levels:

Using the above, \( \hat{T}_k = k \cdot m + C \) for the reference line, the \( k \)th integral nonlinearity is (in LSB):

\[
\text{INL}_k [\text{LSB}] = \frac{T_k - \hat{T}_k}{m}, \quad k = 0, 1, 2, ..., 2^b - 1.
\]

where \( T_k \) is the real value of the \( k \)th transition level (this has to be measured previously).

The differential nonlinearity describes the deviation of the width of each code bin from the reference LSB:

\[
\text{DNL}_k [\text{LSB}] = \frac{T_{k+1} - T_k - m}{m}, \quad k = 0, 1, ..., 2^b - 2.
\]

The global INL and DNL values of the A/D converter are the maximums of the absolute values:

\[
\text{INL} = \max(|\text{INL}_k|) \quad \text{and} \quad \text{DNL} = \max(|\text{DNL}_k|).
\]

Above error sources show that a digital number representing the analog input might have much more error sources than quantization error. Effective number of bits (ENOB) is a quantity which integrates all error sources of the A/D. Its definition is the following: \( \text{ENOB} \) of a real A/D is the number of bits of an ideal A/D converter of whose \( \text{SNR} \) (signal to noise ratio) equals to the \( \text{SNR} \) of the real A/D converter.

### 2.2 D/A conversion

D/A conversion means assigning voltage values to digital codes. A constant voltage value corresponds to every digital code, so there are no transition levels. The relation between the digital codes and voltages assuming an ideal D/A:

\[
U_D = D \cdot q.
\]

The meaning of \( q \) is the same here:

\[
q = \frac{\text{FSR}}{2^b},
\]
2.2.1 Analyzing D/A converters

Similarly to A/D converters, the characteristics of a D/A is the curve connecting the $U_D$ output voltages. The reference line is the line which fits best the $U_D$ points, it can be fit to the highest and lowest voltage or can be calculated using least squares method.

**Characteristics of the ideal D/A**

The characteristics of an ideal D/A is described by the following line:

$$U_D = D \cdot q,$$

where

$$q = \frac{FSR}{2^b}.$$

Since the maximum value of $D$ is $2^b - 1$, the maximum output voltage is:

$$U_{D,\text{max}} = (2^b - 1) \cdot q = FS - q,$$

so the output voltage do not reach the nominal maximum output value, $FS$.

**Characteristics of a nonideal D/A**

The first two error sources are the gain and offset errors which are very similar to the errors of the A/D converters. These two errors are determined by comparing the ideal characteristics and the reference line. The ideal values of the $LSB$ and offset are:

$$q_{id} = \frac{FSR}{2^b} \quad \text{and} \quad C_{id} = 0 \text{ V}.$$

The latter comes from the $U_D = D \cdot q_{id}$ equation.

Let $\hat{U}_D = D \cdot m + C$ be the equation of the reference line. The offset and gain errors are:

$$e_g = m - q_{id}, \quad e_{os} = C - C_{id}.$$

Using the above expression the absolute value of the gain error is determined, however in many cases its relative value is more advantageous:

$$h_g = \frac{m - q_{id}}{q_{id}}.$$

Similarly to A/D converters, D/A-s also have integral and differential nonlinearities. Integral nonlinearity describes the fluctuation of the characteristics around the reference line (see the figure below). Let $\hat{U}_D = m \cdot D + C$ the equation of the reference line, and $U_D$ the exact output voltage for a given $D$. The expression of the INL is:

$$INL_D [LSB] = \frac{U_D - \hat{U}_D}{m}.$$

If the characteristics of the D/A were linear then $U_{D+1} - U_D = m$ would be true for every possible value of $D$, thus the difference between two adjacent output voltages would be the reference $LSB$. The differential nonlinearity describes the deviation of the difference two adjacent outputs from $m$: 

where $FSR$ is the full scale range and $b$ is the number of bits of the D/A converter.
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\[ DNL_D [LSB] = \frac{U_{D+1}-U_D-m}{m}. \]

The global INL and DNL values of the D/A converter are:

\[ INL = \max(|INL_D|) \quad \text{and} \quad DNL = \max(|DNL_D|). \]

Example: D/A of 2 bits converts into the [0 V, 1 V] nominal domain. The exact outputs for each digital code are the following:

\[ U_0 = -0,1 \, V, \quad U_1 = 0,1 \, V, \quad U_2 = 0,4 \, V, \quad U_3 = 0,8 \, V. \]

First we determine the offset and gain errors. For this purpose, the ideal characteristics is needed:

\[ q = \frac{FSR}{2^b} = 0,25 \, V \quad \text{so} \quad U_D = D \cdot q. \]

Now the reference line has to be determined. Instead of performing the least squares method the line is fit to the first and last output:

\[ U_0 = 0 \cdot m + C \quad \text{and} \quad U_3 = 3 \cdot m + C. \]

The parameters of the reference line are:

\[ C = -0,1 \, V \quad \text{és} \quad m = 0,3 \, V. \]

The reference line is:

\[ \hat{U}_D = [-0,1 + 0,3 \cdot D] \, V. \]

So the values of the offset and gain errors are:

\[ e_{er} = 20 \% \quad \text{és} \quad e_o = -0,1 \, V. \]

To determine the INL the differences between the reference line and the real characteristics has to be calculated. Since the line was fit to the endpoints (\( U_0 \) and \( U_3 \)) those don’t have to be examined:

\[ INL_1 [LSB] = \frac{U_1-\hat{U}_1}{m} = -\frac{1}{3} \, LSB, \]
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\[ I_{NL2} [\text{LSB}] = \frac{u_2-u_2}{m} = -\frac{1}{3} \text{LSB}. \]

Thus the \( I_{NL} \) is:

\[ I_{NL} = \frac{1}{3} \text{LSB}. \]

The \( D_{NL_k} \) values are:

\[ D_{NL0} [\text{LSB}] = \frac{u_1-u_0-m}{m} = -\frac{1}{3} \text{LSB}, \]
\[ D_{NL1} [\text{LSB}] = \frac{u_2-u_1-m}{m} = 0 \text{LSB}, \]
\[ D_{NL2} [\text{LSB}] = \frac{u_3-u_2-m}{m} = \frac{1}{3} \text{LSB}. \]

Thus the \( D_{NL} \) is:

\[ D_{NL} = \frac{1}{3} \text{LSB}. \]
3 Measurement tasks

3.1 The development board

The measurement is based on an Analog Devices Microconverter SAR Evaluation Board which contains an AdµC831 microcontroller – it has an 8051 processor core and several 12-bit A/D and D/A converters. The PC communicates with the board through an RS-232 (UART) interface. The program of the microcontroller is pre-written, the DACs and ADCs can be controlled through simple UART commands, which are implemented in MATLAB routines. The schematic diagram of the microcontroller is shown on Figure 1.

![AdµC831 microcontroller](image)

The microcontroller contains a 12-bit, 8-channel multiplexed successive approximation A/D converter and 2 resistor based D/A converters. The schematic diagram of the A/D converter is shown on Figure 2. During the measurements AIN2 channel is used, which is connected to the BNC connector of the development board (ADC2). The signal range of the A/D is 0...2.5V.
The schematic diagram of the resistor based D/A converter is shown on Figure 3. The IC is configured in a way that the D/A converter output connects to the D/A pins through an integrated output buffer. The pins are wired to BNC connectors denoted with DAC0 and DAC1. DAC0 is connected directly to the D/A IC, while DAC1 is connected through an amplifier with amplification ratio of 1. As the amplifier offers protection and higher loadability, use this output during the measurements.

The signal range of the D/A is programmable – the microcontroller sets the reference voltage to 2.5V.

The specifications of the A/D and D/A converters and the schematic diagram of the development board can be found in the appendix.
3.2 Measurement tasks

3.2.1 Measuring static parameters of the D/A

Start MATLAB and run the dac_gui.m function.

![dac_gui.m main window]

The GUI offers the following functions:

- **DC output**: the value written into the Output Code window can be written into the output register of DAC1 by pressing the Enter key.
- **Square Wave**: Generates a square wave on the DAC1 output. The low (L) level is the digital 0, while the high level (H) can be set by entering a value into the text box. The frequency of the generated square wave is ~850 Hz.
- **Glitch**: The DAC1 output is droved with values 2047 and 2048. At the same time a square wave is generated on DAC0 which can be used for synchronization.
- **Quantization Error**: Generates sine waves on both of the DAC outputs. The resolution of the sine wave is 12 bit on DAC0 and 4 bit on DAC1.

**Measuring offset and amplification error**

Drive the DAC1 output with the minimum and maximum values using dac_gui.m. Measure the output voltage of the D/A with a high-precision multimeter (set the multimeter to 6-digit resolution).

Notes: The nominal output range is denoted with FS (full-scale). Usually, digital 0 represents output value of 0V, while FS output value is generated when the D/A digital input is set to 2^n, where n is the resolution of the D/A. Obviously, the maximum digital value which can be set on the input is 2^n-1 (binary 111…111), therefore the maximal output voltage is FS-1LSB, where LSB denotes the potential change when the least significant bit of the input is changed. The reference voltage on the development board is set to 2.5V. Thus the nominal output range is 0…2.5V (FS=2.5V). The 12-bit digital input has a range of 0…4095; the nominal output for input value 0 is 0V, while input value 4095 nominally results in a FS-1LSB=2.4994V. The amplification error is the difference of the measured values and the nominal values at input values of 0 and 4095, represented in percentage or LSB units.
**Integral linearity error**

For the absolute correct measurement of the linearity error, all possible input values should be measured. In the case of the 12-bit D/A this would result in 4096 measurements. Due to the time constraints, only 10-20 different measurements are done – observations show that such amount of measurements are enough to get a relatively exact characteristic.

Measure the output with minimal and maximal input values (0 and 4095). Then select at least 10 input values evenly distributed in the input value range and measure the output for all of these values. Create an excel sheet with two columns: write the set digital values into the left column and the measured output into the right column. In every measured point, compute the difference between the measured values and the theoretical linear characteristic. The integral linearity error is the maximum value given in LSB units.

**Differential linearity error**

The gradient of the characteristic can be represented with the output voltage change when the digital input is incremented with 1. In an ideal case, the voltage change is exactly 1LSB. The differential linearity error is the difference of the measured values from this theoretical value, given in LSB units.

Measure the output voltage at several points of the characteristic using consecutive input values (e.g. 1030 and 1031). Determine the differential linearity error in LSB units.

### 3.2.2 Dynamic parameters of the D/A

**Laytime**

The laytime of the D/A is the time between two representative points. Laytime starts when the output value is already slightly changed (typically, within 1LSB) and ends when the final output value is reached within an error of 1LSB, as shown on Figure 5.

![Definition of laytime](image)

For measuring setup time, use dac_gui.m and generate a square wave. Firstly, measure the laytime when the input is changed from 0 to ~FS, then measure the laytime from 0 to 0.2*FS.

**Quantization error**

The Quantization error function of the dac_gui.m program illustrates the effect of quantization. On DAC0 a sine wave is generated with 12 bit resolution (this signal is considered as the original sine wave), while on DAC1 the resolution is only 4 bits (this signal is considered as the quantized sine wave). The quantization error is the difference of the quantized signal and the original signal. Using
the oscilloscope, generate the quantization error waveform (that is, subtract the quantized signal from the original signal).

3.2.3 3. A/D converter

MATLAB program
For the A/D converter measurements adc_gui.m MATLAB program is used (Figure 6.), so start this program in MATLAB.

Measurement is started with the Measure data button. Sampling frequency is 115200 Hz, the number of samples recorded is 8192. Selecting Time-domain in the Time-domain/Frequency-domain shows the waveform of the recorded data, while Frequency-domain allows the inspection of the spectrum. For the vertical axis, Calculate in Volts or Calculate in LSB options can be selected. Norm. Power displays the square of the AC component effective value.

![adc_gui.m main window in Time-domain mode](image)

Measurements using sine wave

a. Assuring coherent sampling
Coherent sampling means that the measured data contains an integer number of periods from the measured signal. The sampling frequency of the ADC is 115200 Hz and the measured data contains 8192 samples. Let’s assume that the measured data contains k number of periods and the frequency of the input sine wave is fm. In this case fm=k*(115200/8192). E.g. if k=5, the frequency of the sine wave is fm=70.3125 Hz.
The range of the sine wave should be in 0...2.5V. Take care of not to generate larger amplitude, as this may damage the A/D. Always verify the generated waveform with the oscilloscope, before connecting it to the A/D!

b. Time-domain analysis
Inspect the waveform of the measured data (Time-domain). How can you verify that the sampling was coherent?

c. Frequency-domain analysis
In Frequency-domain mode, measure the frequencies for the larger spectrum lines.

**Undersampling**
According to the sampling theorem, with a sampling frequency of fs, signals with frequency less than fs/2 can be correctly sampled. Undersampling is the situation when the input signal has higher frequency components than fs/2.

Generate a sine wave with a frequency close to the sampling frequency of the A/D. Display the waveform of the measured data and analyze the results.

**Measuring with square wave**
Generate a symmetric square wave and connect it to the A/D.

- Analyze the measured data in the time domain.
- Analyze the measured data in the frequency domain. Does it correspond to your expectation?