

# ANONYMITY IN CONTINUOUS-TIME SYSTEMS

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## I. Introduction

Anonymous communication is a rapidly evolving area: it is needed for several privacy related services, such as anonymous electronic voting, payment or consultancy. Numerous techniques have been developed for the main goal: to deliver messages from senders to recipients in a manner that an adversary should find it difficult to identify who communicates with whom.

In this paper we analyze two representatives of a relatively new field of the continuous-time anonymity systems: the SG-Mix [1] and the PROB-channel [2]. We will compare them based on simulations and come to an interesting result: although the SG-Mix was proven to be optimal<sup>1</sup> in [3], our numbers show that the PROB-channel outperforms it in the evaluated scenarios.

## II. Continuous-time Anonymity Systems for Anonymous Communication

### A. Continuous-time Systems in General

Continuous-time anonymity systems realize anonymous communication according to the following model: senders ( $s_i \in S$ ) send encrypted messages ( $\alpha_j \in \varepsilon_S$ ) at times  $t_S(\alpha_j)$  through the continuous-time system (CTS). The CTS receives the sent messages and performs cryptographic operations on them to obtain a different representation. In order to further confuse the adversary, the CTS delays messages. After the delay the CTS delivers the reencoded messages ( $\beta_k \in \varepsilon_R$ ) to the recipients ( $r_l \in R$ ) at times  $t_R(\beta_k)$ . Both the sent and the delivered messages have a common, fixed size not to reveal information about their relationship.

The adversary's aim is to match the delivered messages to senders ( $\beta_k \rightarrow s_i = S(\beta_k)$ ). For that he can eavesdrop on communication channels and see when messages are being sent or delivered.

### B. Delay Strategies

The heart of the anonymous communication is the delay of the messages. In case of continuous-time systems this delay is randomized according to a probability variable  $\delta$  with a given density function  $f(\delta)$ . For the SG-Mix the delay density is chosen to be exponential with parameter  $\mu$ :  $f^{SG}(\delta) = \mu \cdot e^{-\mu \cdot \delta}$ . From this it follows that the mean delay of such a system is  $\frac{1}{\mu}$ . In order to achieve real-time properties, the density function for the PROB-channel is limited by a maximal delay  $\delta_{\max}^{\text{PROB}}$ . To indicate that the channel needs a certain amount of time for message processing, a minimal delay ( $\delta_{\min}^{\text{PROB}}$ ) was also introduced. The density function of the delay was chosen to be uniform, thus  $f^{\text{PROB}}(\delta) = \frac{1}{\delta_{\max}^{\text{PROB}} - \delta_{\min}^{\text{PROB}}}$

## III. Simulation results

### A. Simulation Environment

For the simulation MIN/MAX senders were used: the solution from [2] requires that each sender sends at least one message in  $\tau_{\max}$  time (minimal message sending frequency) and no sender sends more than one message in  $\tau_{\min}$  time (maximal message sending frequency). If a sender does not have real messages to send, he should send dummy messages to randomly chosen recipients.

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<sup>1</sup>Under special conditions: the authors assumed Poisson-distributed incoming messages and quantified anonymity with the help of entropy.

For the adversary a *locally back-tracing passive* one was chosen [2]. The idea is to match each delivered message to the senders individually. With the following two sets ( $\mu_{\beta_k} = \{\alpha_j | (t_R(\beta_k) - \delta_{\max}) < t_S(\alpha_j) < (t_R(\beta_k) - \delta_{\min})\}$ ) and  $\eta_{\beta_k, s_l} = \{\alpha_j | (\alpha_j \in \mu_{\beta_k}) \wedge (S(\alpha_j) = s_l)\}$ ) equation (1) gives his guess, what is the probability of a certain message  $\beta_k$  being sent by a certain sender  $s_l$ :

$$P_{\beta_k, s_l} = \frac{\sum_{\alpha_j \in \eta_{\beta_k, s_l}} f(t_R(\beta_k) - t_S(\alpha_j))}{\sum_{\alpha_j \in \mu_{\beta_k}} f(t_R(\beta_k) - t_S(\alpha_j))} \quad (1)$$

## B. Numerical Results

The figures below show the difference between the SG-Mix and the PROB-channel. In the simulations 20 senders sent messages during a 5000 seconds long interval. The mean delay for both systems was the same, and  $\tau_{\min}$  was 1 second in all cases. Measured was the ratio of correctly matched messages while the adversary assigned probabilities to messages and senders according to (1). Figure 1 shows the absolute difference of the success ratios, positive values indicate that the PROB-channel outperformed the SG-Mix. Figure 2 shows the relative differences.

Figure 1: Absolute difference

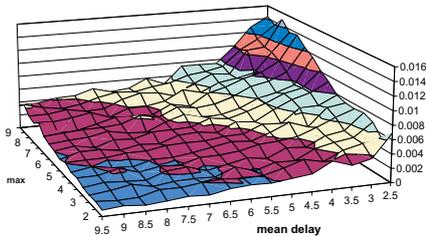
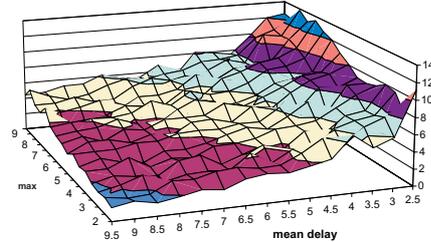


Figure 2: Relative difference (%)



## IV. Conclusion

In this paper two continuous-time anonymity systems, the SG-Mix and the PROB-channel were compared. In several publications anonymity systems are evaluated under the assumption of Poisson-distributed incoming messages: e.g. in [3] the SG-Mix was proven to be optimal. However from [4] we know, messages to real systems do not follow this approximation. Furthermore, entropy is often used as anonymity metrics, although in [5] it was shown that this may lead to false results. The simulation results presented here show also, despite the proven optimality of the SG-Mix the PROB-channel outperforms it under equal conditions. With this in mind the task ahead is significant: the assumption of the Poisson-distributed messages and entropy based anonymity metrics should be reconsidered.

## References

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